Superconducting qubits:
Quantum Circuits as Artificial Atoms

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Funding: RIKEN, NSA, LPS, ARO, NSF, CREST

Quick overview
(just few slides!)

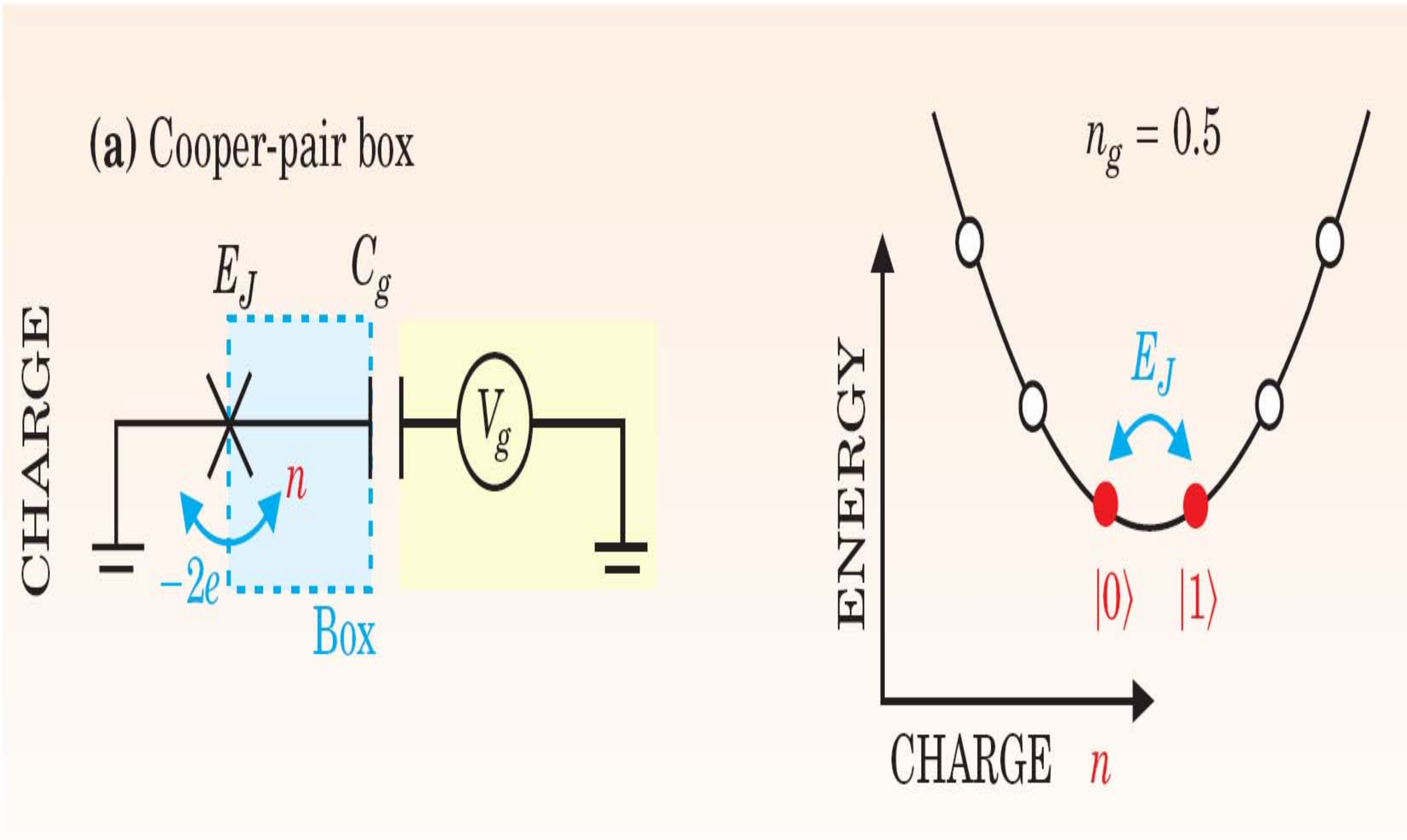
# of several types of superconducting qubits

Short pedagogical review in:

You and Nori, *Physics Today* (November 2005)

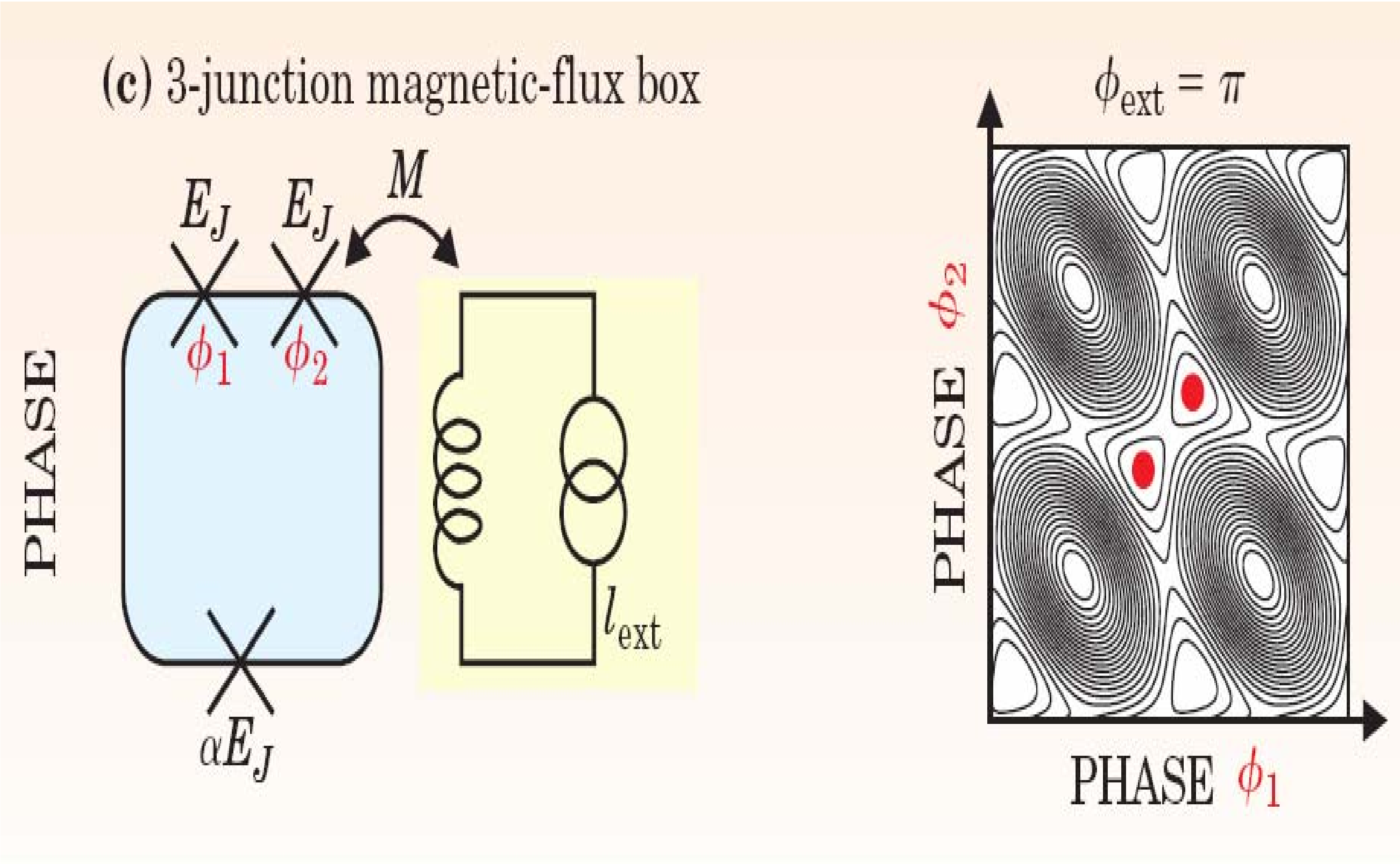
Color reprints available online at dml.riken.jp

Charge qubit

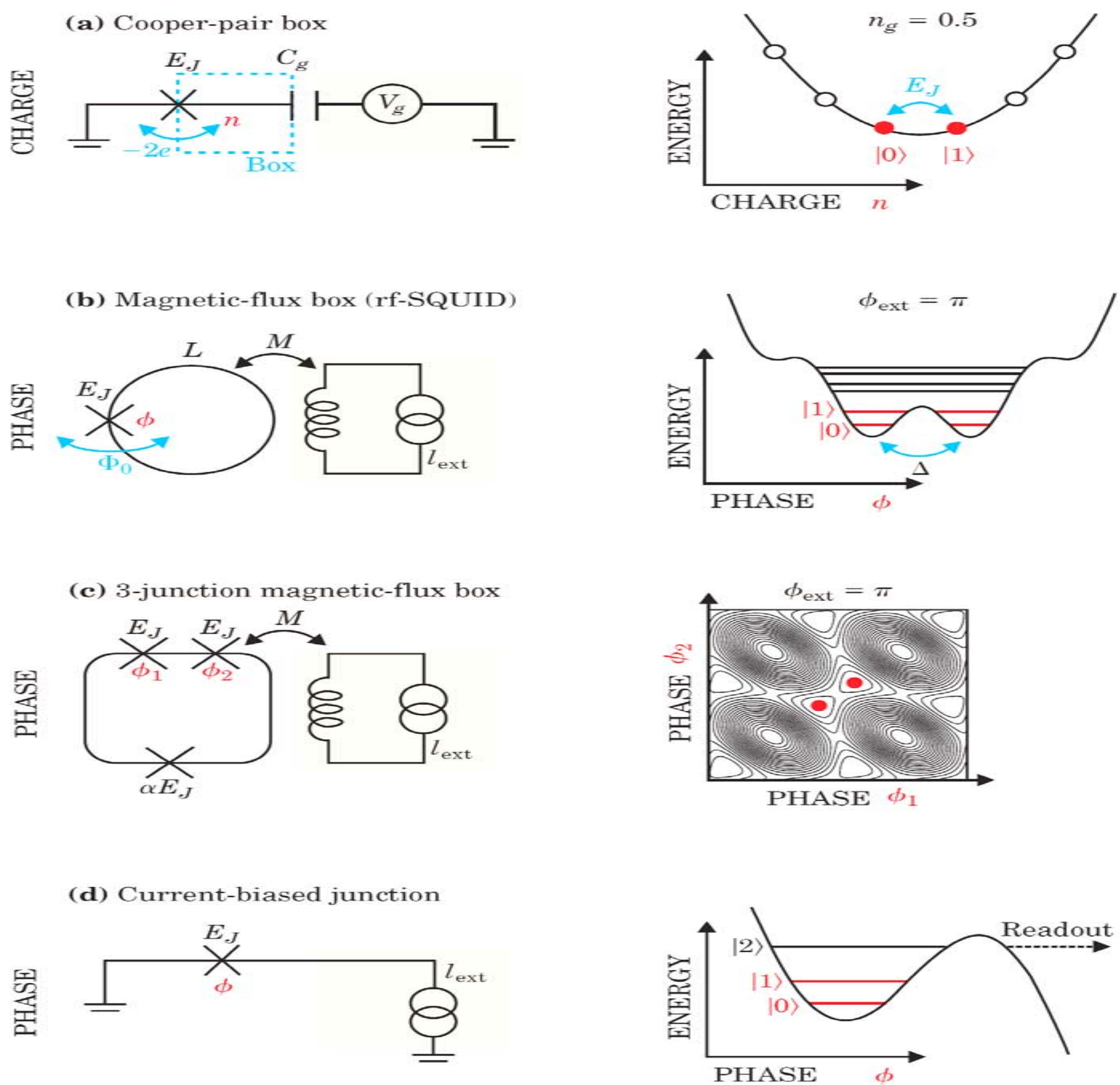


Figures from: You and Nori, *Physics Today* (November 2005)

Flux qubit



Figures from: You and Nori, *Physics Today* (November 2005)



Figures from: You and Nori, *Physics Today* (November 2005)

End of the Quick overview of several types of superconducting qubits

For a short pedagogical overview, please see:

You and Nori, *Physics Today* (November 2005)

Now we will focus on a few results from our group

No time today to cover other results

Contents

- Flux qubits as artificial atoms
- Cavity QED on a chip
- Coupling qubits
- Controllable coupling between qubits:
  - via variable frequency magnetic fields
  - via data buses
- Scalable circuits
- Testing Bell inequalities. Generating GHZ states.
- Quantum tomography
- Conclusions

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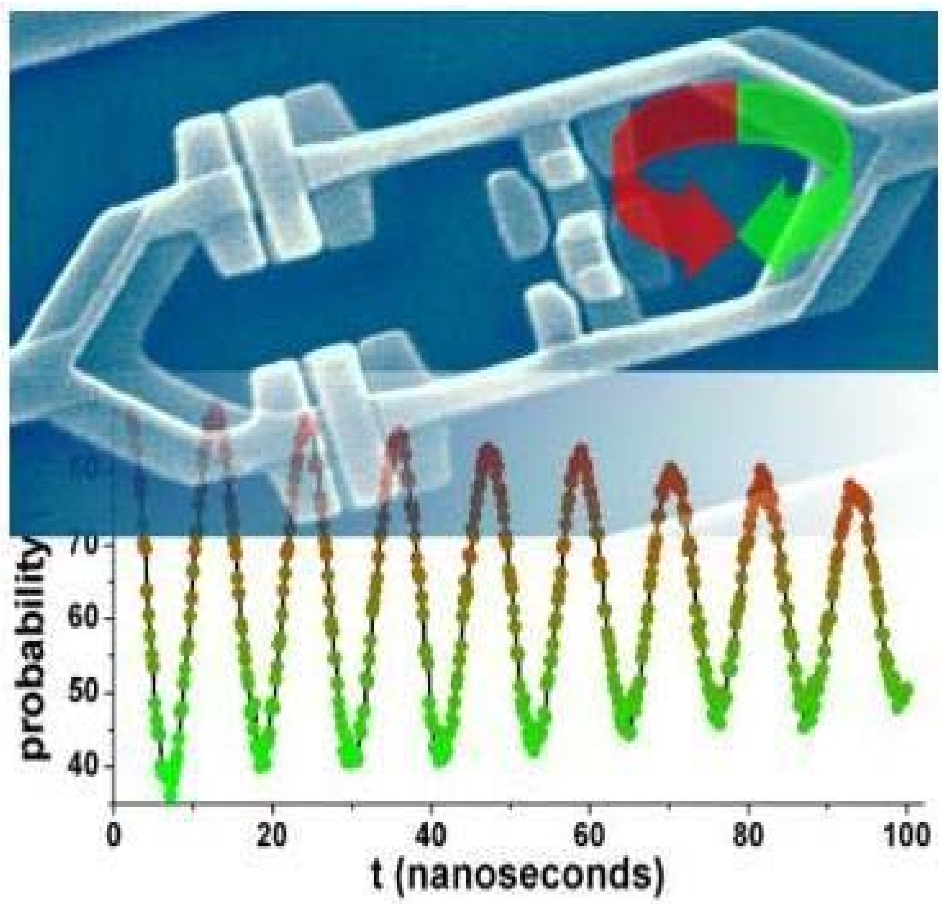
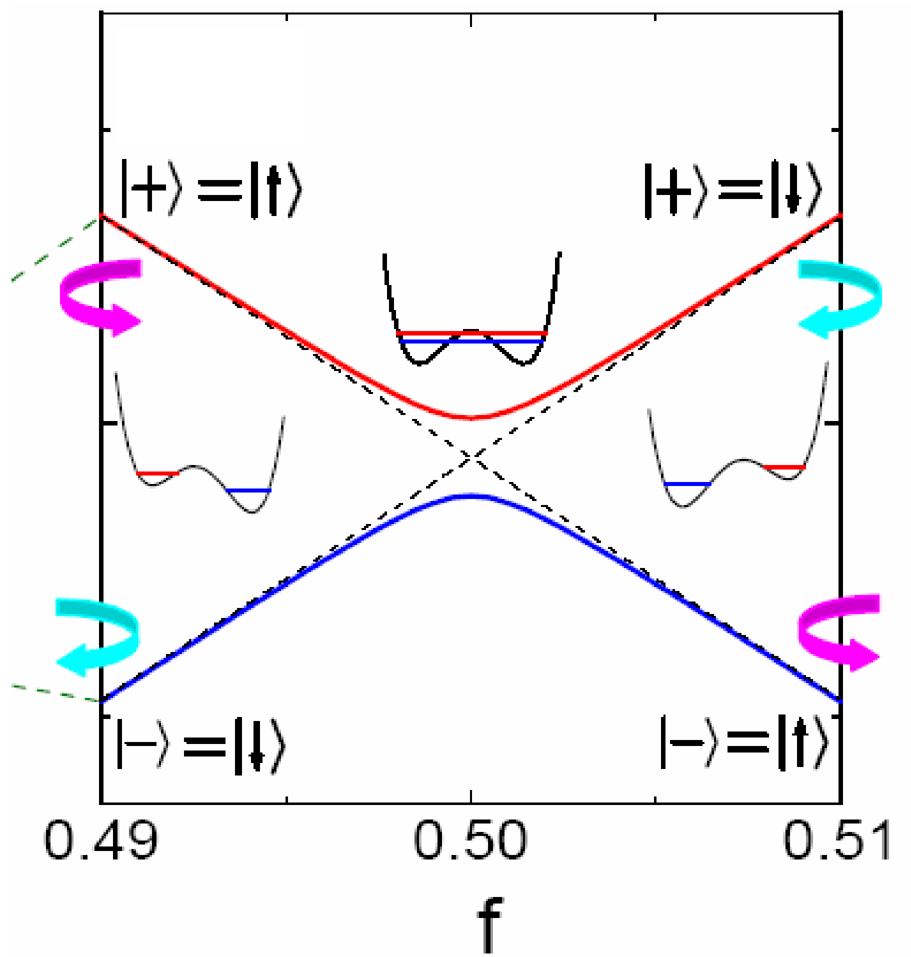
Qubit = Two-level quantum system



Chiorescu et al, *Science* 299, 1869 (2003)



You and Nori, *Phys. Today* 58 (11), 42 (2005)

Reduced magnetic flux: f $= \Phi_e / \Phi_0$. Here: Φ_e = external DC bias flux

Flux qubit: Symmetry and parity

In standard atoms, electric-dipole-induced selection rules for transitions satisfy :

$$\Delta l = \pm 1 \quad \text{and} \quad \Delta m = 0, \ \pm 1$$

In superconducting qubits, there is no obvious analog for such selection rules.

Here, we consider an analog based on the symmetry of the potential $U(\varphi_m, \varphi_p)$ and the interaction between:

-) superconducting qubits (usual atoms) and the

-) magnetic flux (electric field).

Liu, You, Wei, Sun, Nori, *PRL* (2005)

Superconducting flux qutrits:

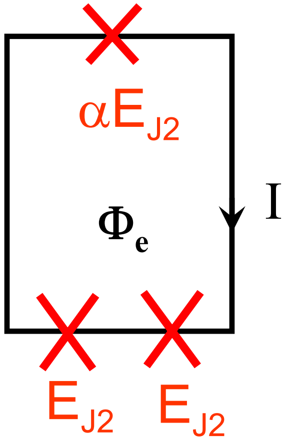
Symmetry of potential energy

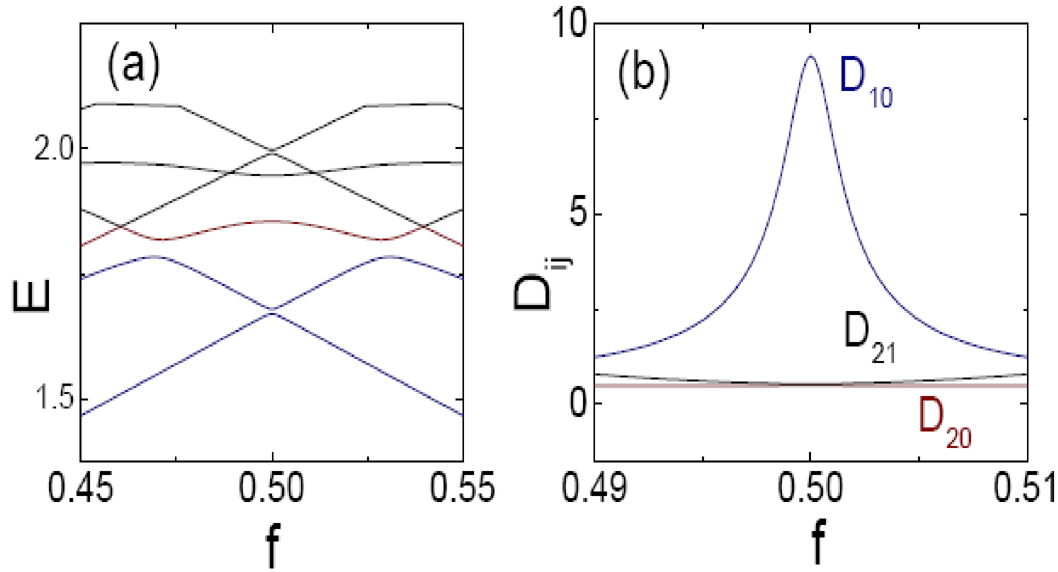
Creating cyclic transitions
(not allowed for normal atoms)

Turning on-and-off inter-level transitions

Using this for generating photons on demand

Flux qutrit (here we consider the three lowest energy levels)





$$H_0 = \frac{P_p^2}{2M_p} + \frac{P_m^2}{2M_m} + U(\varphi_p, \varphi_m, f)$$

$$U = 2E_J\left(1-\cos\varphi_p\cos\varphi_m\right) + \alpha E_J\left[1-\cos\left(2\varphi_m + 2\pi f\right)\right] \qquad f = \frac{\Phi_e}{\Phi_0}$$

Phases and momenta (conjugate variables) are (see, e.g., Orlando et al, *PRB* (1999))

$$\varphi_p = (\phi_1+\phi_2)/2; \quad \varphi_m = (\phi_1-\phi_2)/2; \quad P_k = -i\hbar\,\partial/\partial\varphi_k \quad (k=p,\ m)$$

Effective masses

$$M_p = \left(\Phi_0/2\pi\right)^2 2C; \quad M_m = 2M_p(1+2\alpha) \ \text{ with capacitance } C \text{ of the junction}$$

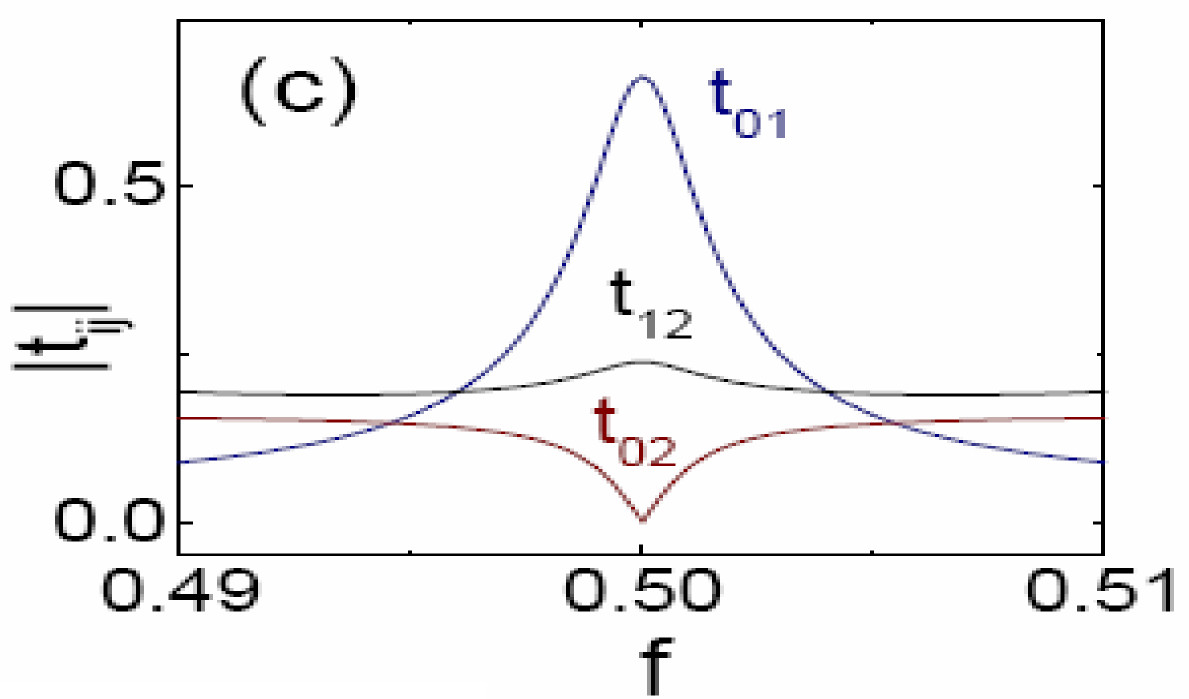
Flux qubit: Symmetry and parity

Parity of $U(\varphi_m, \varphi_p) \equiv U$

$$U = 2E_J\left(1-\cos\varphi_p\cos\varphi_m\right) + \alpha E_J\left[1-\cos\left(2\varphi_m + 2\pi f\right)\right]$$

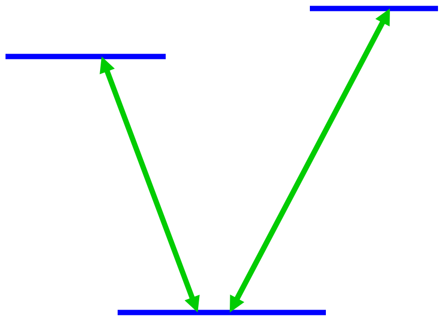
$f = 1/2 \Rightarrow U(\varphi_m, \varphi_p)$ even function of φ_m and φ_p

$$U = 2E_J\left(1-\cos\varphi_p\cos\varphi_m\right) + \alpha E_J\left[1+\cos\left(2\varphi_m\right)\right]$$

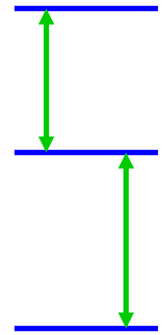


Liu, You, Wei, Sun, Nori, *PRL* 95, 087001 (2005)

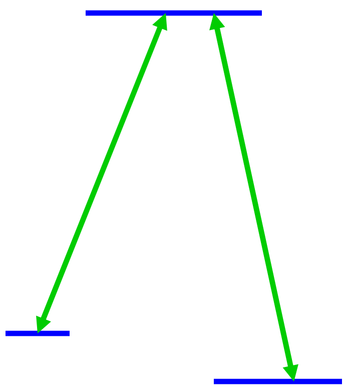
Allowed three-level transitions in natural atoms



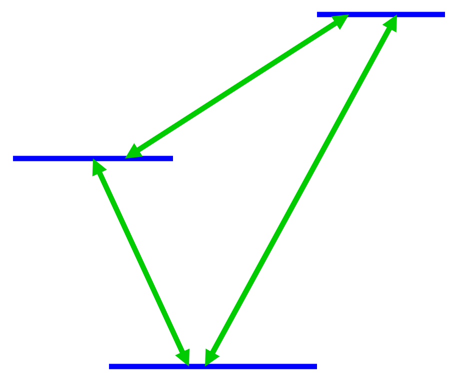
V - type



Ξ - type or ladder



Λ - type



No Δ – type because of the electric-dipole selection rule.

Some differences between artificial and natural atoms

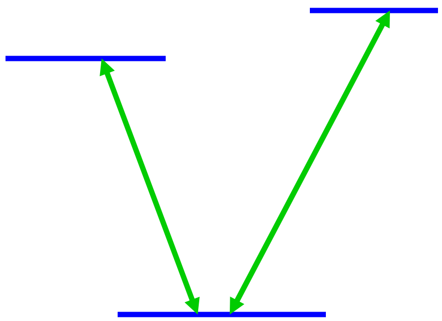
In natural atoms, it is *not* possible to obtain cyclic transitions by only using the electric-dipole interaction, due to its well-defined symmetry.

However, these transitions can be obtained in a flux qutrit circuit, due to the broken symmetry of the potential of the flux qubit, when the bias flux deviates from the optimal point.

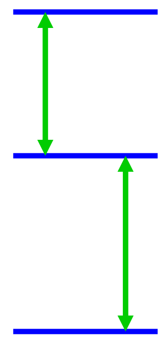
The magnetic-field-induced transitions in the flux qutrit are similar to atomic electric-dipole-induced transitions.

Liu, You, Wei, Sun, Nori, *PRL* (2005)

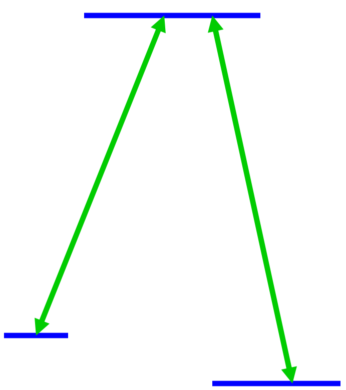
Different transitions in three-level systems



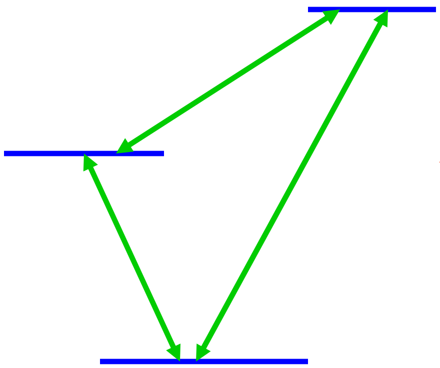
V - type



Ξ - type or ladder
(f = 1/2)



Λ - type



Δ – type
can be obtained
using flux qubits
(*f* away from 1/2)

Liu, You, Wei, Sun, Nori, *PRL* (2005)

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Superconducting flux qutrits:

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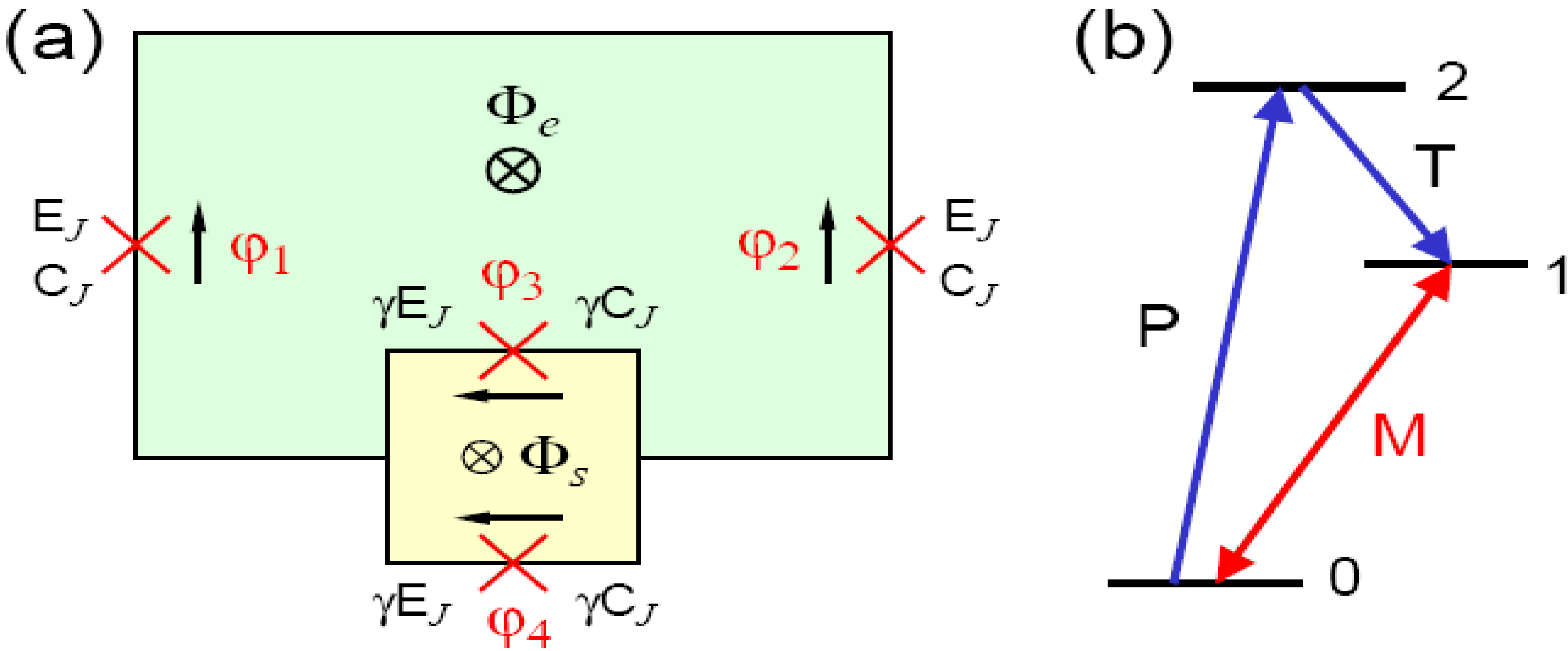
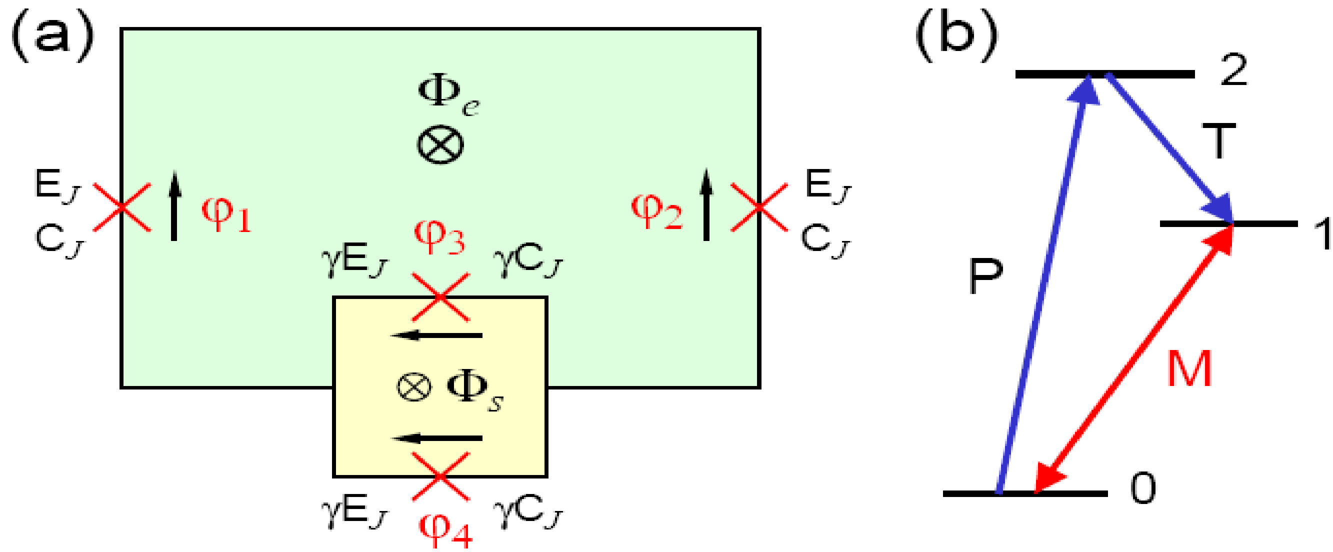
Flux qubit as a micromaser and single-photon source



You, Liu, Sun, Nori, Phys. Rev. B (2007). cond-mat (2005).

Flux qutrit as a micromaser and single-photon source

You, Liu, Sun, Nori,
Phys. Rev. B (2007)



We propose a *tunable* on-chip *photon-generator* using a superconducting quantum circuit.

By taking advantage of externally controllable state transitions, a <u>state population inversion</u> can be achieved and preserved.
When needed, the circuit can *generate a single photon.*

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Circuit QED Comparison with atoms

Circuit QED: Superconducting qubit in a cavity	Atom in a cavity
Josephson junction device	Atom
Current and voltage sources	Light sources
Voltmeters and ammeters	Detectors
T = 30 mK	T = 300 K
Electrodynamic environment	Cavity
Strong JJ-environment coupling	Weak atom-field coupling
Dissipation in environment	Photon losses

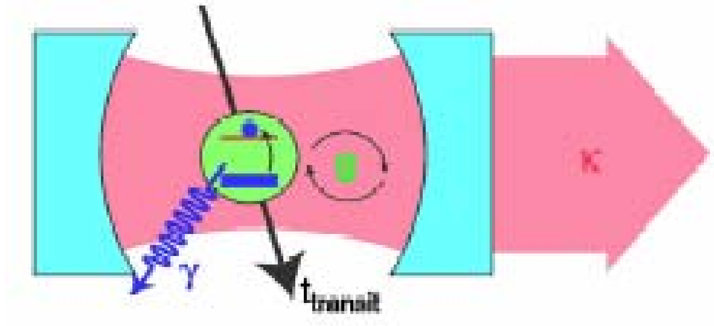
Circuit QED

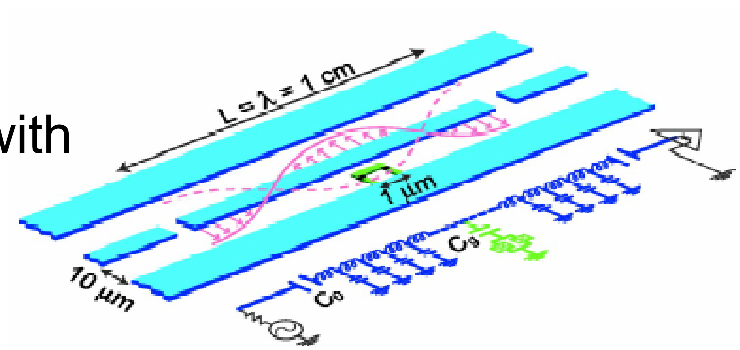
Qubit-field interaction

Hamiltonian

$$H = 4E_c(n - C_g V_g / 2e)^2 - E_J \cos\varphi$$

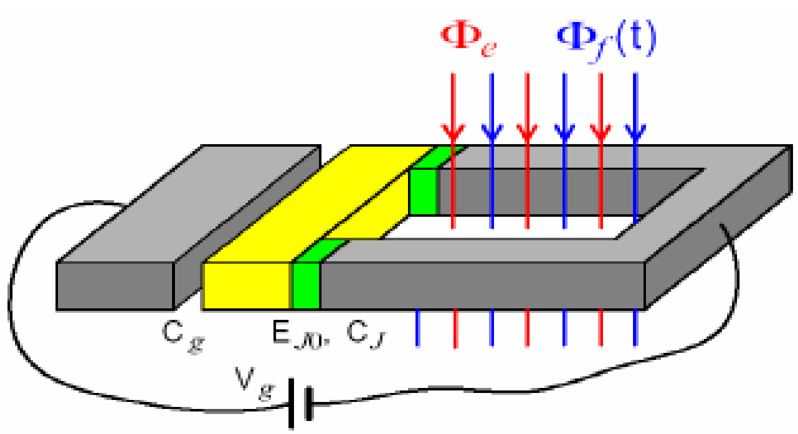
$$E_J(\Phi_X) = 2E_J \cos(\pi \Phi_X / \Phi_0)$$



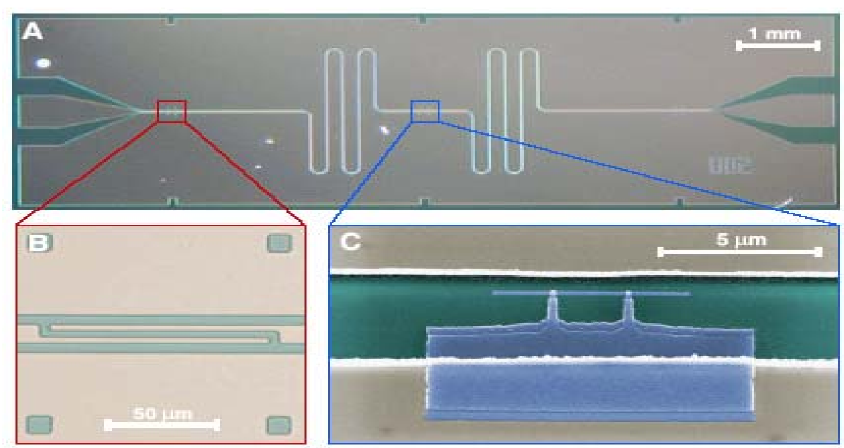


The charge qubit has two ways of interacting with a cavity field:

1) quantized field through the SQUID loop
2) quantized field applied to the gate voltage



You and Nori, PRB 68, 064509 (2003)



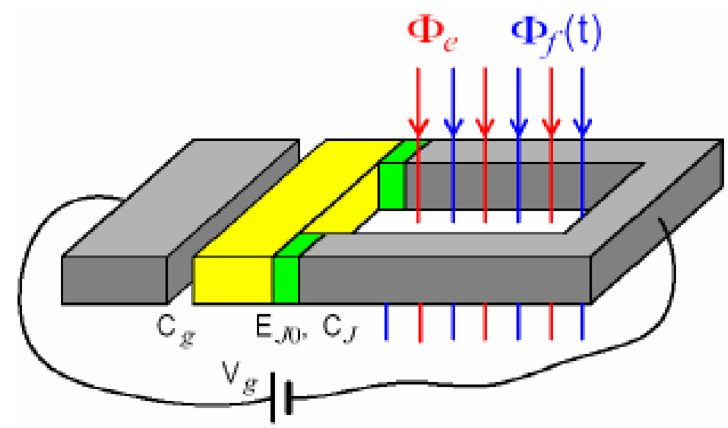
Yale group

Circuit QED

Qubit-field interaction

Hamiltonian

$$H = 4E_c(n - C_g V_g / 2e)^2 - E_J \cos\varphi$$



Quantized field is through the SQUID loop

$$E_J(\Phi_X) = 2E_J \cos(\pi\Phi_X / \Phi_0)$$

Then the magnetic flux Φ_X is replaced by

$$\Phi_X = \Phi_c + \Phi_q$$

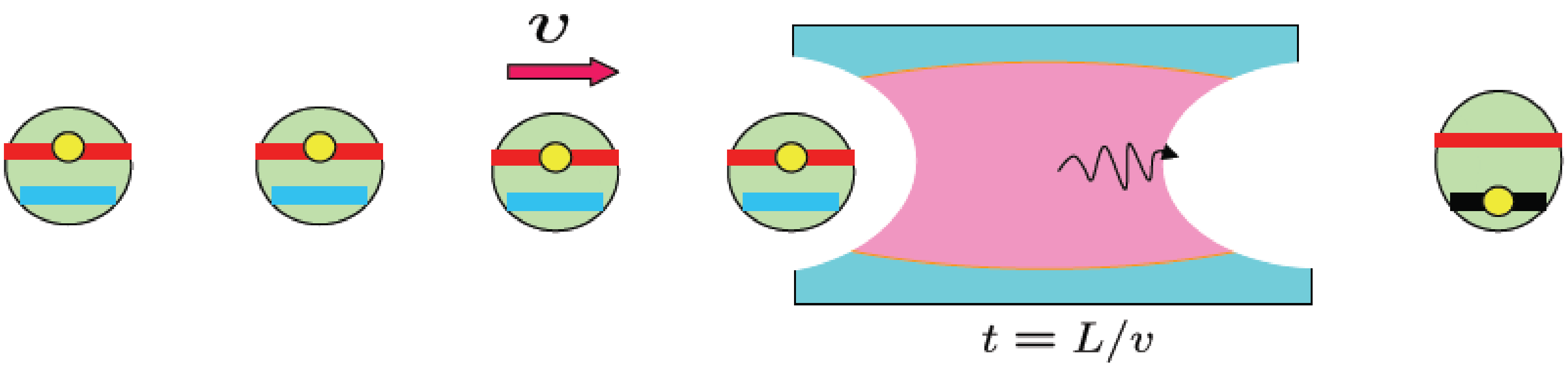
and we have

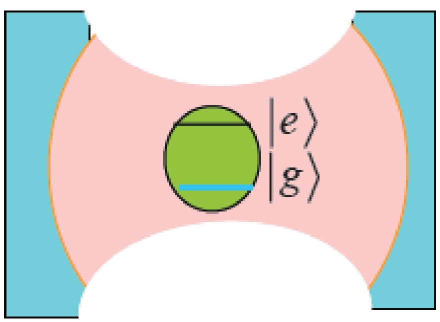
$$H = -\frac{1}{2}\omega_z \sigma_z - \frac{1}{2}\mathrm{E_J} \cos\left[\frac{\pi}{\Phi_0}\left(\Phi_C I + ga + g^* a^+\right)\right]\sigma_x$$

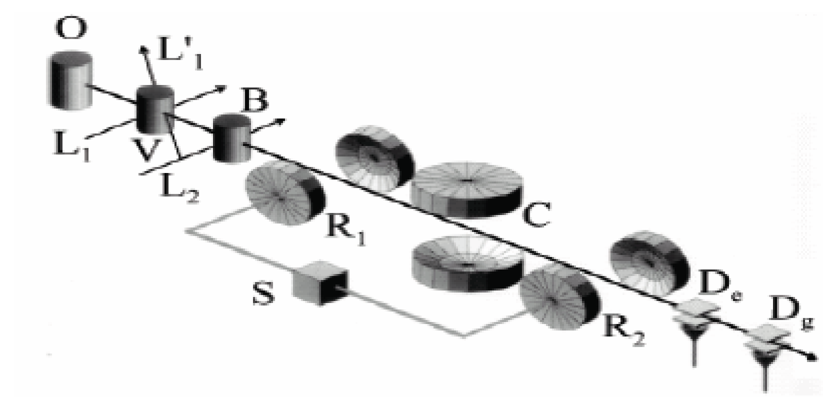
Comparison of our proposal with a micromaser

Carrier process: thermal excitation for micromaser

First red sideband excitation: the excited atoms enter the cavity, decay, and emit photons







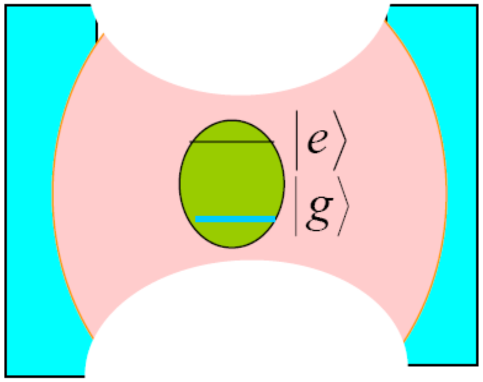
X. Maitre, et al., PRL 79, 769 (1997)

Comparison of our proposal with a micromaser

	JJ qubit photon generator	Micromaser
Before	JJ qubit in its ground state then excited via $n_g = 1/2, \quad \Phi_C = \Phi_0$	Atom is thermally excited in oven
Interaction with microcavity	JJ qubit interacts with field via $n_g = 1, \quad \Phi_C = \Phi_0/2$	Flying atoms interact with the cavity field
After	Excited JJ qubit decays and emits photons	Excited atom leaves the cavity, decays to its ground state providing photons in the cavity.

Liu, Wei, Nori, *EPL* (2004); *PRA* (2005); *PRA* (2005)

Interaction between the JJ qubit and the cavity field



Liu, Wei, Nori,
EPL 67, 941 (2004);
PRA 71, 063820 (2005);
PRA 72, 033818 (2005)

$$H = \underbrace{\hbar\omega a^{\dagger}a}_{\text{cavity field}} \underbrace{-2E_C(1-2n_g)\sigma_z}_{\text{charging energy}}$$
$$\underbrace{-E_J\cos\left[\frac{\pi}{\Phi_0}(\Phi_c + ga + g^*a^{\dagger})\right]\sigma_x}_{\text{interaction term}}$$

with $g = i\int_S u(r)\cdot ds$ and $\hbar\omega = 2E_C$

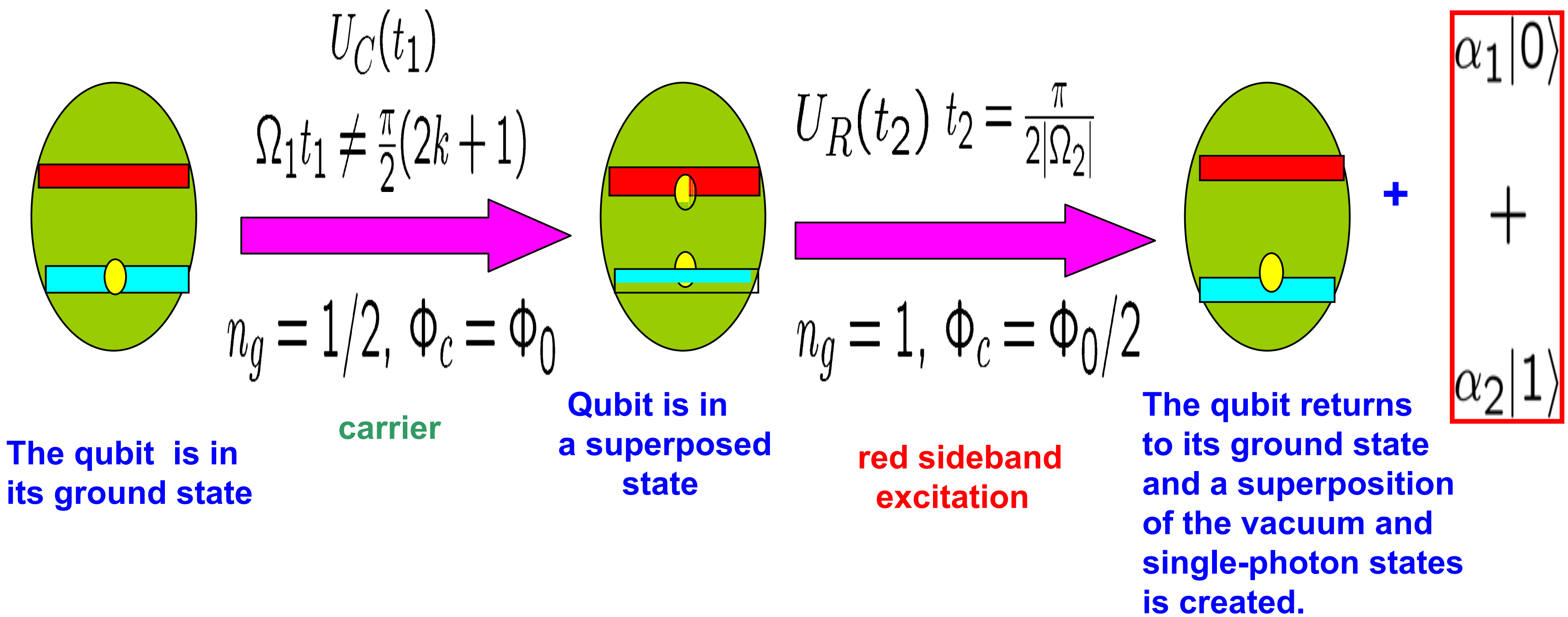
(1) The interaction between the cavity field and the SQUID is controlled by the gate charge n_g and the dc applied flux Φ_C.
(2) S is the area of the SQUID.
(3) $u(r)$ is a mode function of a single-mode cavity field.

details

Cavity QED on a chip

How to create superpositions of photon states

$\alpha_1|0\rangle + \alpha_2|1\rangle$ with $\alpha_1 = \cos(\Omega_1 t_1)$ and $\alpha_2 = e^{-i\theta}\sin(\Omega_1 t_1)$



When the red sideband excitation satisfies the condition $t_2 = \pi/2|\Omega_2|$, it creates a superposition of the vacuum and single photon states.

Contents

Qubits can be ***coupled***

either directly

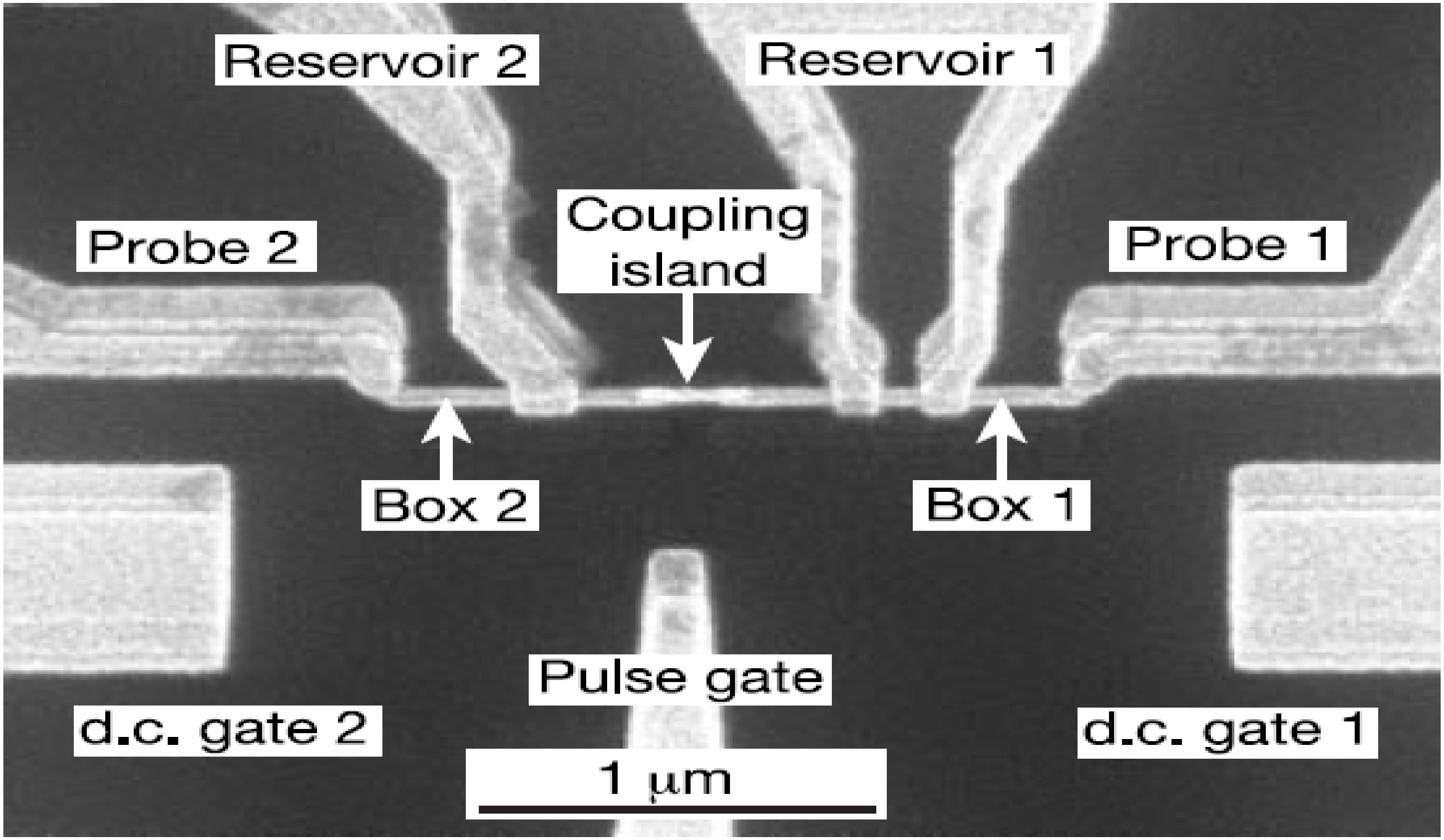
or

indirectly, using a data bus

(i.e., an “intermediary”)

Let us now very quickly
(fasten your seat belts!)
see a few experimental
examples of qubits
coupled directly

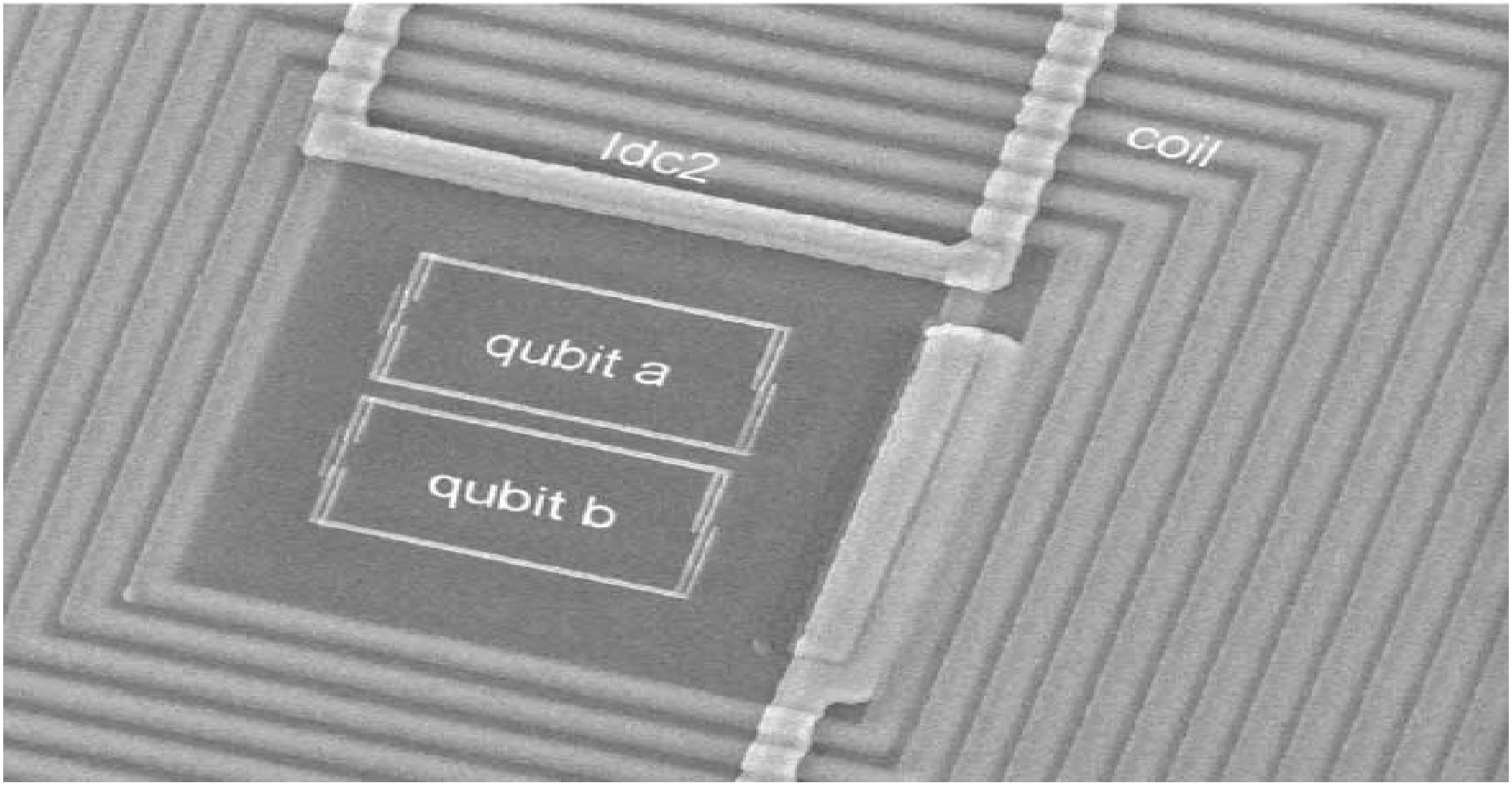
Capacitively coupled charge qubits



NEC-RIKEN

Entanglement; conditional logic gates

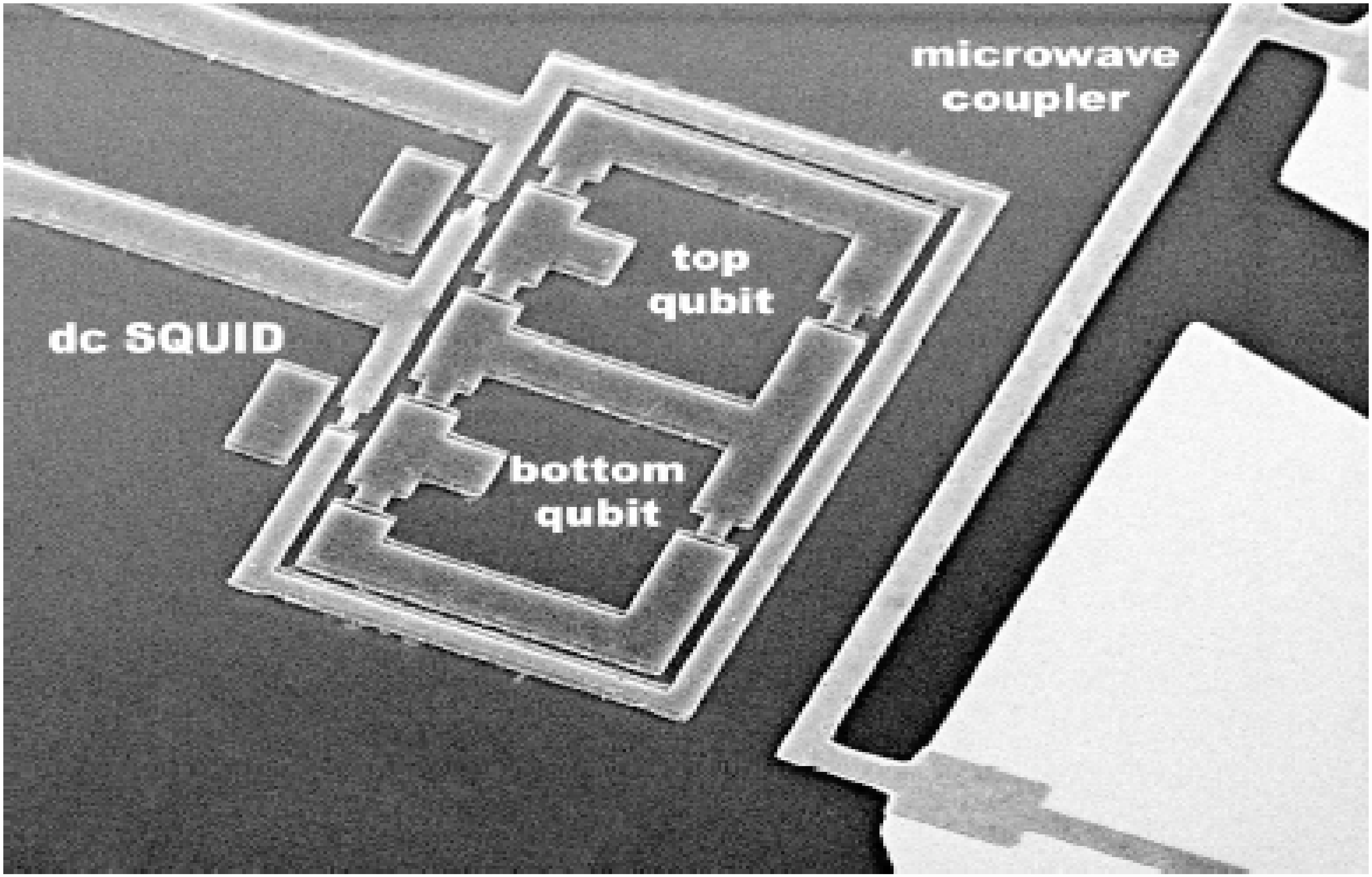
Inductively coupled flux qubits



A. Izmalkov et al., PRL 93, 037003 (2004)

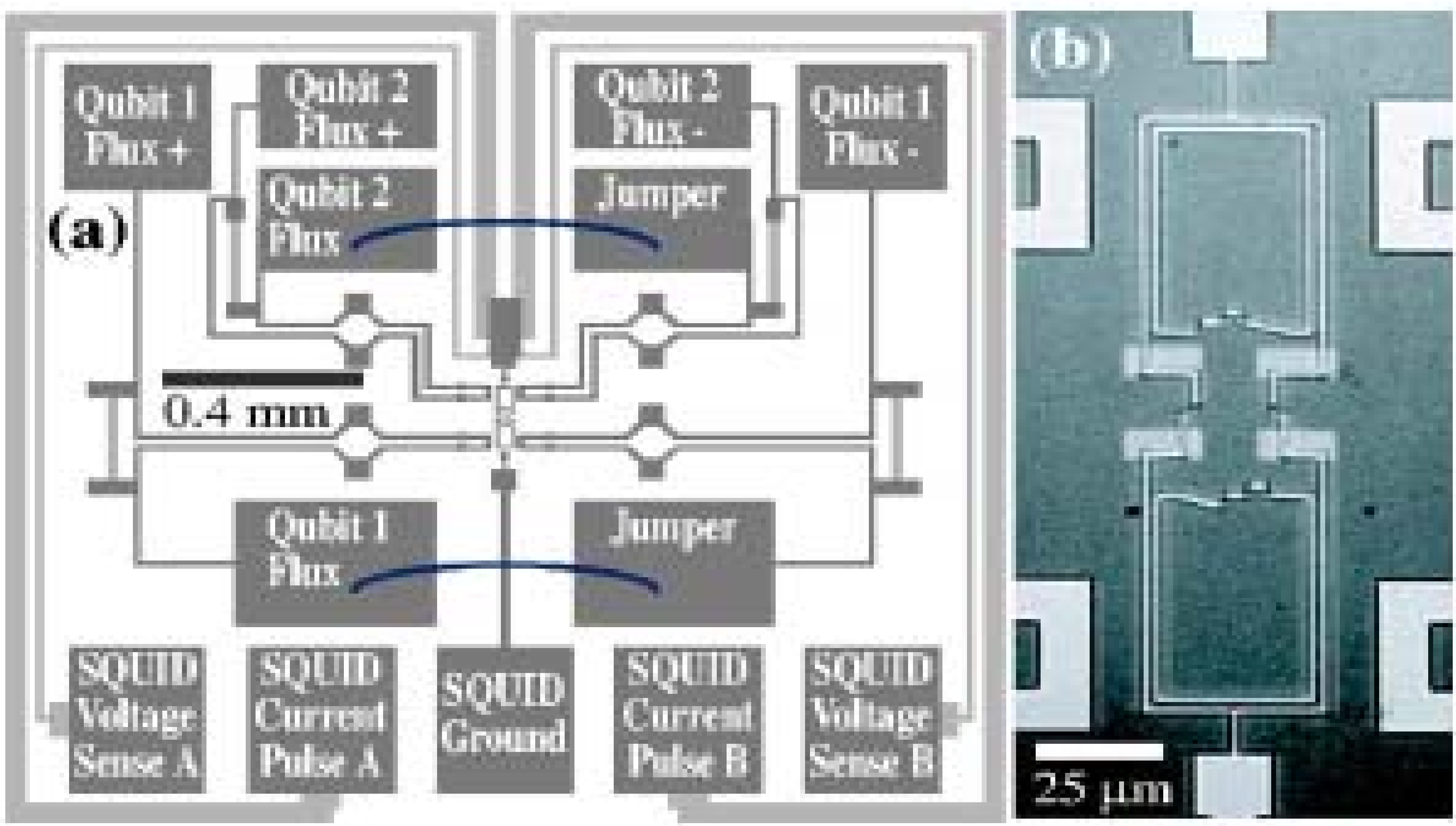
Entangled flux qubit states

Inductively coupled flux qubits



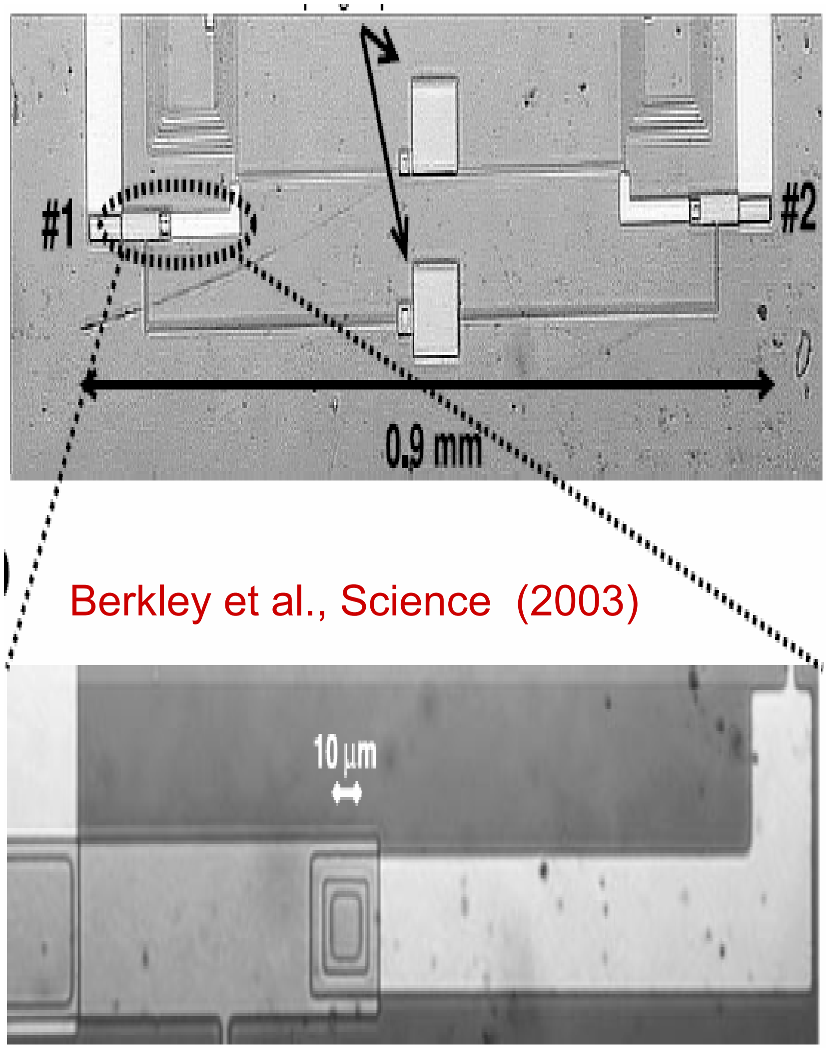
J.B. Majer et al., *PRL* 94, 090501 (2005). Delft group

Inductively coupled flux qubits

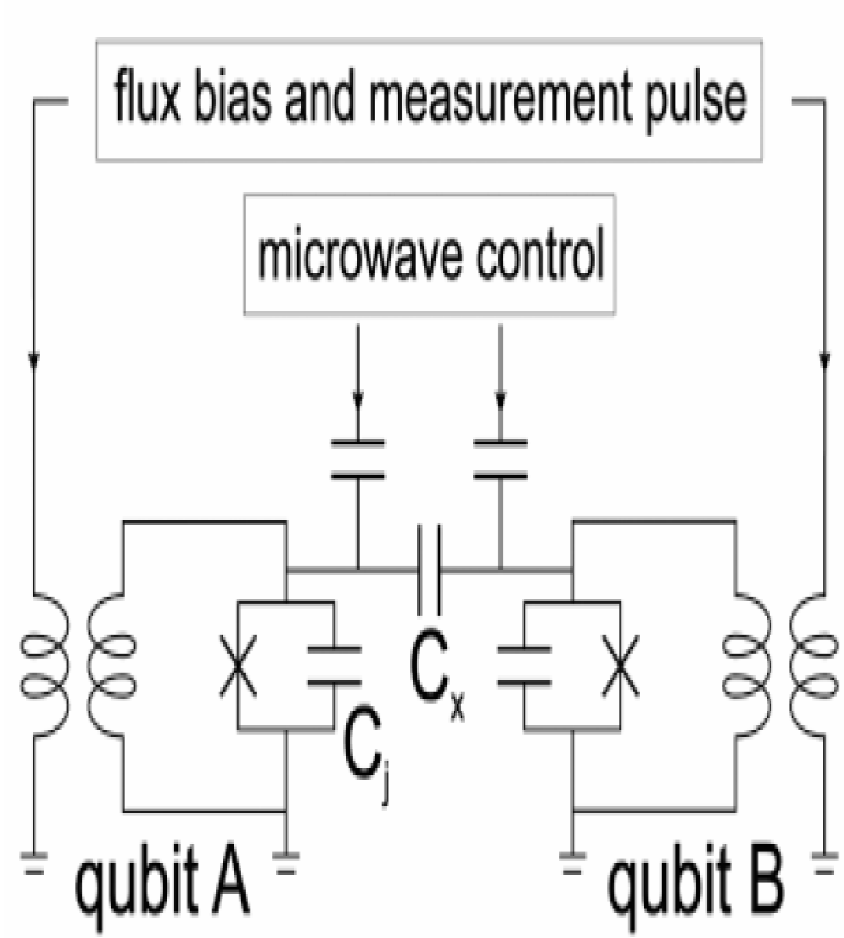


J. Clarke's group, Phys. Rev. B 72, 060506 (2005)

Capacitively coupled phase qubits



Berkley et al., Science (2003)



McDermott et al., Science (2005)

Entangled phase qubit states

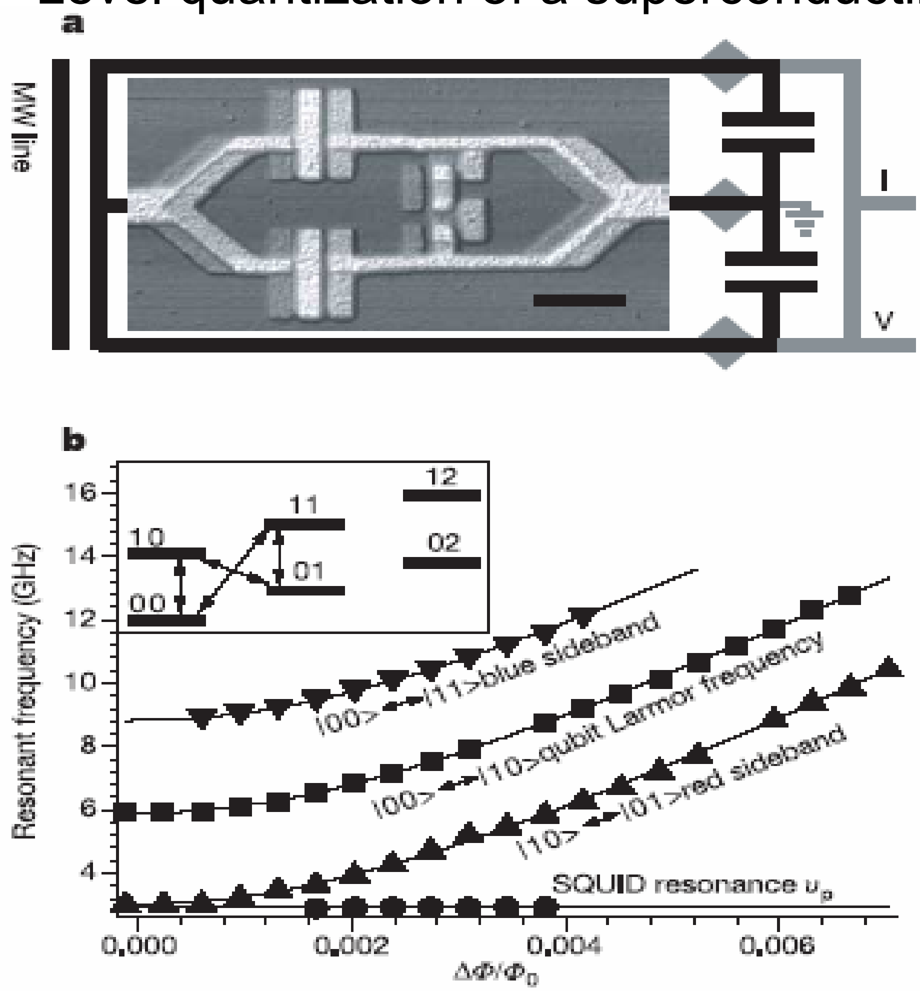
Let us now consider qubits

coupled *indirectly*

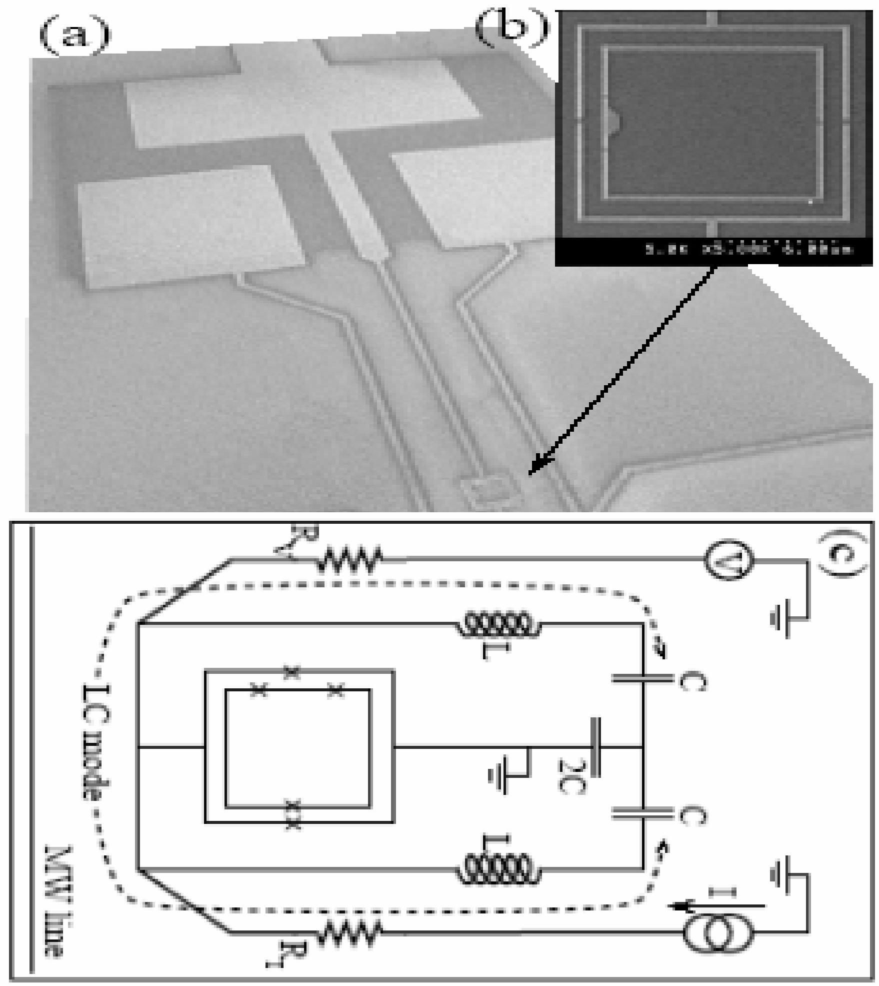
Scalable circuits: using an LC data bus

LC-circuit-mediated interaction between qubits

Level quantization of a superconducting LC circuit has been observed.



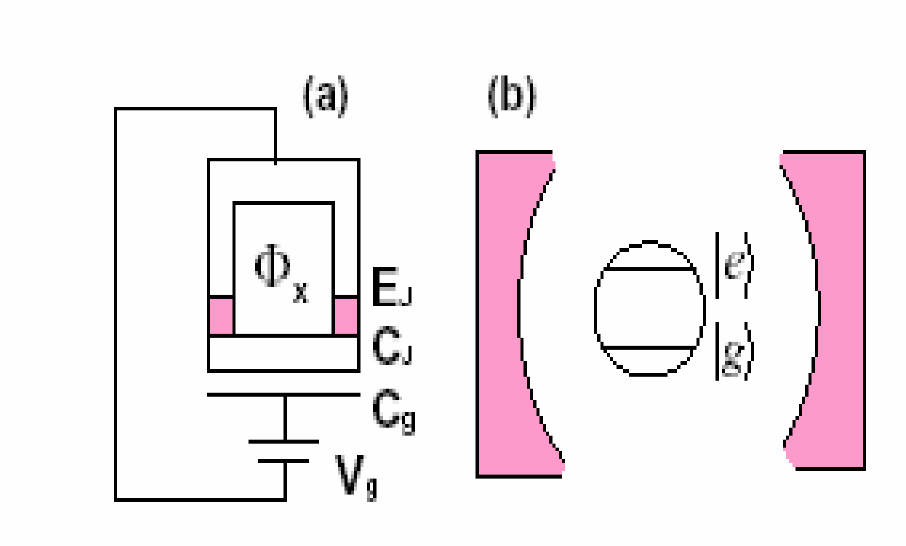
Delft, Nature, 2004

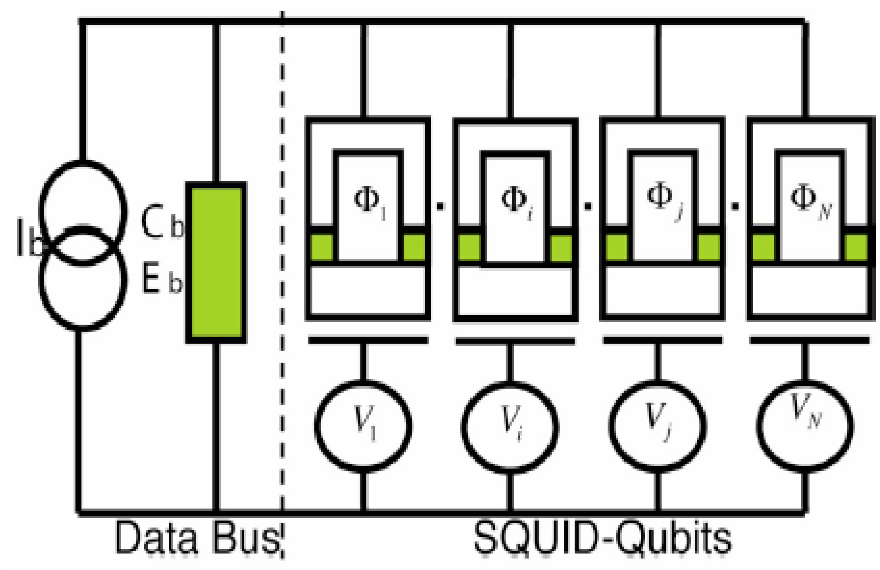


NTT, PRL 96, 127006 (2006)

Switchable coupling: data bus

A switchable coupling between the qubit and a data bus could also be realized by changing the magnetic fluxes through the qubit loops.





Liu, Wei, Nori, EPL 67, 941 (2004)

Wei, Liu, Nori, PRB 71, 134506 (2005)

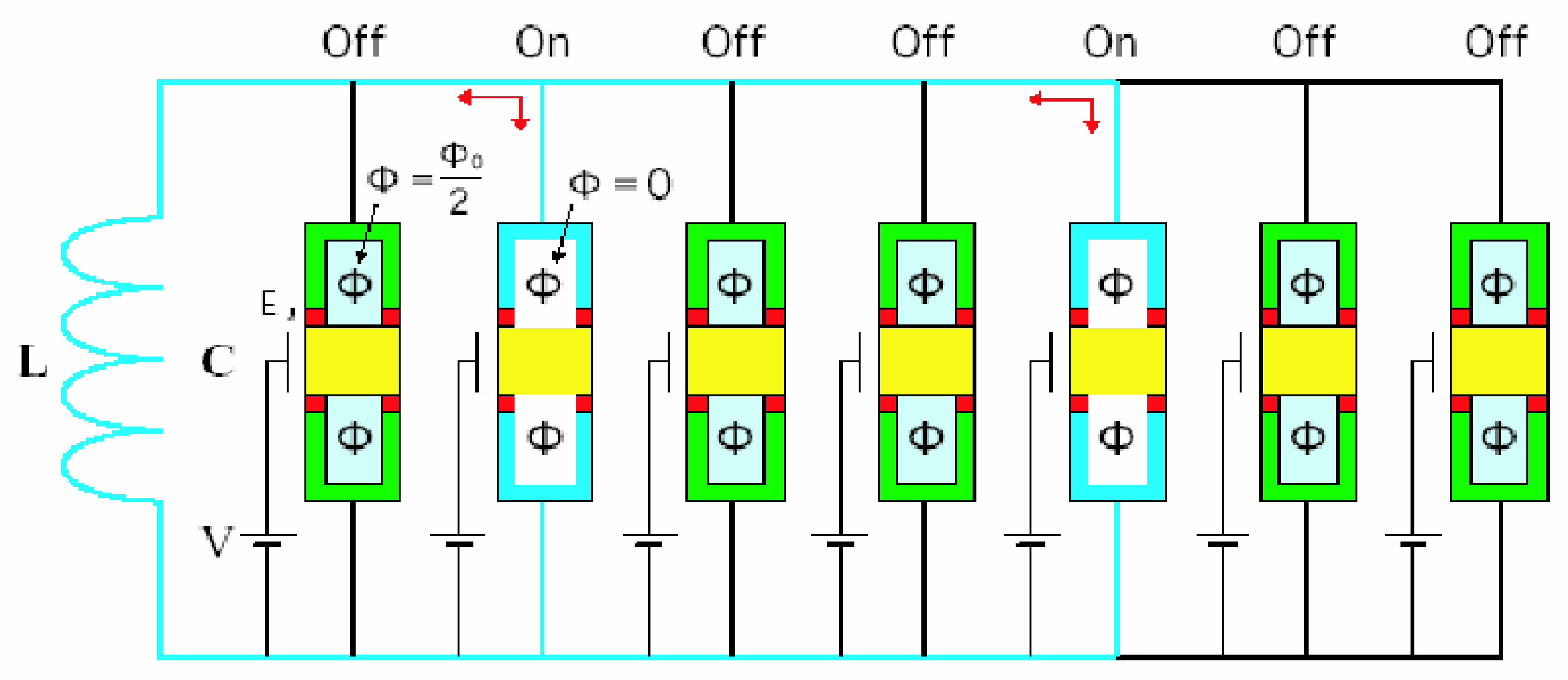
Single-mode cavity field

Current-biased junction

The bus-qubit coupling is proportional to $\cos\left(\pi \frac{\Phi_X}{\Phi_0}\right)$

Scalable circuits

Couple qubits *directly* via a common inductance



You, Tsai, and Nori, *Phys. Rev. Lett.* 89, 197902 (2002)

Switching on/off the SQUIDs connected to the Cooper-pair boxes, can couple any selected charge qubits by the common inductance (*not* using LC oscillating modes).

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  - via data buses
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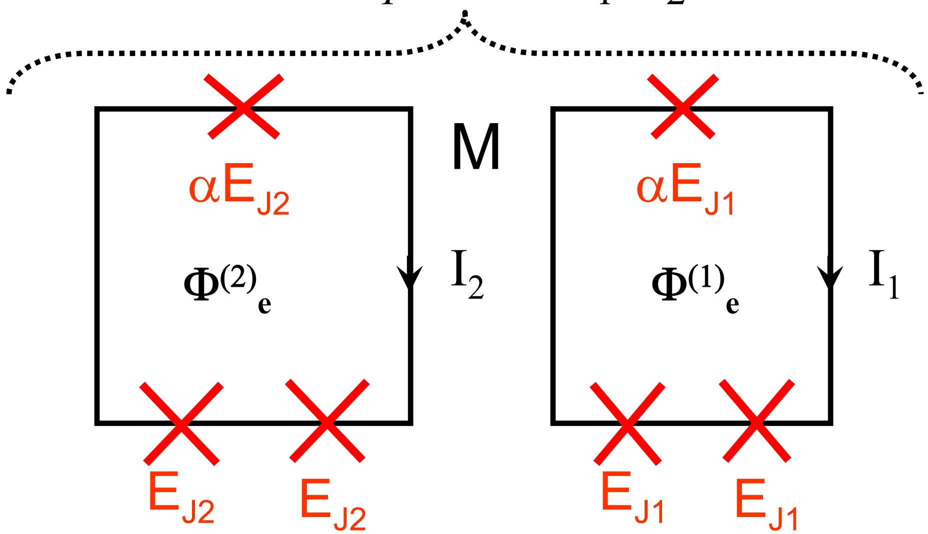
Coupling qubits directly and (first) without VFMF (Variable Frequency Magnetic Flux)

$$H_0 = H_{q1} + H_{q2} + H_I$$ = Total Hamiltonian

$$H_I = M \; I_1 \; I_2$$



$$H_{ql} = \frac{P_{ml}^2}{2M_{ml}} + \frac{P_{pl}^2}{2M_{pl}} + 2E_{Jl} + \alpha E_{Jl} - 2E_{Jl}\cos\varphi_m^{(l)}\cos\varphi_P^{(l)} - \alpha E_{Jl}\cos\left(2\pi f_1 + 2\varphi_m^{(l)}\right)$$

l=1,2

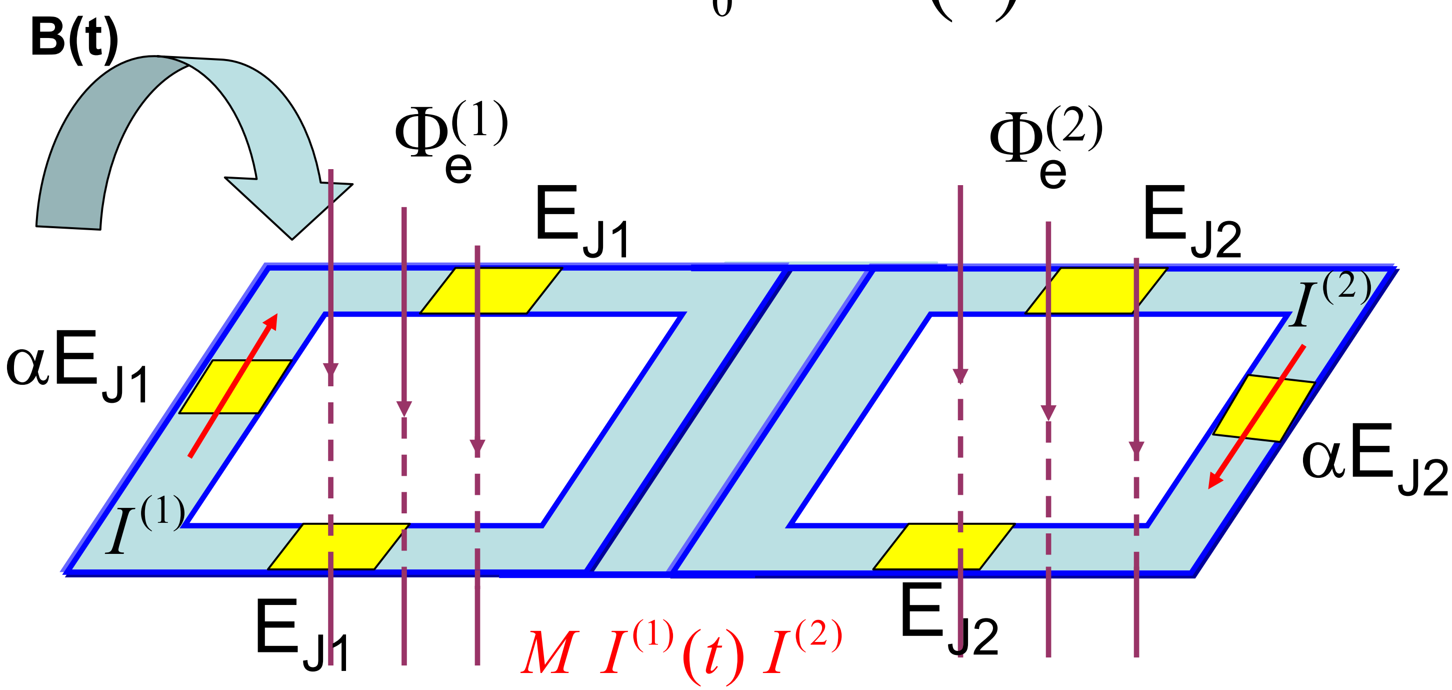
Now: let’s consider a

Variable-Frequency-Magnetic-Flux (VFMF)

Controllable couplings via VFMFs

Applying a Variable-Frequency Magnetic Flux (VFMF)

$$H = H_0 + H(t)$$

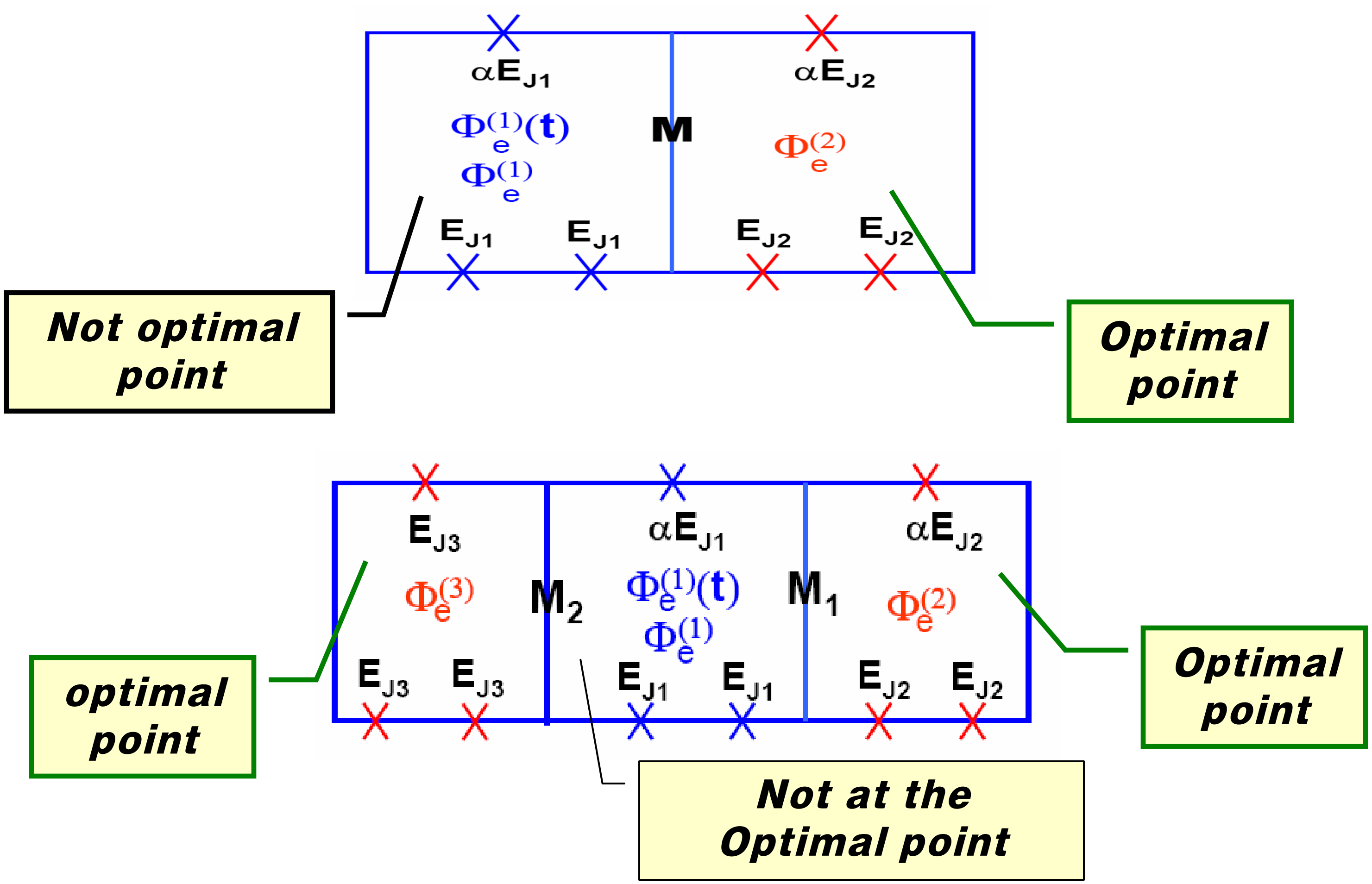


$$I^{(1)}(t) \approx I^{(1)} + \tilde{I}(t)$$

Liu, Wei, Tsai, and Nori, *Phys. Rev. Lett.* 96, 067003 (2006)

Controllable couplings via VFMFs

The couplings in these two circuits work similarly



back

Grajcar, Liu, Nori, Zagoskin, *Switchable resonant coupling of flux qubits,* Phys. Rev. B **74**, 172505 (Nov. 2006)

Switchable coupling proposals

(without using data buses)

Proposal \ Feature →	Weak fields	Optimal point	No additional circuitry
Rigetti et al. (Yale)	No	Yes	Yes
Liu et al. (RIKEN-Michigan)	OK	No	Yes
Bertet et al. (Delft) Niskanen et al. (RIKEN-NEC) Grajcar et al. (RIKEN-Michigan)	OK	Yes	No
Ashhab et al. (RIKEN-Michigan)	OK	Yes	Yes

Depending on the experimental parameters, our proposals might be useful options in certain situations.

details

Now: let’s consider a

Variable-Frequency-Magnetic-Flux (VFMF)

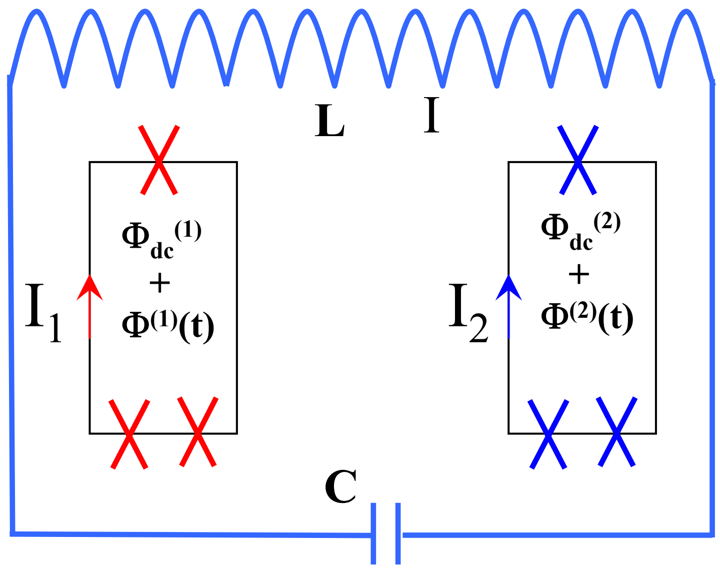
and also

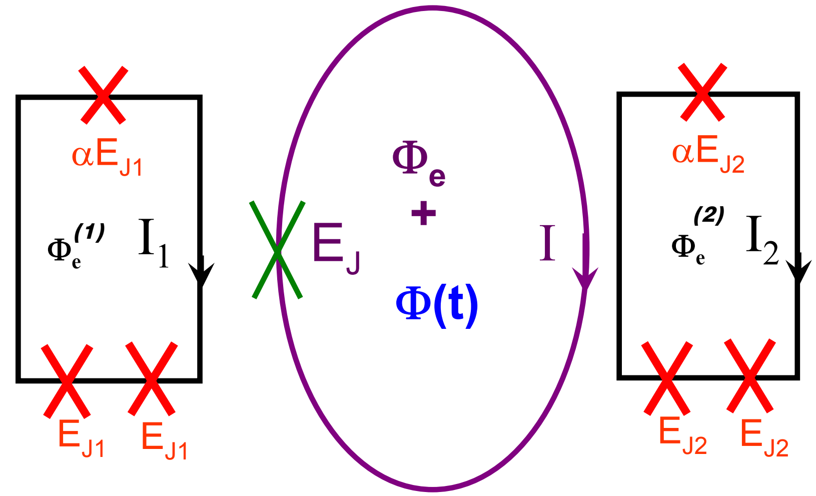
a data bus

Contents

- Flux qubits
- Cavity QED on a chip
- Coupling qubits
- **Controllable coupling between qubits:**
  - **via variable frequency magnetic fields**
  - **via data buses**
- **Scalable circuits**
- Testing Bell inequalities. Generating GHZ states.
- Quantum tomography
- Conclusions

A data bus using TDMF to couple several qubits





A data bus could couple several tens of qubits.

The TDMF introduces a nonlinear coupling between the qubit, the LC circuit, and the TDMF.

Liu, Wei, Tsai, Nori, *PRL* (2006)

details

Comparison between SC qubits and trapped ions

Qubits	Trapped ions	Superconducting circuits
Quantized bosonic mode	Vibration mode	LC circuit
Classical fields	Lasers	Magnetic fluxes

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- Flux qubits
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Dynamical decoupling

Main idea:

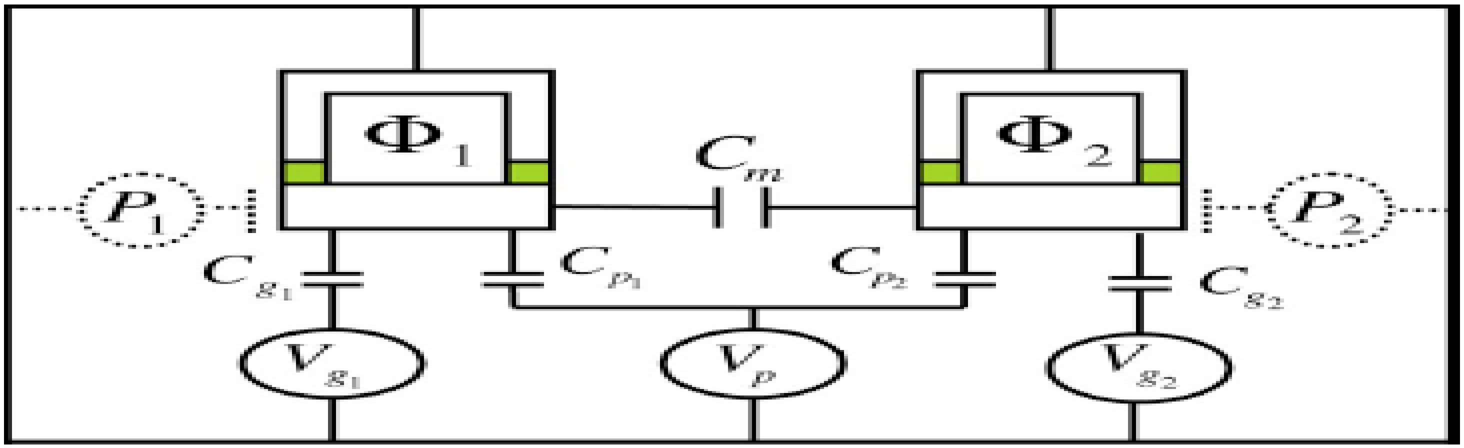
Let us assume that the coupling between qubits is not very strong (coupling energy < qubit energy)

Then the interaction between qubits can be effectively incorporated into the single qubit term (as a perturbation term)

Then single-qubit rotations can be approximately obtained, even though the qubit-qubit interaction is fixed.

Wei, Liu, Nori, *Phys. Rev. B* 72, 104516 (2005)

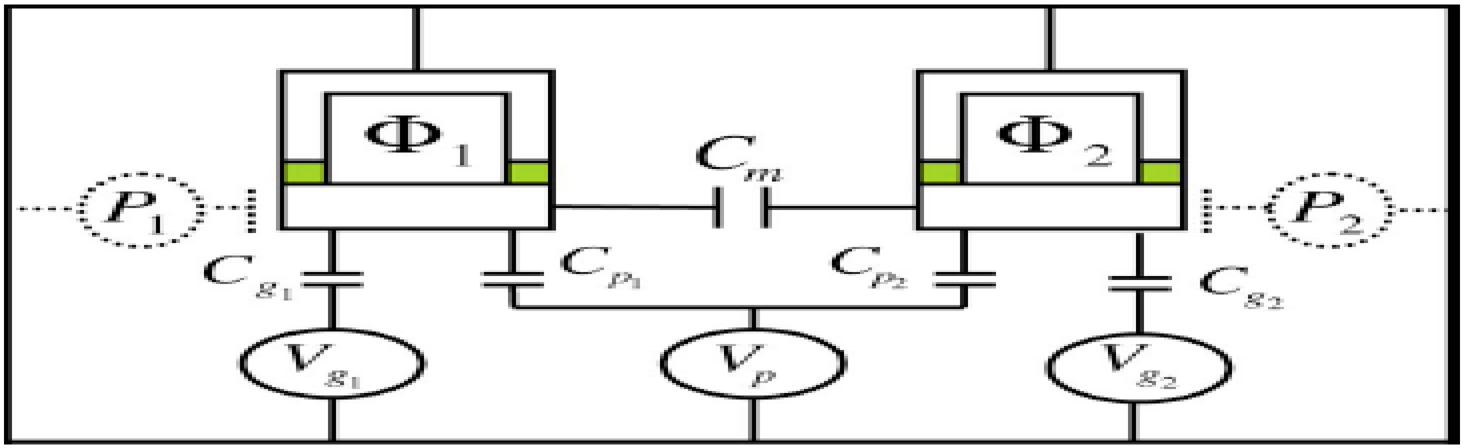
Testing Bell's inequality



We propose how to use coupled Josephson qubits to test Bell's inequality

Wei, Liu, Nori, *Phys. Rev. B* (2005)

Testing Bell's inequality



Wei, Liu, Nori, *Phys. Rev. B* (2005)

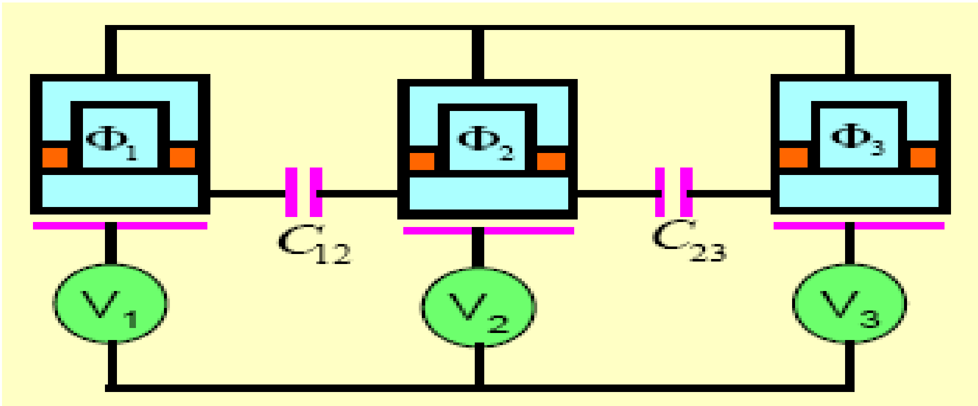
1) Propose an effective dynamical decoupling approach to overcome the "fixed-interaction" difficulty for effectively implementing elemental logical gates for quantum computation.

2) The proposed single-qubit operations and local measurements should allow testing Bell's inequality with a pair of capacitively coupled Josephson qubits.

Generating GHZ states

We propose an efficient approach to produce and control the quantum entanglement of three macroscopic coupled superconducting qubits.

Wei, Liu, Nori,
Phys. Rev. Lett. (June 2006)



We show that their Greenberger-Horne-Zeilinger (GHZ) entangled states can be deterministically generated by appropriate conditional operations.

Contents

Quantum tomography

We propose a method for the *tomographic reconstruction of qubit states* for a general class of solid state systems in which the Hamiltonians are represented by spin operators, e.g., with Heisenberg-, XXZ-, or XY- type exchange interactions.

We analyze the implementation of the projective operator measurements, or spin measurements, on qubit states. All the qubit states for the spin Hamiltonians can be reconstructed by using experimental data.

This general method has been applied to study how to reconstruct any superconducting charge qubit state.

Liu, Wei, Nori, *Europhysics Letters* 67, 874 (2004); *Phys. Rev. B* 72, 014547 (2005); preprint (2007).

Quantum tomography

Quantum states

A single qubit state can be expressed in the basis $\{|0\rangle, |1\rangle\}$ as a density matrix

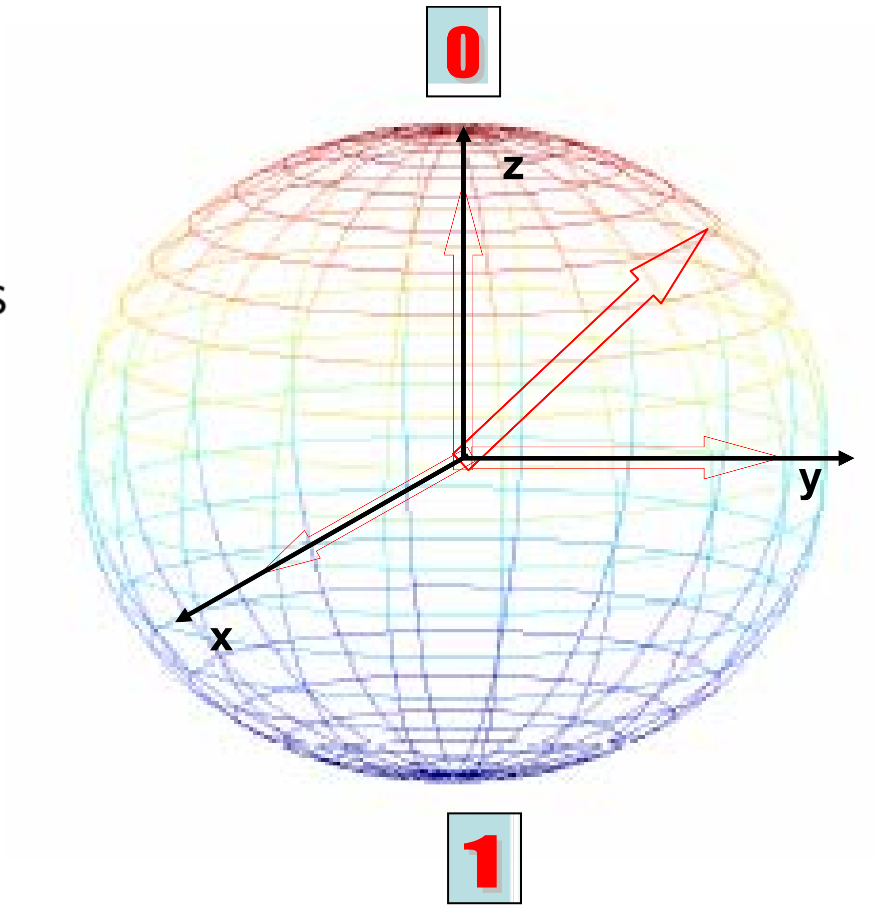
$$\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix},$$

which can be rewritten as

$$\rho = \frac{1}{2}(1 + \sum_k r_k \sigma_k)$$

with three Pauli matrices σ_k (k=x, y, z), and

$$r_z = \rho_{00} - \rho_{11},$$
$$r_x = \rho_{01} + \rho_{10},$$
$$r_y = i(\rho_{01} - \rho_{10}).$$



Liu, Wei, and Nori, Europhys. Lett. 67, 874 (2004)

Quantum tomography

r_k can be determined via measurements of σ_k: $r_k = Tr(\rho\,\sigma_k)$

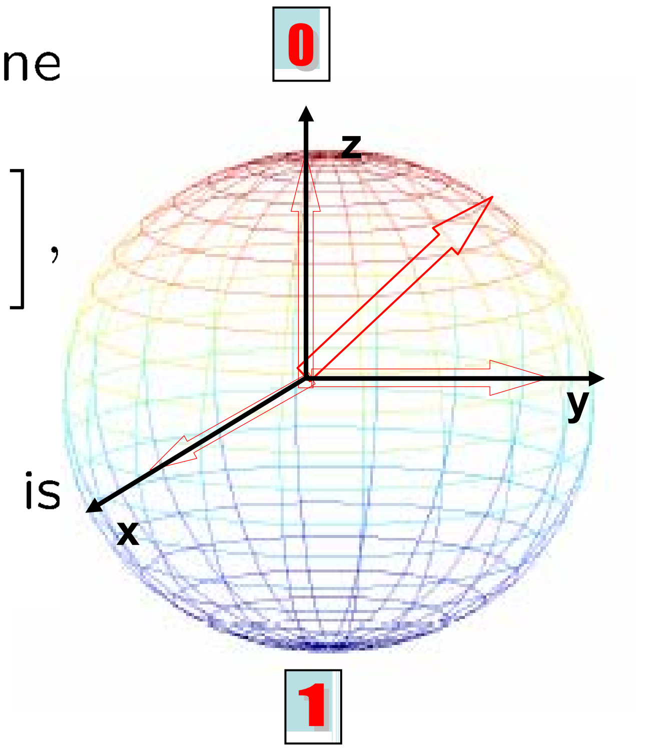
r_z determines the probabilities of $|0\rangle$ and $|1\rangle$.

r_x and r_y determine the relative phase of the state.

The experimental measurement $|1\rangle\langle 1|$ is done along the z axis, that is,

$$|1\rangle\langle 1| = \frac{1}{2}(I - \sigma_z) = \frac{1}{2}\left[\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\right],$$

which is used to obtain r_z.



The resulting probability of measuring $|1\rangle\langle 1|$ is $p = Tr(\rho|1\rangle\langle 1|) = \frac{1}{2}(1 - r_z) = \rho_{11}$.

Quantum tomography

1. r_x and r_y cannot be directly obtained via the experimentally realizable measurement $|1\rangle\langle 1|$.

2. A quantum operation (rotation) W needs to be performed so that the r_x and r_y are transformed to a measurable position.

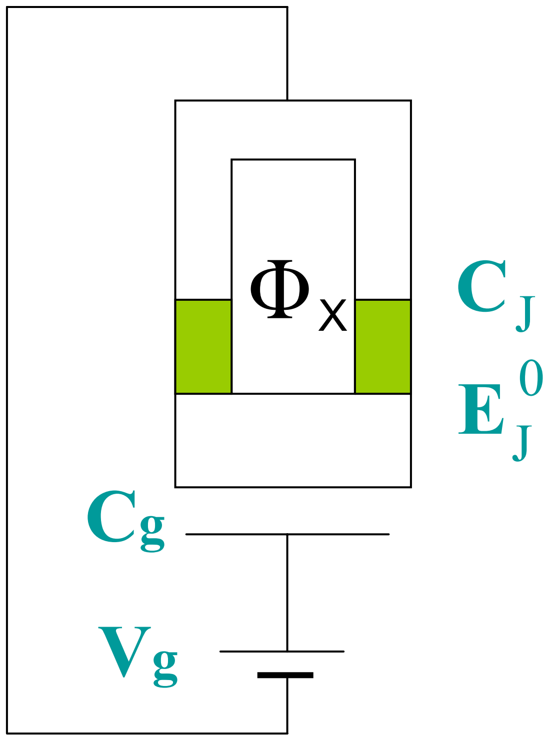
3. After the operation W is made on the qubit state, the measured probability is

$$p = Tr(W\rho W^{\dagger}|1\rangle\langle 1|).$$

4. r_y (r_x) can be obtained by a rotation $\pi/2$ around the x (y) axis.

back

Superconducting charge qubit



Hamiltonian

$$H = -E_{\text{ch}}\,\sigma_z - E_{\text{J}}\,\sigma_x$$

with

$$E_{\text{ch}} = \frac{e^2}{C_{\text{g}}+2C_{\text{J}}}(1-2n_g)$$

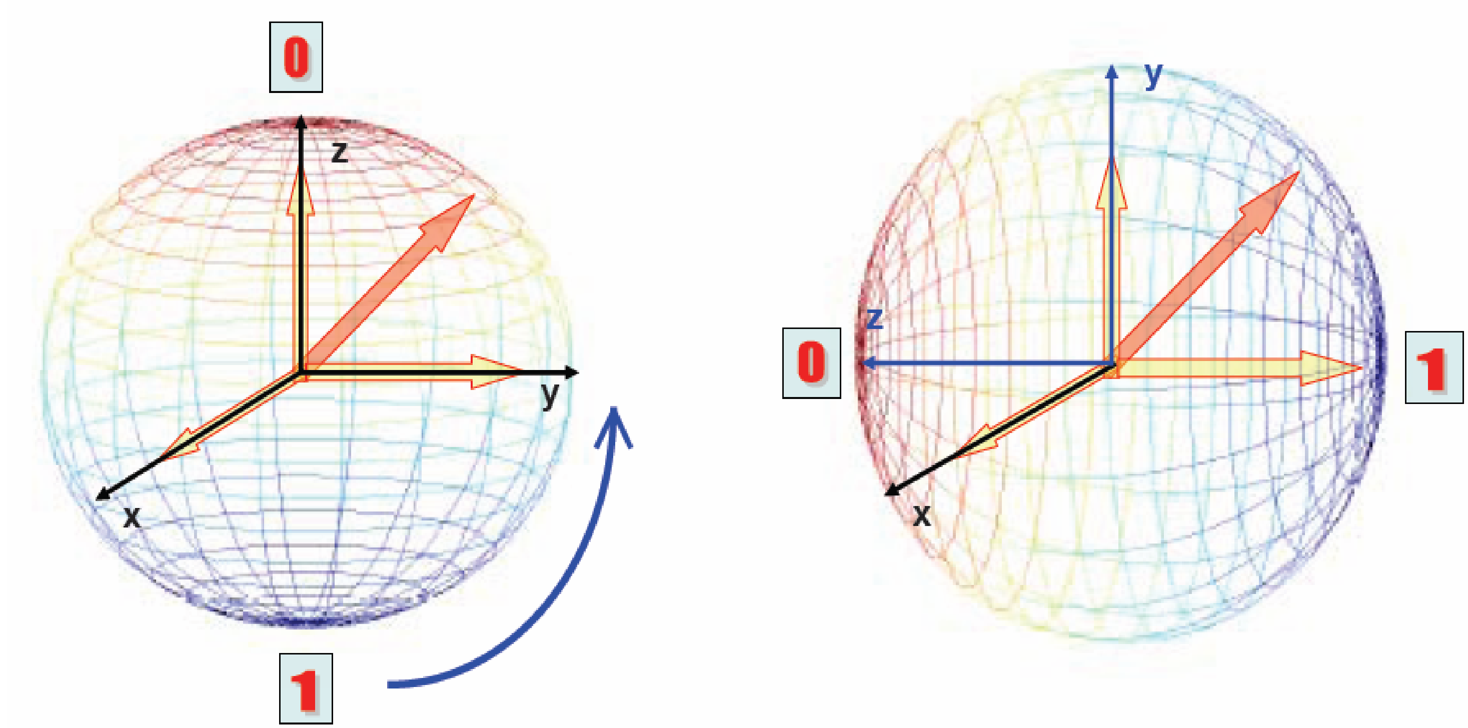
$$E_{\text{J}} = E_{\text{J}}^0 \cos\left(\pi \frac{\Phi_X}{\Phi_0}\right)$$

Quantum tomography for superconducting charge qubits

Liu, Wei, Nori, *Europhysics Letters* 67, 874 (2004); *Phys. Rev. B* 72, 014547 (2005); preprint (2007).

Quantum tomography

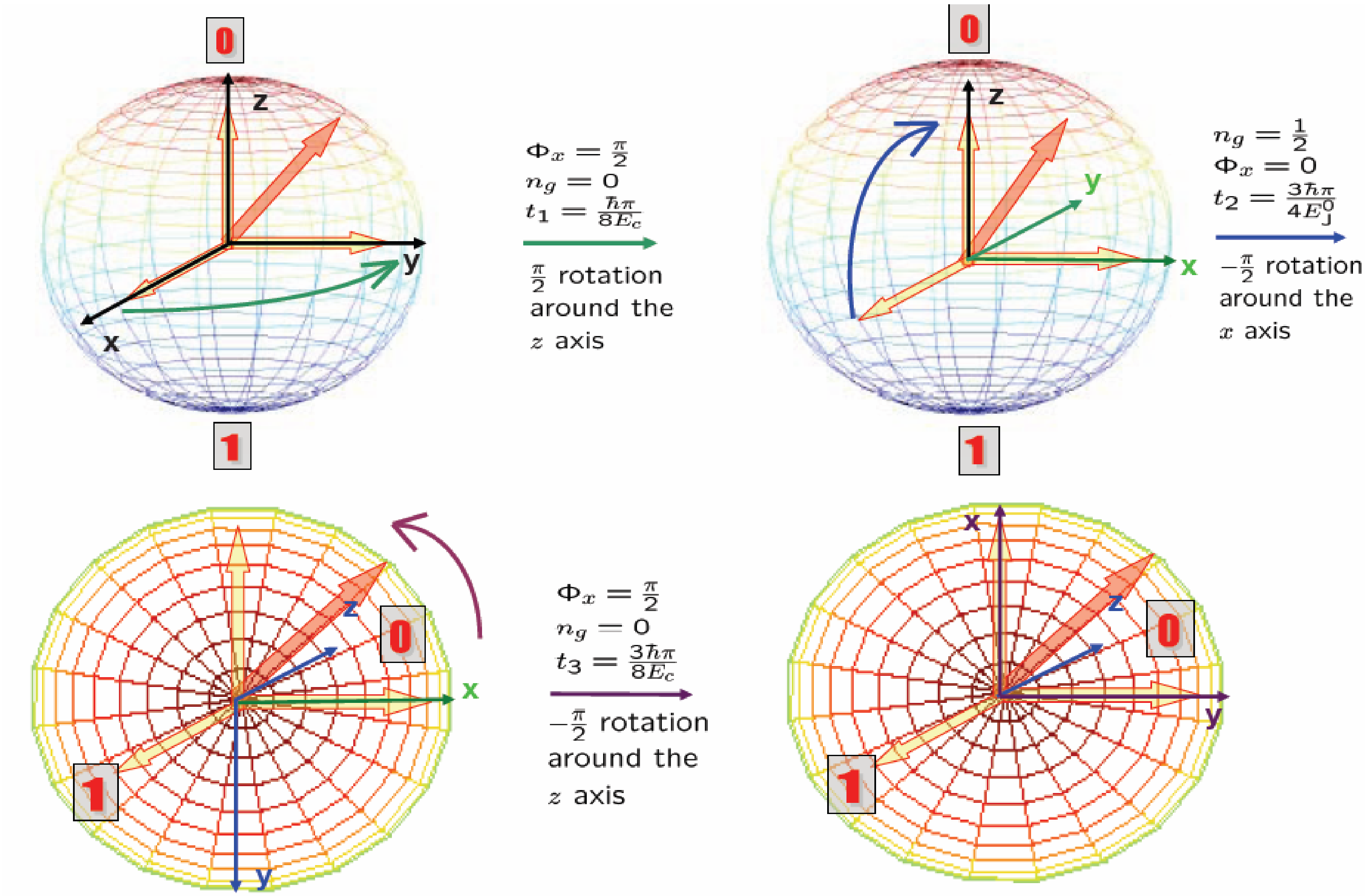
$\frac{\pi}{2}$ rotation around the x axis



This rotation can be realized by setting $\Phi_x = 0$ and $n_{\mathrm{C}} = \frac{1}{2}$ with an evolution time $t_x = \frac{\hbar\pi}{4E_{\mathrm{J}}^0}$.

Quantum tomography

$\frac{\pi}{2}$ rotation around the y axis



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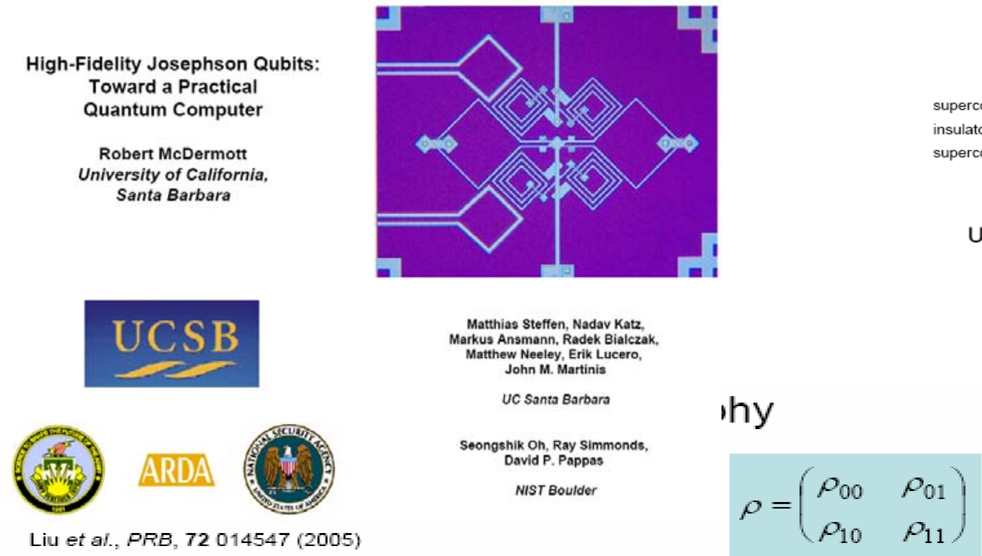
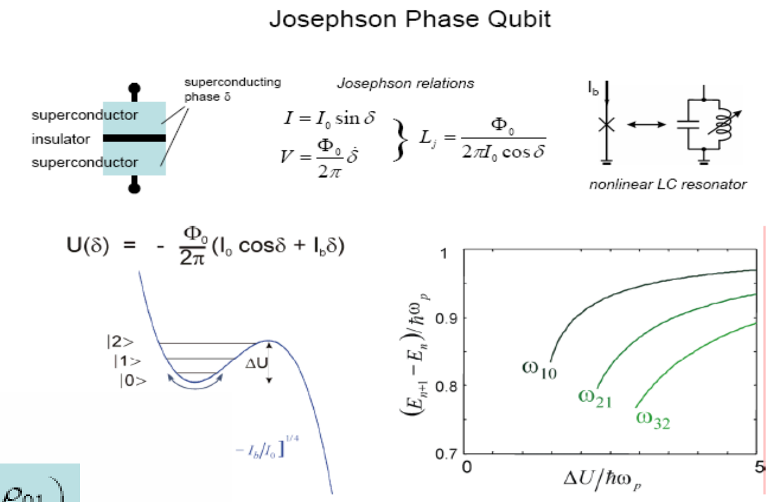
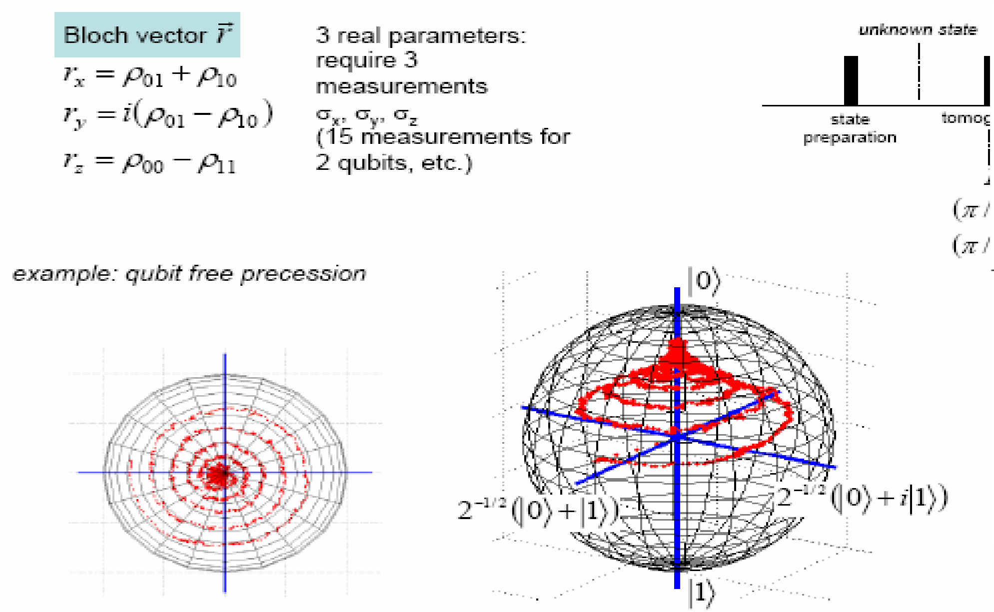
Liu, Wei, Nori, *Europhysics Letters* 67, 874 (2004); *Phys. Rev. B* 72, 014547 (2005); preprint (2007).

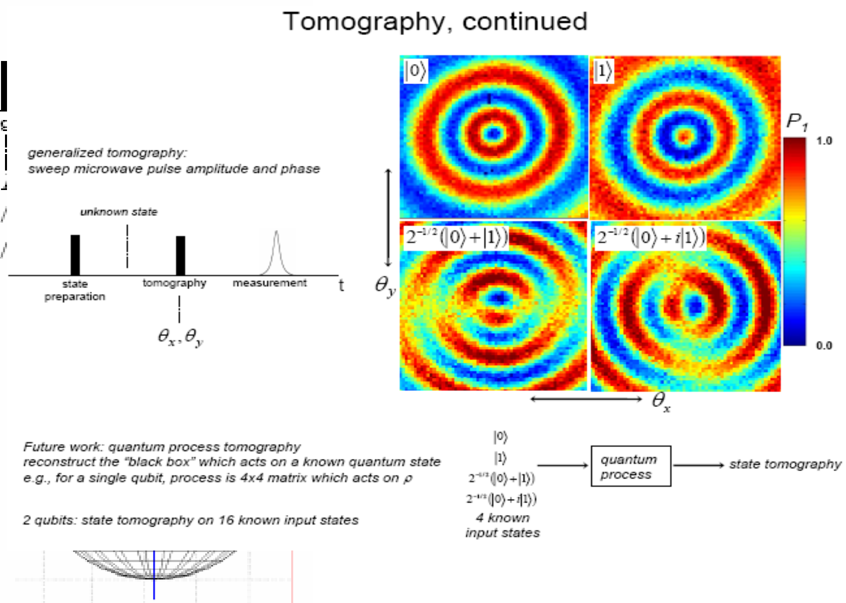
Experiments on quantum tomography in SQs

PRL (2006), Martinis group









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Summary

- Studied SC charge, flux, charge-flux, and phase qubits
- Studied many analogies between atomic physics and SC qubits including ion traps, on-chip micromasers, cyclic transitions, generating photon states using SC qubits
- We proposed and studied circuit QED. It has been verified experimentally, years after our prediction
- Proposed several methods of controllable couplings between different qubits
- Studied how to dynamically decouple qubits with always-on interactions
- Introduced solid state quantum tomography