General marginal deformations in open string field theory

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SFT 07 at RIKEN

Analytic solutions for marginal deformations

bosonic string | **superstring**

Schnabl, hep-th/0701248
Kiermaier, Okawa, Rastelli
and Zwiebach, hep-th/0701249

↓ general super ↘

Erler, arXiv: 0704.0930
Okawa, arXiv: 0704.0936
Okawa, arXiv: 0704.3612
(The Seitaro Nakamura Prize)

↓ general

Kiermaier & Okawa
arXiv: 0707.4472
(Fuchs, Kroyter & Potting
arXiv: 070~~8~~4. 2222)

super ↘

Kiermaier & Okawa
arXiv: 0708.3394
(Fuchs & Kroyter
arXiv: 0706.0717)

Background independence

The equation of motion of the spacetime theory

↕

Conformal invariance of the world-sheet theory

Open bosonic string field theory

Ground state

tachyonic scalar $T(p)$ in 26 dimensions

First-excited state

massless vector field $A_\mu(p)$

$\vdots$

Degrees of freedom of string field theory

$$\{ T(p),\ A_\mu(p),\ \cdots \}$$

$$\downarrow$$

$$S[\, T(p),\ A_\mu(p),\ \cdots \,]$$

SU(2) gauge fields $\qquad$ A single 2×2 matrix field

$$A_\mu^a(x),\ a=1,2,3 \longrightarrow A_\mu(x) = \frac{1}{2}\sum_{a=1}^{3} A_\mu^a(x)\,\sigma^a$$

$$\{ T(p),\ A_\mu(p),\ \cdots \} \longrightarrow \text{String field } \Psi$$

= a state in a two-dimensional conformal field theory

$$\Psi = \int \frac{d^{26}p}{(2\pi)^{26}} \left[\frac{1}{\sqrt{\alpha'}}\, T(p)\, c_1 |0;p\rangle + \frac{1}{\sqrt{\alpha'}}\, A_\mu(p)\, \alpha_{-1}^\mu c_1 |0;p\rangle + \cdots \right]$$

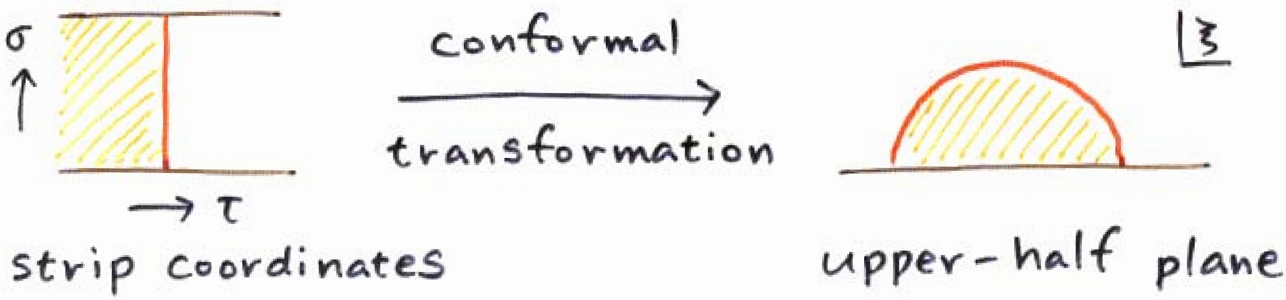
$$A_\mu^a(x) = \mathrm{tr}\, \sigma^a A_\mu(x)$$

Ψ can be specified by giving $\langle \phi, \Psi \rangle$

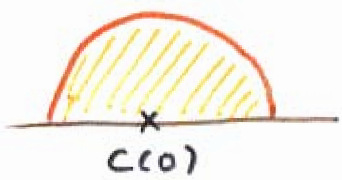
for all ϕ in the Fock space.

CFT description

string field = state in the 2D CFT



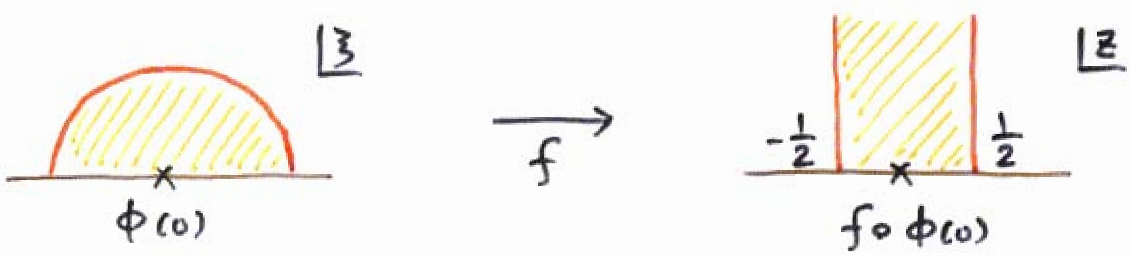
State-operator correspondence

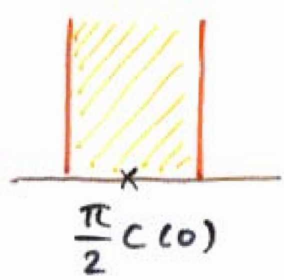
$c_1|0\rangle \longleftrightarrow$ 

Useful coordinate (sliver frame)

Rastelli & Zwiebach, hep-th/0006240

$$\boxed{z = f(\xi) = \frac{2}{\pi}\arctan\xi}$$

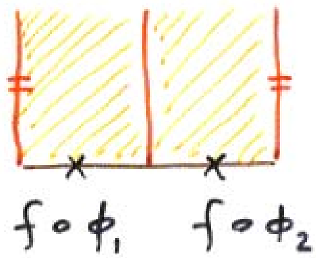


$c_1|0\rangle \longleftrightarrow$ 

$$\left[\begin{aligned} f\circ c(\xi) &= \left(\frac{df(\xi)}{d\xi}\right)^{-1} c(f(\xi)) \\ f\circ c(0) &= \frac{\pi}{2}\, c(0) \end{aligned}\right]$$

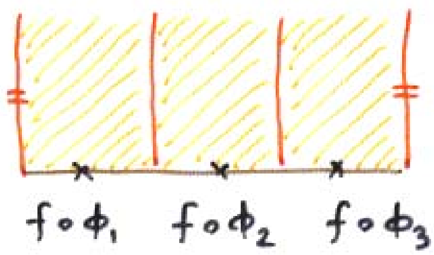
BPZ inner product

$$\langle \phi_1, \phi_2 \rangle =$$



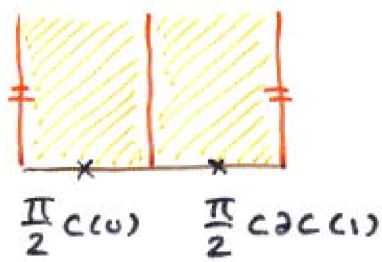
Star product

$$\langle \phi_1, \phi_2 * \phi_3 \rangle =$$



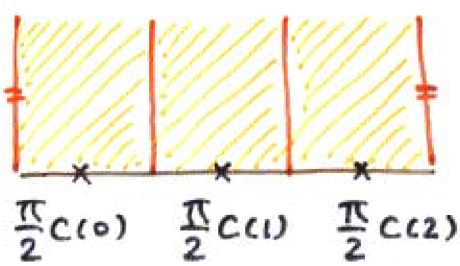
Examples $\quad T = c_1 |0\rangle$

$$\langle T, Q_B T \rangle =$$



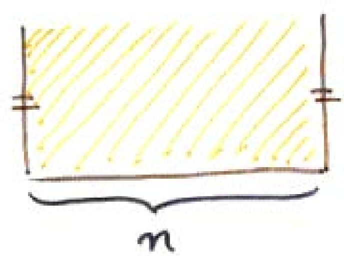
$$= -1$$

$$\langle T, T * T \rangle =$$



$$= -\frac{81\sqrt{3}}{64}$$

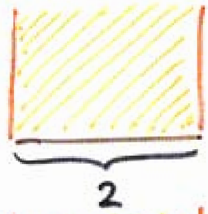
$$\langle c(z_1)\, c(z_2)\, c(z_3) \rangle \quad \text{on}$$



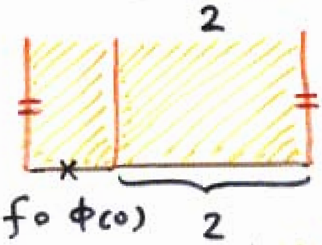
$$= \left(\frac{n}{\pi}\right)^3 \sin\frac{\pi(z_1 - z_2)}{n} \sin\frac{\pi(z_1 - z_3)}{n} \sin\frac{\pi(z_2 - z_3)}{n}$$

Wedge state W_α

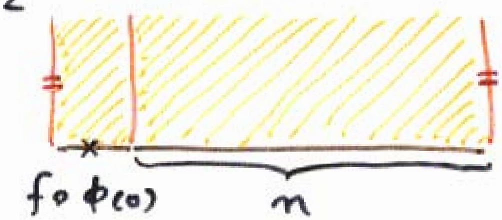
$|0 * 0\rangle \longleftrightarrow$



$\langle \phi, 0 * 0 \rangle =$

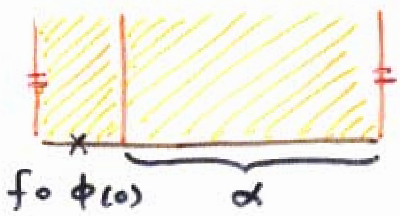


$\langle \phi, \underbrace{0 * 0 * 0 * \cdots * 0}_{n} \rangle =$



$\langle \phi, W_\alpha \rangle =$

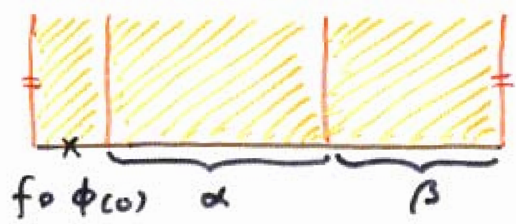
(α : real)



$W_1 = |0\rangle$

$W_2 = |0 * 0\rangle$

$\langle \phi, W_\alpha * W_\beta \rangle =$

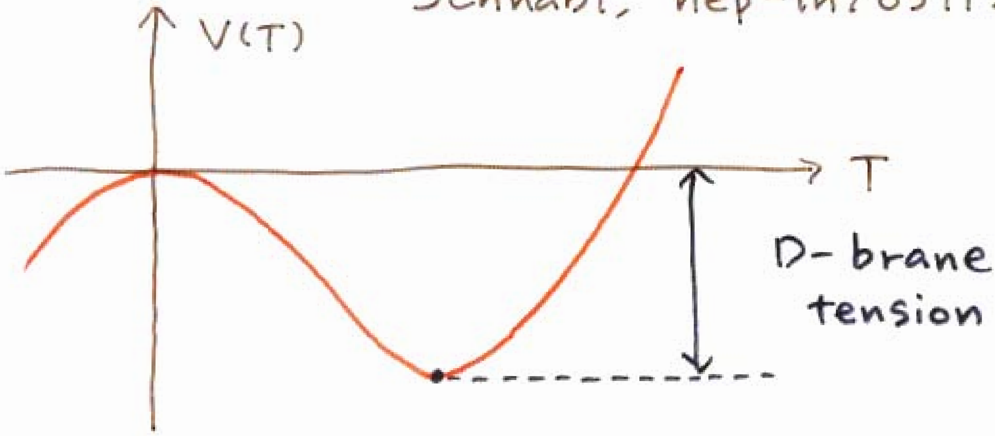


$= \langle \phi, W_{\alpha+\beta} \rangle$

$$W_\alpha * W_\beta = W_{\alpha+\beta}$$

Schnabl's analytic solution for tachyon condensation

Schnabl, hep-th/0511286

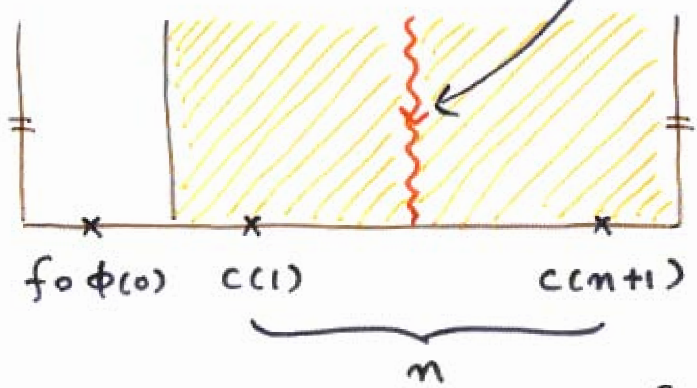


$$\Psi = \lim_{N\to\infty}\left[\sum_{n=0}^{N}\psi_n' - \psi_N\right]$$

$$\psi_n' = \frac{d}{dn}\psi_n$$

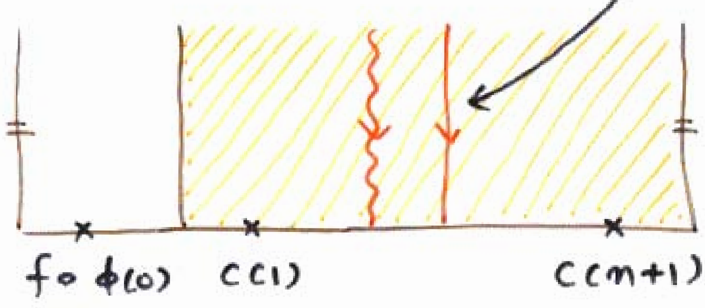
$$\mathcal{B} = \oint \frac{dz}{2\pi i}\, b(z)$$

$$\langle \phi, \psi_n \rangle =$$



$$\mathcal{L} = \oint \frac{dz}{2\pi i}\, T(z)$$

$$\langle \phi, \psi_n' \rangle =$$



$$[\mathcal{B}, \mathcal{L}] = 0$$

$$\left(\text{cf. } L_0 e^{-tL_0} = -\partial_t e^{-tL_0}\right)$$

Marginal deformations

The deformation of the boundary CFT

$$S \rightarrow S + \lambda \int dt \; V(t)$$

is marginal if V is a primary field of dimension one.

Examples

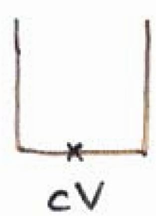
— $V(z) = i\sqrt{\frac{2}{\alpha'}}\, \partial X^{\mu}(z)$

constant mode of the gauge field
or transverse coordinate of the D-brane

— $V(t) = :\cos \frac{X(t)}{\sqrt{\alpha'}}:$

For any dimension-one matter primary field V, cV is BRST closed.

$\Psi^{(1)} =$



$\Rightarrow \quad Q_B \Psi^{(1)} = 0$

When the deformation is exactly marginal, we expect a solution of the form

$$\Psi = \sum_{n=1}^{\infty} \lambda^n \Psi^{(n)} \qquad \lambda : \text{deformation parameter}$$

to the nonlinear equation of motion:

$$Q_B \Psi + \Psi * \Psi = 0 .$$

$$Q_B \Psi^{(1)} = 0$$

$$Q_B \Psi^{(2)} = - \Psi^{(1)} * \Psi^{(1)}$$

$$\vdots$$

$$Q_B \Psi^{(n)} = - \sum_{m=1}^{n-1} \Psi^{(m)} * \Psi^{(n-m)}$$

Formally,

$$\Psi^{(2)} = - \frac{b_0}{L_0} [\Psi^{(1)} * \Psi^{(1)}]$$

$$b_0 = \oint \frac{d\xi}{2\pi i} \xi \, b(\xi) , \quad L_0 = \oint \frac{d\xi}{2\pi i} \xi \, T(\xi)$$

Marginal deformations with regular operator products

$$V(t_1)\, V(t_2) \cdots V(t_n) \quad : \quad \text{regular}$$

Schnabl, hep-th/0701248 ; KORZ, hep-th/0701249

$$\Psi^{(2)} = -\frac{B}{L}\left[\Psi^{(1)} * \Psi^{(1)}\right]$$

$$= -\int_0^\infty dT\, B\, e^{-TL}\left[\Psi^{(1)} * \Psi^{(1)}\right]$$

subtle (later)

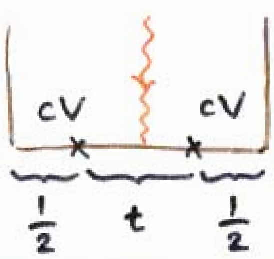
where

$$B = \oint \frac{dz}{2\pi i}\, z\, b(z) = \oint \frac{d\xi}{2\pi i}\, \frac{f(\xi)}{f'(\xi)}\, b(\xi)$$

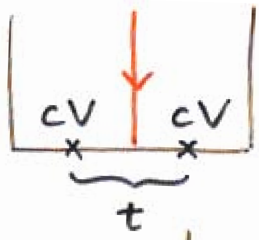
$$= b_0 + \frac{2}{3} b_2 - \frac{2}{15} b_4 + \cdots$$

$$L = \{ Q_B,\, B \}$$

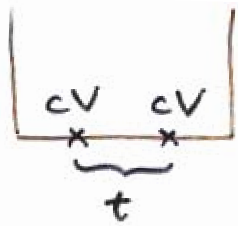
$$\boxed{\Psi^{(2)} = \int_0^1 dt}$$



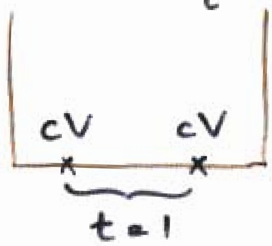
$Q_B \Psi^{(2)} = -\int_0^1 dt$



$= -\int_0^1 dt \, \frac{\partial}{\partial t}$



$= -$



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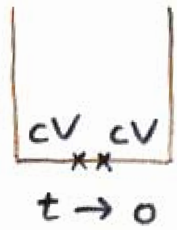


$\| $

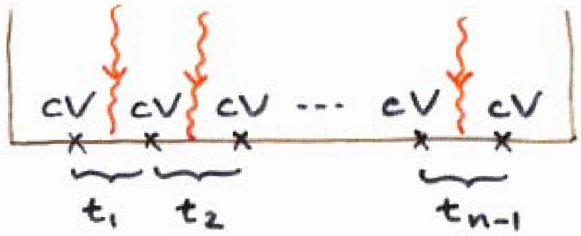
$- \Psi^{(1)} * \Psi^{(1)}$

$$\lim_{\epsilon \to 0} cV(0)\, cV(\epsilon) = 0 \quad \Longrightarrow \quad Q_B \Psi^{(2)} = -\Psi^{(1)} * \Psi^{(1)}$$

If $\lim_{\epsilon \to 0} V(0)\, V(\epsilon) =$ finite or vanishing,

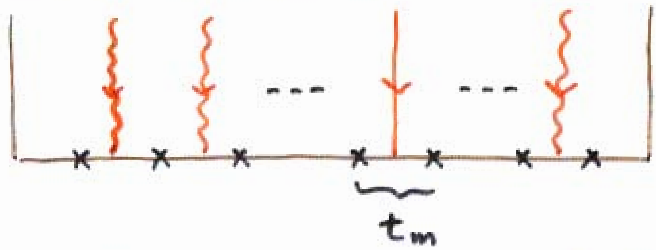
$\Psi^{(2)}$ is finite and $Q_B \Psi^{(2)} = -\Psi^{(1)} * \Psi^{(1)}$.

$$\Psi^{(n)} = \int d^{n-1}t$$

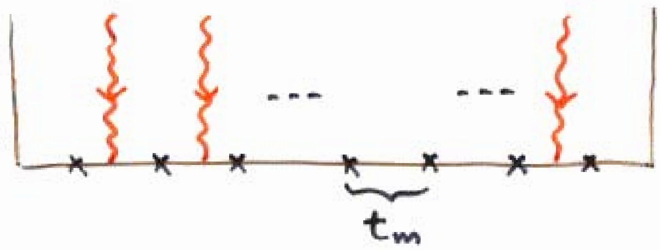


where $\int d^{n-1}t = \int_0^1 dt_1 \int_0^1 dt_2 \cdots \int_0^1 dt_{n-1}$

$$Q_B \Psi^{(n)} = -\sum_m \int d^{n-1}t$$

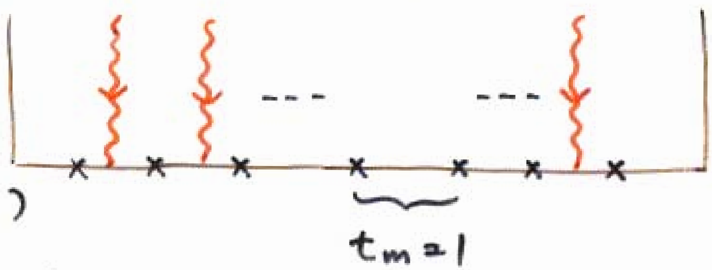


$$= -\sum_m \int d^{n-1}t \frac{\partial}{\partial t_m}$$



$$= -\sum_m \int d^{n-2}t$$

(no dt_m)



$$= -\sum_m \Psi^{(m)} * \Psi^{(n-m)}$$

Example

$$V(t) = \exp\left[\frac{1}{\sqrt{\alpha'}} X^0(t)\right]$$

rolling tachyon

half S-brane decay

An exact time-dependent solution

incorporating all α' corrections.

In general, $V(t_1)\,V(t_2)\cdots V(t_n)$ are singular.

$$\left(\text{e.g.} \quad V(z)\,V(w) \sim \frac{1}{(z-w)^2} \right)$$

$\Psi^{(2)}$ and $\Psi^{(3)}$ for the singular case were constructed in hep-th/0701249

— regularization

— counterterms

Not all marginal deformations are exactly marginal.

$\longrightarrow$ obstruction for the construction of $\Psi^{(2)}$ when the deformation is not exactly marginal at $O(\lambda^2)$.

Generalization to higher orders does not seem so easy.

$\Psi^{(n)}$
- n unintegrated vertex operators
- $n-1$ integrals of moduli
- $n-1$ b-ghost insertions

Integrated vertex operators are more closely related to finite deformations of boundary CFT.

New strategy

$\Psi^{(n)}$
- one unintegrated vertex operator
- $n-1$ integrated vertex operators

General marginal deformations

Kiermaier & Okawa, arXiv: 0707.4472

Unintegrated vertex operator

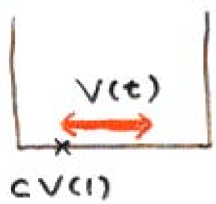
$$Q_B \cdot cV(t) = 0$$

Integrated vertex operator

$$Q_B \cdot \int_a^b dt\, V(t) = \int_a^b dt\, \partial_t [\, cV(t)\,] = cV(b) - cV(a)$$

A solution (when the OPE is regular)

$$\Psi_L^{(2)} = \int_1^2 dt$$



$$Q_B \cdot \int_1^2 dt\; cV(1)\, V(t) = - \int_1^2 dt\; cV(1)\, \partial_t [\, cV(t)\,]$$

$$= - cV(1)\, cV(2)$$

$$\Psi_L^{(3)} \quad \frac{1}{2}\, cV(1) \left[\int_1^3 dt\; V(t) \right]^2$$

$$- \frac{1}{2}\, cV(1) \left[\int_2^3 dt\; V(t) \right]^2 \quad \text{on } W_3$$

$$\text{or} \quad \int_1^2 dt_1 \int_{t_1}^3 dt_2\; cV(1)\, V(t_1)\, V(t_2) \quad \text{on } W_3$$

$\Psi_L^{(n)}$ ingredients $e^{\lambda V(a,b)}$, $\lambda cV(a)\, e^{\lambda V(a,b)}$

where $V(a,b) \equiv \int_a^b dt\, V(t)$

fixed wedge state W_n

We only need the relations

$$Q_B \cdot e^{\lambda V(a,b)}$$
$$= e^{\lambda V(a,b)}\, \lambda cV(b) - \lambda\, cV(a)\, e^{\lambda V(a,b)}$$

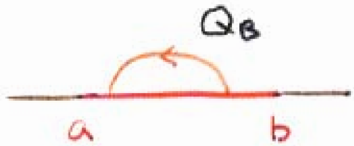
and

$$Q_B \cdot [\lambda cV(a)\, e^{\lambda V(a,b)}]$$
$$= -\lambda cV(a)\, e^{\lambda V(a,b)}\, \lambda cV(b).$$

These relations generalize to the singular case.

Ψ_L does not satisfy the reality condition.

$\Psi_L \to$ a real solution by a gauge transformation



$= 0$

Change of boundary conditions

$$e^{\lambda V(a,b)} = 1 + \lambda \int_a^b dt\, V(t) + \frac{\lambda^2}{2} \int_a^b dt_1 \int_a^b dt_2\, V(t_1) V(t_2) + \cdots$$

$$\downarrow$$

$[\, e^{\lambda V(a,b)} \,]_r$ singular case

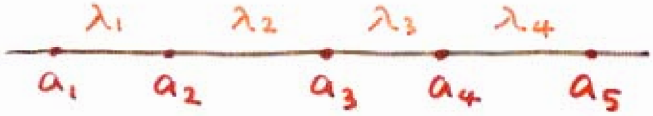
$\nwarrow$ renormalization

$$Q_B \cdot [\, e^{\lambda V(a,b)} \,]_r$$
$$= [\, e^{\lambda V(a,b)}\, \mathcal{O}_R(b) \,]_r - [\, \mathcal{O}_L(a)\, e^{\lambda V(a,b)} \,]_r$$

$$Q_B \cdot [\, \mathcal{O}_L(a)\, e^{\lambda V(a,b)} \,]_r = - [\, \mathcal{O}_L(a)\, e^{\lambda V(a,b)}\, \mathcal{O}_R(b) \,]_r$$

$$\mathcal{O}_L = \lambda\, cV + O(\lambda^2)\,, \quad \mathcal{O}_R = \lambda\, cV + O(\lambda^2)$$

Assumptions



$$\left[\prod_{i=1}^{n} e^{\lambda_i V(a_i, a_{i+1})} \right]_r$$

$$a_i < a_{i+1}$$

(I) $\quad Q_B \cdot [\, e^{\lambda V(a,b)} \,]_r = [\, e^{\lambda V(a,b)} O_R(b) \,]_r - [\, O_L(a)\, e^{\lambda V(a,b)} \,]_r$

(II) $\quad Q_B \cdot [\, O_L(a)\, e^{\lambda V(a,b)} \,]_r = - [\, O_L(a)\, e^{\lambda V(a,b)} O_R(b) \,]_r$

(III) Replacement

$$[\, \dots \; e^{\lambda_i V(a_i, a_{i+1})} e^{\lambda_i V(a_{i+1}, a_{i+2})} \; \dots \,]_r$$

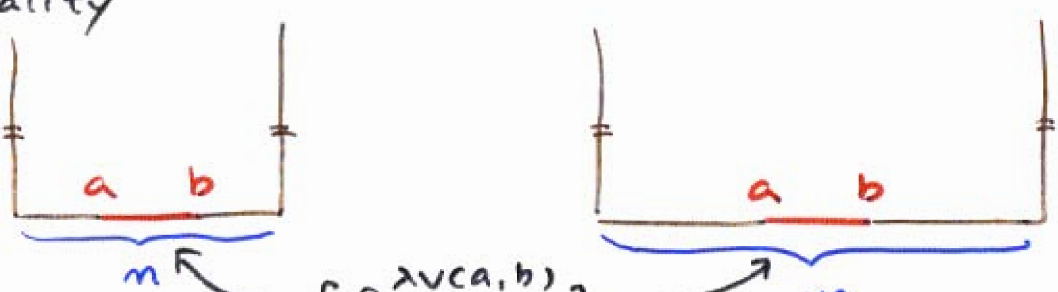
$$= [\, \dots \; e^{\lambda_i V(a_i, a_{i+2})} \; \dots \,]_r$$

(IV) Factorization

$$[\, \dots \; e^{\lambda_{j-1} V(a_{j-1}, a_j)} e^{\lambda_{j+1} V(a_{j+1}, a_{j+2})} \dots \,]_r$$

$$= [\, \dots \; e^{\lambda_{j-1} V(a_{j-1}, a_j)} \,]_r \, [\, e^{\lambda_{j+1} V(a_{j+1}, a_{j+2})} \dots \,]_r$$

(V) Locality



$[\, e^{\lambda V(a,b)} \,]_r$

the same operator

(VI) Reflection

$$\left[\exp\left(\lambda \int_a^b dt \; V(a+b-t)\right) \right]_r = \left[\exp\left(\lambda \int_a^b dt \; V(t)\right) \right]_r$$

[(III), (IV), and (V) with O_L and/or O_R as well.]

Solutions

$$[\, e^{\lambda V(a,b)} \,]_r = \sum_{n=0}^{\infty} \lambda^n [\, V^{(n)}(a,b) \,]_r$$

Define U by

$$U = 1 + \sum_{n=1}^{\infty} \lambda^n U^{(n)}$$

$$U^{(n)} : \quad [\, V^{(n)}(1,n) \,]_r \quad \text{on } W_n$$

Assumption (I)

$$\rightarrow \quad Q_B \cdot [\, V^{(n)}(a,b) \,]_r$$

$$= \sum_{r=1}^{n} [\, V^{(n-r)}(a,b)\, O_R^{(r)}(b) \,]_r - \sum_{\ell=1}^{n} [\, O_L^{(\ell)}(a)\, V^{(n-\ell)}(a,b) \,]_r$$

$$\left(O_L = \sum_{n=1}^{\infty} \lambda^n O_L^{(n)}, \quad O_R = \sum_{n=1}^{\infty} \lambda^n O_R^{(n)} \right)$$

$$Q_B U = A_R - A_L$$

$$A_L = \sum_{n=1}^{\infty} \lambda^n A_L^{(n)}, \quad A_R = \sum_{n=1}^{\infty} \lambda^n A_R^{(n)}$$

$$A_L^{(n)} : \quad \sum_{\ell=1}^{n} [\, O_L^{(\ell)}(1)\, V^{(n-\ell)}(1,n) \,]_r \quad \text{on } W_n$$

$$A_R^{(n)} : \quad \sum_{r=1}^{n} [\, V^{(n-r)}(1,n)\, O_R^{(r)}(n) \,]_r \quad \text{on } W_n$$

Define Ψ_L and Ψ_R by

$$\Psi_L = A_L * U^{-1}, \quad \Psi_R = U^{-1} * A_R$$

We claim that

$$Q_B \Psi_L + \Psi_L * \Psi_L = 0, \quad Q_B \Psi_R + \Psi_R * \Psi_R = 0$$

$\Psi_L \longrightarrow \Psi_R$ by a gauge transformation

$$\Psi_R = U^{-1} * \Psi_L * U + U^{-1} * Q_B U$$

Real solution

$$\Psi_{real} = \frac{1}{\sqrt{U}} * \Psi_L * \sqrt{U} + \frac{1}{\sqrt{U}} * Q_B \sqrt{U}$$

U^{-1}, $\sqrt{U}$, and $\frac{1}{\sqrt{U}}$ are well defined perturbatively because $U = 1 + O(\lambda)$.

The construction of solutions is divided into two steps

Step 1 (boundary CFT)
Construction of $[\, e^{\lambda V(a,b)}\,]_r$.

Step 2 (String field theory)
$[\, e^{\lambda V(a,b)}\,]_r \longrightarrow \Psi_L, \Psi_R, \Psi_{real}$

We have explicitly constructed $[\, e^{\lambda V(a,b)}\,]_r$ for a class of marginal deformations which include

$$V(t) = \frac{i}{\sqrt{2\alpha'}} \partial_t X^\mu(t), \; \sqrt{2} : \cos \frac{X^\mu(t)}{\sqrt{\alpha'}} : , \; \sqrt{2} : \cosh \frac{X^0(t)}{\sqrt{\alpha'}} :$$

$$\begin{aligned} Q_B \Psi_L &= Q_B (A_L * U^{-1}) \\ &= Q_B A_L * U^{-1} + A_L * U^{-1} * Q_B U * U^{-1} \\ &= Q_B A_L * U^{-1} + A_L * U^{-1} * (A_R - A_L) * U^{-1} \\ &= (Q_B A_L + A_L * U^{-1} * A_R) * U^{-1} - \Psi_L * \Psi_L \end{aligned}$$

$$\boxed{- Q_B A_L = A_L * U^{-1} * A_R}$$

$$\downarrow$$

$$Q_B \Psi_L + \Psi_L * \Psi_L = 0$$

Generalization of wedge states

Define U_α $(\alpha \geq 0)$ by

$$U_\alpha = \sum_{n=0}^{\infty} \lambda^n U_\alpha^{(n)} \qquad U_\alpha^{(n)} : [\, V^{(n)}(1, n+\alpha) \,]_r \ \text{ on } \ W_{n+\alpha}$$

$$U_0 = U$$

$$U_\alpha = W_\alpha + O(\lambda)$$

Assumptions (III), (IV), and (V) $\rightarrow$ $U_{\alpha+\beta} = U_\alpha * U^{-1} * U_\beta$

If we introduce $A \star B \equiv A * U^{-1} * B$,

$$\boxed{U_{\alpha+\beta} = U_\alpha \star U_\beta}$$

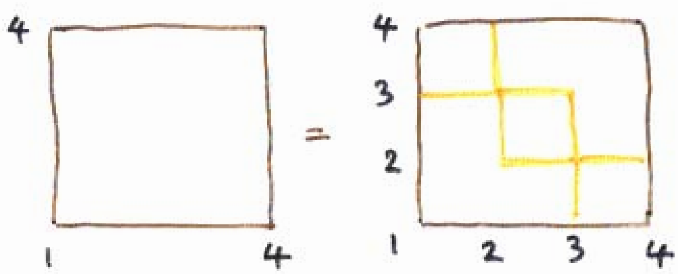
Example : $U_2 = U_1 * U^{-1} * U_1$ at $O(\lambda^2)$

$$U_2^{(2)} = U_1^{(0)} * U_1^{(2)} + U_1^{(1)} * U_1^{(1)} + U_1^{(2)} * U_1^{(0)} - U_1^{(0)} * U^{(2)} * U_1^{(0)}$$

$$[\, V(1,4)^2 \,]_r$$

$$= [\, V(2,4)^2 \,]_r + 2 [\, V(1,2) \,]_r [\, V(3,4) \,]_r + [\, V(1,3)^2 \,]_r$$

$$- [\, V(2,3)^2 \,]_r$$



$1 < 2 < 3 < 4$ on W_4

Outline of the proof

$$-Q_B A_L \sim \sum_{l,r} [\, U_{l+r} \text{ with } \lambda^l O_L^{(l)} \text{ and } \lambda^r O_R^{(r)} \,]$$

$$A_L * U^{-1} * A_R \sim \sum_l [\, U_l \text{ with } \lambda^l O_L^{(l)} \,] * U^{-1} * \sum_r [\, U_r \text{ with } \lambda^r O_R^{(r)} \,]$$

$$\sim \sum_{l,r} [\, \underbrace{U_l * U^{-1} * U_r}_{= \, U_{l+r}} \text{ with } \lambda^l O_L^{(l)} \text{ and } \lambda^r O_R^{(r)} \,]$$

$$\Rightarrow \ -Q_B A_L = A_L * U^{-1} * A_R$$

String field theory around the deformed background

$$S = -\frac{1}{g^2}\left[\frac{1}{2}\langle \Psi, Q_B \Psi \rangle + \frac{1}{3}\langle \Psi, \Psi * \Psi \rangle\right]$$

$$\Psi = \Psi_0 + \delta\Psi \qquad \Psi_0 : \text{ a solution}$$

$$S = S_0 - \frac{1}{g^2}\left[\frac{1}{2}\langle \delta\Psi, \tilde{Q}_B \delta\Psi \rangle + \frac{1}{3}\langle \delta\Psi, \delta\Psi * \delta\Psi \rangle\right]$$

where $\tilde{Q}_B A = Q_B A + \Psi_0 * A - (-1)^A A * \Psi_0$

We find that

$$S = S_0 - \frac{1}{g^2}\left[\frac{1}{2}\langle\langle \Phi, \mathcal{Q}\Phi \rangle\rangle + \frac{1}{3}\langle\langle \Phi, \Phi \star \Phi \rangle\rangle\right]$$

where

$$A \star B \equiv A * U^{-1} * B$$

$$\mathcal{Q} A \equiv Q_B A + A_L \star A - (-1)^A A \star A_R$$

$$\langle\langle A, B \rangle\rangle \equiv \langle A, U^{-1} * B * U^{-1} \rangle$$

$$\Psi = \Psi_{real} + \frac{1}{\sqrt{U}} * \Phi * \frac{1}{\sqrt{U}}$$

Open superstring field theory

Berkovits, hep-th/9503099

The world-sheet SCFT

$$\begin{cases} c = 15 \text{ matter SCFT} \\ bc \text{ ghosts} \\ \beta\gamma \text{ ghosts} \longrightarrow \eta, \xi, \phi \quad (\gamma \cong \eta e^{\phi}, \ \beta \cong \partial\xi e^{-\phi}) \end{cases}$$

Unintegrated vertex operator in the -1 picture

$c e^{-\phi} \hat{V}_{1/2}$ $\quad \hat{V}_{1/2}$: a superconformal primary field of dimension $1/2$

$$Q_B \cdot c e^{-\phi} \hat{V}_{1/2}(t) = 0$$

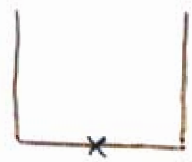
Include ξ_0 and η_0 $\longrightarrow$ the large Hilbert space

The equation of motion of the free theory

$$Q_B \eta_0 \Phi = 0$$

Gauge transformations

$$\delta\Phi = Q_B \Lambda + \eta_0 \Omega$$

$\Phi^{(1)}$ = 

Grassmann even

$\mathcal{V} = c\xi e^{-\phi} \hat{V}_{1/2}$

The equation of motion of the interacting theory

$$\eta_0 (e^{-\Phi} Q_B e^{\Phi}) = 0$$

WZW-like $\quad \partial_z \leftrightarrow Q_B, \quad \partial_{\bar{z}} \leftrightarrow \eta_0$

(All string products are defined using the star product.)

Marginal deformations

$$S \longrightarrow S + \lambda \int dt \, V_1(t)$$

where $V_1 = G_{-1/2} \cdot \hat{V}_{1/2}$

$$\Phi = \lambda \Phi^{(1)} + O(\lambda^2)$$

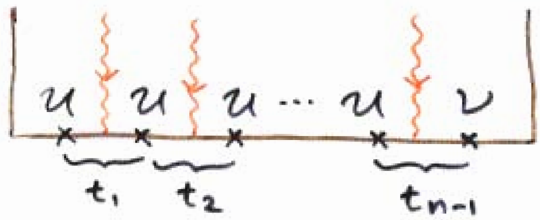
Marginal deformations for the superstring with regular operator products

Analytic solutions
Erler, arXiv: 0704.0930
Okawa, arXiv: 0704.0936

$$\Phi = -\ln(1-H)$$

where

$$H = \sum_{n=1}^{\infty} \lambda^n \int d^{n-1}t$$



$$\mathcal{U} \equiv Q_B \cdot \mathcal{V}$$

Erler's idea

Replace cV in the bosonic solution by $Q_B \cdot \mathcal{V}$

$$\left.\begin{array}{l} \Phi = \lambda\, Q_B \Phi^{(1)} + O(\lambda^2) \\ Q_B \Phi + \Phi^2 = 0 \end{array}\right\} \rightarrow \text{pure gauge}$$

$$\eta_0 \Phi = 0$$

Solutions to $e^{-\Phi} Q_B e^{\Phi} = \Phi$ also solve the equation of motion.

Real solutions

① Find a real solution to

$$Q_B G + [\Phi, G] = \frac{d}{d\lambda}\Phi$$

$$\Rightarrow e^{\Phi_{real}} = P\exp\left[\int_0^{\lambda} d\lambda' G(\lambda')\right]$$

Okawa, arXiv: 0704.3612

② $$e^{\Phi_{real}} = \frac{1}{\sqrt{e^{\Phi}(e^{\Phi})^{\ddagger}}}\, e^{\Phi} \qquad \left(Q_B [e^{\Phi}(e^{\Phi})^{\ddagger}] = 0 \right)$$

Erler, arXiv: 0704.0930v2

General marginal deformations for the superstring

Kiermaier & Okawa, arXiv: 0708.3394

Integrated vertex operator in the 0 picture

$$V(a,b) = \int_a^b dt\, V_1(t) = \int_a^b dt\, G_{-1/2} \cdot \hat{V}_{1/2}(t)$$

$$Q_B \cdot V(a,b)$$
$$= [\, cV_1(b) + \eta e^{\phi} \hat{V}_{1/2}(b) \,] - [\, cV_1(a) + \eta e^{\phi} \hat{V}_{1/2}(a) \,]$$

Any BRST-closed state is BRST exact in the large Hilbert space because

$\exists\, R(t)$ such that $Q_B \cdot R(t) = 1$

We choose $R(t) = -\, c\xi\partial\xi e^{-2\phi}(t)$

$$\lim_{\epsilon \to 0} R(t-\epsilon)\,[\, cV_1(t) + \eta e^{\phi} \hat{V}_{1/2}(t) \,] = c\xi e^{-\phi} \hat{V}_{1/2}(t)$$

Solution to the linearized equation of motion

$$Q_B \cdot [\, e^{\lambda V(a,b)} \,]_r = [\, e^{\lambda V(a,b)} O_R(b) \,]_r - [\, O_L(a)\, e^{\lambda V(a,b)} \,]_r$$

as in the bosonic case.

Ingredients: $[\, e^{\lambda V(a,b)} \,]_r$, $[\, O_L(a)\, e^{\lambda V(a,b)} \,]_r$, $[\, e^{\lambda V(a,b)} O_R(b) \,]_r$, and $R(t)$

Solutions

Define U, A_L, A_R, Ψ_L, and Ψ_R as before.

Define $\hat{A}_L$ and $\hat{A}_R$ by

$$\hat{A}_L = \sum_{n=1}^{\infty} \lambda^n \hat{A}_L^{(n)}, \qquad \hat{A}_R = \sum_{n=1}^{\infty} \lambda^n \hat{A}_R^{(n)}$$

$$\hat{A}_L^{(n)} : \lim_{\epsilon \to 0} \sum_{\ell=1}^{n} R(1-\epsilon) \left[O_L^{(\ell)}(1)\, V^{(n-\ell)}(1,n) \right]_r \text{ on } W_n$$

$$\hat{A}_R^{(n)} : -\lim_{\epsilon \to 0} \sum_{r=1}^{n} \left[V^{(n-r)}(1,n)\, O_R^{(r)}(n) \right]_r R(n+\epsilon) \text{ on } W_n$$

$$Q_B A_L = -A_L U^{-1} A_R \longrightarrow Q_B \hat{A}_L = A_L + \hat{A}_L U^{-1} A_R$$

Define Φ_L and Φ_R by

$$e^{\Phi_L} = 1 + \hat{A}_L U^{-1}, \qquad e^{-\Phi_R} = 1 - U^{-1} \hat{A}_R$$

$$\begin{aligned} Q_B e^{\Phi_L} &= (A_L + \hat{A}_L U^{-1} A_R) U^{-1} - \hat{A}_L U^{-1} (A_R - A_L) U^{-1} \\ &= (1 + \hat{A}_L U^{-1}) A_L U^{-1} \\ &= e^{\Phi_L} \Psi_L \end{aligned}$$

$$\eta_0 (e^{-\Phi_L} Q_B e^{\Phi_L}) = \eta_0 \Psi_L = 0$$

Similarly, $\eta_0 (e^{-\Phi_R} Q_B e^{\Phi_R}) = 0$

Real solution

$$e^{\Phi_{real}} = \frac{1}{\sqrt{e^{\Phi_L} U e^{-\Phi_R}}}\, e^{\Phi_L} \sqrt{U}$$

$$\left(Q_B (e^{\Phi_L} U e^{-\Phi_R}) = 0, \quad \eta_0 U = 0 \right)$$

Conclusions and discussion

We have constructed analytic solutions for any exactly marginal deformation in any boundary (S)CFT when $[\, e^{\lambda V(a,b)} \,]_r$ is given.

New characterization of exact marginality using the BRST formalism
(cf. Recknagel & Schomerus, hep-th/9811237)

Study explicit solutions.
- rolling tachyons
- lower-dimensional D-branes

Backgrounds not connected by marginal deformations

Background independence

The equation of motion of the spacetime theory
$\longleftrightarrow$ Conformal invariance of the world-sheet theory

Our new solutions have little relevance to Schnabl's tachyon vacuum solution.
— Let's see how far we can get!