超対称性の自発的破れの

格子シミュレーションによる測定

2007 年 12 月 23 日 理研シンポジウム「量子場の理論と対称性」

金森 逸作 (理研)

Based on: I.K., Fumihiko Sugino and Hiroshi Suzuki,
arXiv:0711.2132 [hep-lat], arXiv:0711.2099 [hep-lat]

1. Introduction
2. Idea
3. Example: SQM
4. Two-dimensional $N = (2, 2)$ SYM
5. Conclusion

1 Introduction

Spontaneous SUSY breaking should be treated non-perturbatively:

not broken in tree level $\Rightarrow$ not broken in all orders

- Witten index: *spontaneously broken/unbroken*
- *Not* available in some models:
  - 2-dim $N = (2, 2)$ pure SYM (maybe broken? Hori-Tong)

Lattice regularization of supersymmetric field theories:

- formulation

  (SYM) CKKU, Sugino, Catterall, DKKN, ST,...
- Monte Carlo simulation $\Rightarrow$ it does work

  Catterall, Suzuki, FKST

Our work:

Observe spontaneous (non-)breaking of SUSY via lattice simulation

Plan of this talk

2 Idea

The order parameter of the SUSY breaking: Hamiltonian

$H = 0$: SUSY $\qquad$ $H > 0$: ~~SUSY~~

The conjugate applied field: temperature $\qquad Z = \mathrm{tr}\, e^{-\beta H}$

$\Rightarrow$ *anti*-periodic boundary condition for fermions

$H = 0$ comes form the SUSY algebra ($\langle \Omega | H | \Omega \rangle = \left|\left| Q | \Omega \rangle \right|\right|^2$)

- algebra: $\{Q, \overline{Q}\} = i\partial_0$
- Noether charge for $\overline{Q}$: $\overline{\mathcal{Q}}$

$\Rightarrow$ $\boxed{H \equiv \frac{i}{2} Q \overline{\mathcal{Q}}}$

$H = H_{\mathrm{canonical}} + (\text{e.o.m})$

gauge invariant H

(later)

We should measure

$$\boxed{E_0 \equiv \lim_{\beta \to \infty} \langle H \rangle_{\mathrm{aPBC}}} \left(= \lim_{\beta \to \infty} \frac{\mathrm{tr}\, H e^{-\beta H}}{\mathrm{tr}\, e^{-\beta H}}\right)$$

The action must be Q-invariant (Q can be kept on the lattice)

Note: periodic boundary condition

Under the periodic condition, $\langle H \rangle$ is always 0 or indefinite

$$Z_{PBC} = N \int_{\mathrm{PBC}} d\mu\, e^{-S} = \mathrm{tr}(-1)^F e^{-\beta H}$$

Fujikawa Z.Phys.C15(1982)275

$$= \text{Witten index} = \#(E = 0 \text{ state})$$

$$\Rightarrow \langle H \rangle_{\mathrm{PBC}} = \frac{N \int_{\mathrm{PBC}} d\mu\, H e^{-S}}{Z_{\mathrm{PBC}}}$$

$$= \frac{\mathrm{tr}(-1)^F H e^{-\beta H}}{Z_{PBC}} = \frac{-\frac{\partial}{\partial\beta}\mathrm{tr}(-1)^F e^{-\beta H}}{Z_{\mathrm{PBC}}} = \frac{0}{Z_{\mathrm{PBC}}}$$

If $Z_{\mathrm{PBC}} = 0$, $\langle H \rangle_{\mathrm{PBC}}$ is indefinite; simulation does not work

Q-exact H: If the action and the measure are invariant under Q,

$$\int_{\mathrm{PBC}} d\mu\, H e^{-S} = \int_{\mathrm{PBC}} d\mu\, Q\left(\frac{i}{2}\overline{Q} e^{-S}\right) = 0$$

3 Example: SQM

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}(W')^2 + \overline{\psi}(i\partial - W'')\psi + \frac{1}{2}F^2 = Q\left(\frac{1}{2}\overline{\psi}(F - i\partial\phi + W')\right) + \partial(\dots)$$

superpotential $W = W(\phi(x))$

$W(-\infty)W(+\infty) < 0$: SUSY is *broken*

$W(-\infty)W(+\infty) > 0$: SUSY is *kept*

algebra: $Q^2 = \overline{Q}^2 = 0 \qquad \{Q, \overline{Q}\} = i\partial_0$

Noether charge:

$\overline{\mathcal{Q}} = -\overline{\psi}(\partial\phi - iW')$

$$\left[\begin{array}{ll} Q\phi = \psi & Q\psi = 0 \\ Q\overline{\psi} = F + i\partial\phi - W' & QF = -i\partial\psi + W''\psi \\ \overline{Q}\,\phi = \overline{\psi} & \overline{Q}\,\overline{\psi} = 0 \\ \overline{Q}\psi = -F + i\partial\phi + W' & \overline{Q}F = i\partial\overline{\psi} + W''\overline{\psi} \end{array}\right]$$

Construction of Hamiltonian

Hamiltonian

$$H = iQ\overline{\mathcal{Q}}/2$$

$$= \underbrace{\frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(W')^2 + \overline{\psi}W''\psi - \frac{1}{2}F^2}_{=H_{\text{canonical}}} + \overbrace{\frac{1}{2}F(F - i\partial\phi - W')}^{\text{(e.o.m)}}$$

$$+ \frac{1}{2}\overline{\psi}(i\partial - W'')\psi$$

After Wick rotation, we obtain the Euclidean Hamiltonian (real time $\rightarrow$ imaginary time)

- Nilpotent Q
- Q-exact action
- Q-exact Hamiltonian

SQM on the lattice

(Euclidean)

Nilpotent Q

$$Q\phi(x) = \psi(x) \qquad Q\psi(x) = 0$$

$$Q\overline{\psi}(x) = F(x) - \partial\phi(x) - W'(\phi(x)) \qquad QF(x) = \partial\psi(x) + W''(\phi(x))\psi(x)$$

$$\partial\phi(x) = \phi(x+a) - \phi(x)$$

Q-exact action

Catterall JHEP 05(2003)038

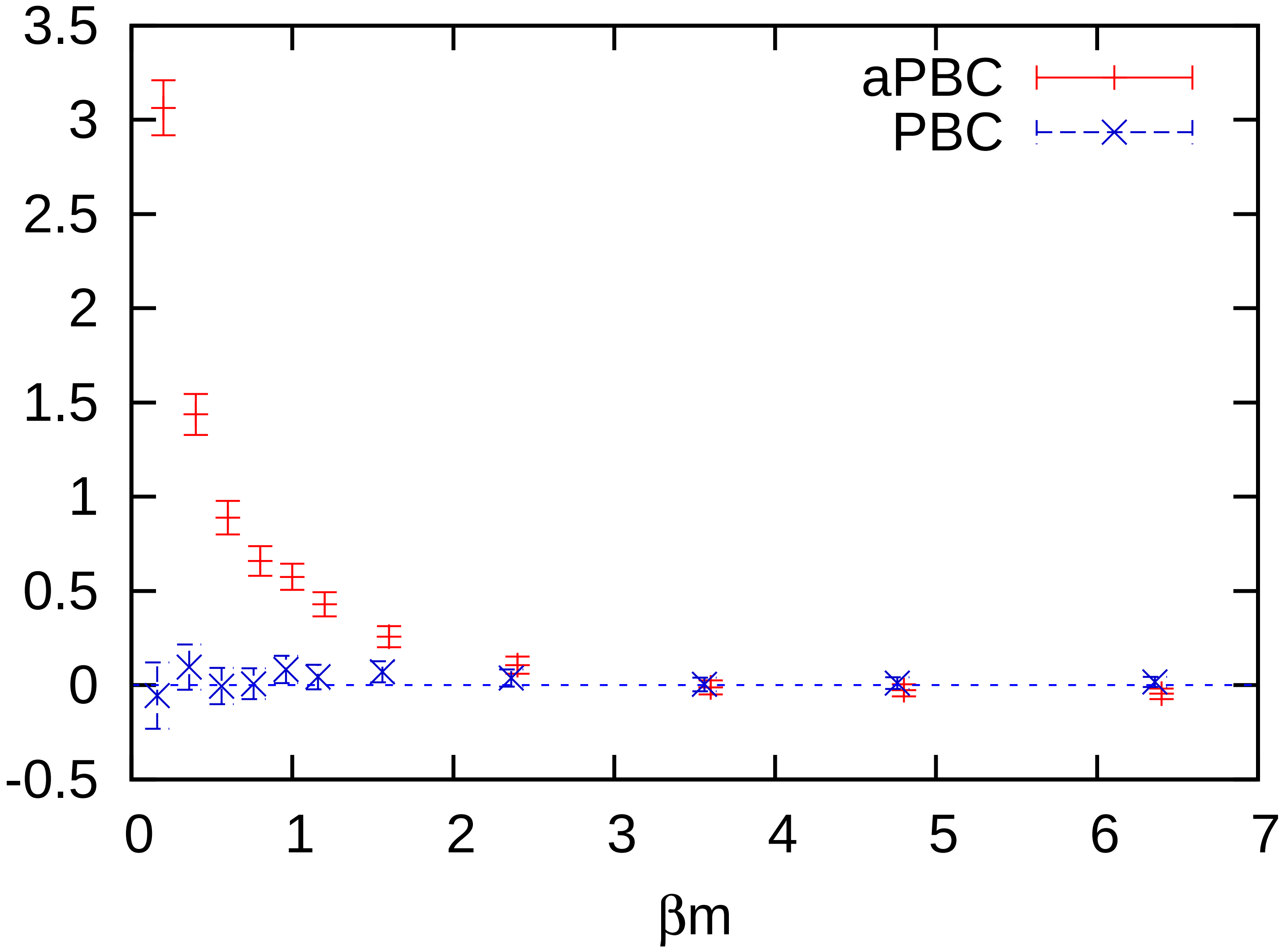
$$S \equiv -Q\sum_x \frac{1}{2}\overline{\psi}(x)\left[F(x) + \partial\phi(x) + W'(\phi(x))\right]$$

Q-exact Hamiltonian

$$H(x) \equiv \frac{i}{2}Q\underbrace{\left[-\frac{1}{a}\overline{\psi}(x)\left\{i\partial\phi(x) - iW'(\phi(x))\right\}\right]}_{\text{descritization of continuum } \overline{\mathcal{Q}}}$$

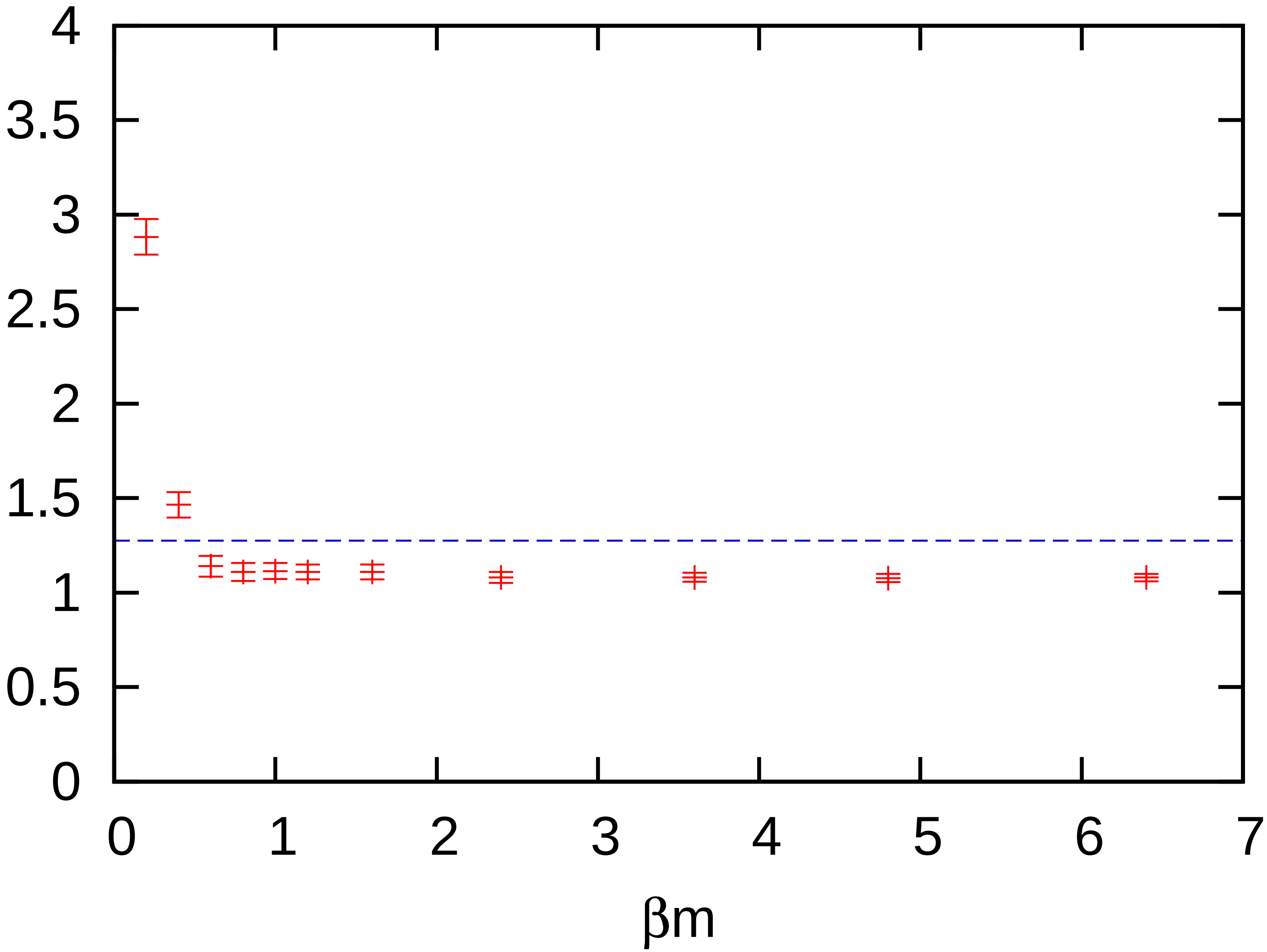
Result of SQM (no SUSY breaking)

$$W = \frac{1}{4} m^2 \phi^4$$



Result of SQM (SUSY breaking)

$$W = \frac{1}{2}m\phi^2 + \frac{1}{3}g\phi^3$$



4 Two-dimensional $N=(2,2)$ SYM

Q-exact Lagrangian (continuum, twisted fermion basis)

$$\mathcal{L} = Q(\dots) = \frac{1}{g^2}\,\mathrm{tr}\Big\{-\frac{1}{4}[\phi,\overline{\phi}]^2 - H^2 + 2HF_{01} + D_0\phi D_0\overline{\phi} - D_1\phi D_1\overline{\phi}$$
$$+\frac{1}{4}\eta[\phi,\eta] + \cdots - \psi_0 D_0\eta + \dots\Big\}$$

nilpotent Q (Twisted) SUSY Algebra

$$Q^2 = \delta_{\phi}^{(\mathrm{gauge})} \qquad Q_0^2 = -\delta_{\overline{\phi}}^{(\mathrm{gauge})} \qquad \{Q, Q_0\} = -2\partial_0 - 2i\delta_{A_0}^{(\mathrm{gauge})}$$

$$QA_0 = i\psi_0 \qquad Q\psi_0 = D_0\phi \qquad Q_0A_0 = \frac{i}{2}\eta \qquad Q_0\eta = -2D_0\overline{\phi}$$
$$QA_1 = \psi_1 \qquad Q\psi_1 = iD_1\phi \qquad Q_0A_1 = -\chi \qquad Q_0\chi = iD_1\overline{\phi}$$
$$Q\phi = 0 \qquad \dots \qquad Q_0\overline{\phi} = 0 \qquad \dots$$

$Q_0 \Rightarrow$ Noether current $\mathcal{J}_0$

Hamiltonian density

$$\mathcal{H} = Q\mathcal{J}_0^0/2 = Q\frac{1}{g^2}\,\mathrm{tr}\left\{\frac{1}{2}\eta[\phi,\overline{\phi}] + 2\chi H + 2\psi_0 D_0\overline{\phi} - 2i\psi_1 D_1\overline{\phi}\right\}$$

$$= \frac{1}{g^2}\,\mathrm{tr}\left\{\frac{1}{4}[\phi,\overline{\phi}]^2 + H^2 + D_0\phi D_0\overline{\phi} + D_1\phi D_1\overline{\phi} - \psi_0 D_0\eta + i\psi_1 D_1\eta \right.$$

$$\left. - \frac{1}{4}\eta[\phi,\eta] - \chi[\phi,\chi] - \psi_0[\overline{\phi},\psi_0] + \psi_1[\overline{\phi},\psi_1]\right\}$$

This construction gives Q-invariant and gauge invariant Hamiltonian

cf. $\mathcal{H} = \mathcal{H}_{\mathrm{canonical}} + ($e.o.m$)$

Hamiltonian density(cont.)

$$\mathcal{H} = \mathcal{H}_{\text{canonical}} + (\text{e.o.m})$$

$$= \frac{1}{g^2}\,\mathrm{tr}\Big(\frac{1}{4}[\phi,\overline{\phi}]^2 + H^2 + \partial_0\phi\partial_0\phi + \ldots\Big) \qquad \mathcal{H}_{\text{cannonical}}$$

$$+ 2\,\mathrm{tr}(A_0\mathcal{G}) + \frac{1}{g^2}\,\mathrm{tr}\big\{\psi_0(-2[\overline{\phi},\psi_0] + 2iD_1\chi - D_0\eta)\big\} \qquad (\text{e.o.m})$$

$\mathcal{G}$: Gauss law constraint

$$\mathcal{G} = \frac{1}{g^2}\left\{D_1H + \frac{i}{2}[\phi, D_0\overline{\phi}] + \frac{i}{2}[\overline{\phi}, D_0\phi] + i\{\psi_1,\chi\} + \frac{i}{2}\{\eta,\psi_0\}\right\}$$

canonical momentum: *not* gauge covariant

$$\pi_A = \frac{\partial}{\partial(\partial_0\phi_A)}\int dx\mathcal{L}$$

SYM on the lattice

Sugino, JHEP 01(2004)067

Nilpotent Q $Q^2 = [\phi, \ \cdot\ ] = \delta_\phi^{(\text{gauge})}$

$$QU(x,\mu) = i\psi_\mu(x)U(x,\mu)$$

$$Q\psi_\mu(x) = i\psi_\mu(x)\psi_\mu(x) - i\big(\phi(x) - U(x,\mu)\phi(x+a\hat{\mu})U(x,\mu)^{-1}\big)$$

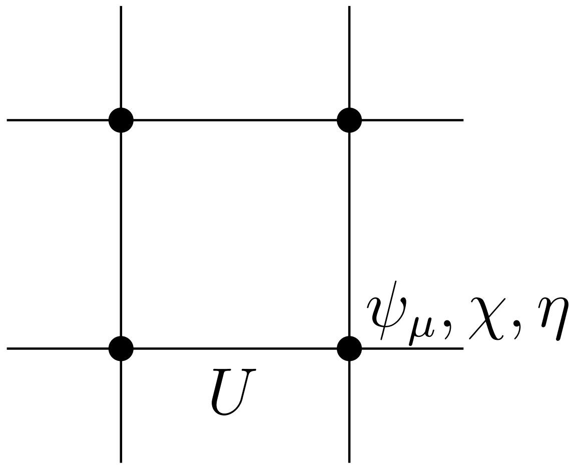
$$Q\phi(x) = 0$$

$$Q\chi(x) = H(x)$$

$$QH(x) = [\phi(x), \chi(x)]$$

$$Q\overline{\phi}(x) = \eta(x)$$

$$Q\eta(x) = [\phi(x), \overline{\phi}(x)]$$



SYM on the lattice

Q-invariant action ($SU(N_C)$)

$$S = Q\frac{1}{a^2g^2}\sum_x \mathrm{tr}\Big[\chi(x)H(x) + \frac{1}{4}\eta(x)[\phi(x),\overline{\phi}(x)] - i\chi(x)\hat{\Phi}(x)$$

$$+ i\sum_{\mu=0,1}\left\{\psi_\mu(x)\left(\overline{\phi}(x) - U(x,\mu)\overline{\phi}(x+a\hat{\mu})U(x,\mu)^{-1}\right)\right\}\Big]$$

$$= \frac{1}{a^2g^2}\sum_x \mathrm{tr}\left[\frac{1}{4}\hat{\Phi}_{\mathrm{TL}}(x)^2 + \dots \qquad\right]$$

$$i\hat{\Phi}(x) = \frac{U(x,0,1) - U(x,0,1)^{-1}}{1 - \frac{1}{\epsilon^2}||1 - U(x,0,1)||^2} \sim 2iF_{01} \quad \Rightarrow ||1 - U(x,0,1)|| < \epsilon$$

To suppress lattice artifact “vacua”, we need:

$0 < \epsilon < 2\sqrt{2}$ for $N_C = 2, 3, 4$

$0 < \epsilon < 2\sqrt{N_C}\sin(\pi/N_C)$ for $N_C > 5$

SYM on the lattice

Q-exact Hamiltonian

Discretized “Noether current”:

$$
\begin{aligned}
\mathcal{J}_0^0(x) = \frac{1}{a^4 g^2}\,\mathrm{tr}\Big\{ &\eta(x)[\phi(x),\overline{\phi}(x)]^2 + 2\chi(x)H(x) \\
&- 2i\psi_0(x)\big(\overline{\phi}(x) - U(x,0)\overline{\phi}(x+a\hat{0})U(x,0)^{-1}\big) \\
&+ 2i\psi_1(x)\big(\overline{\phi}(x) - U(x,1)\overline{\phi}(x+a\hat{1})U(x,1)^{-1}\big)\Big\}
\end{aligned}
$$

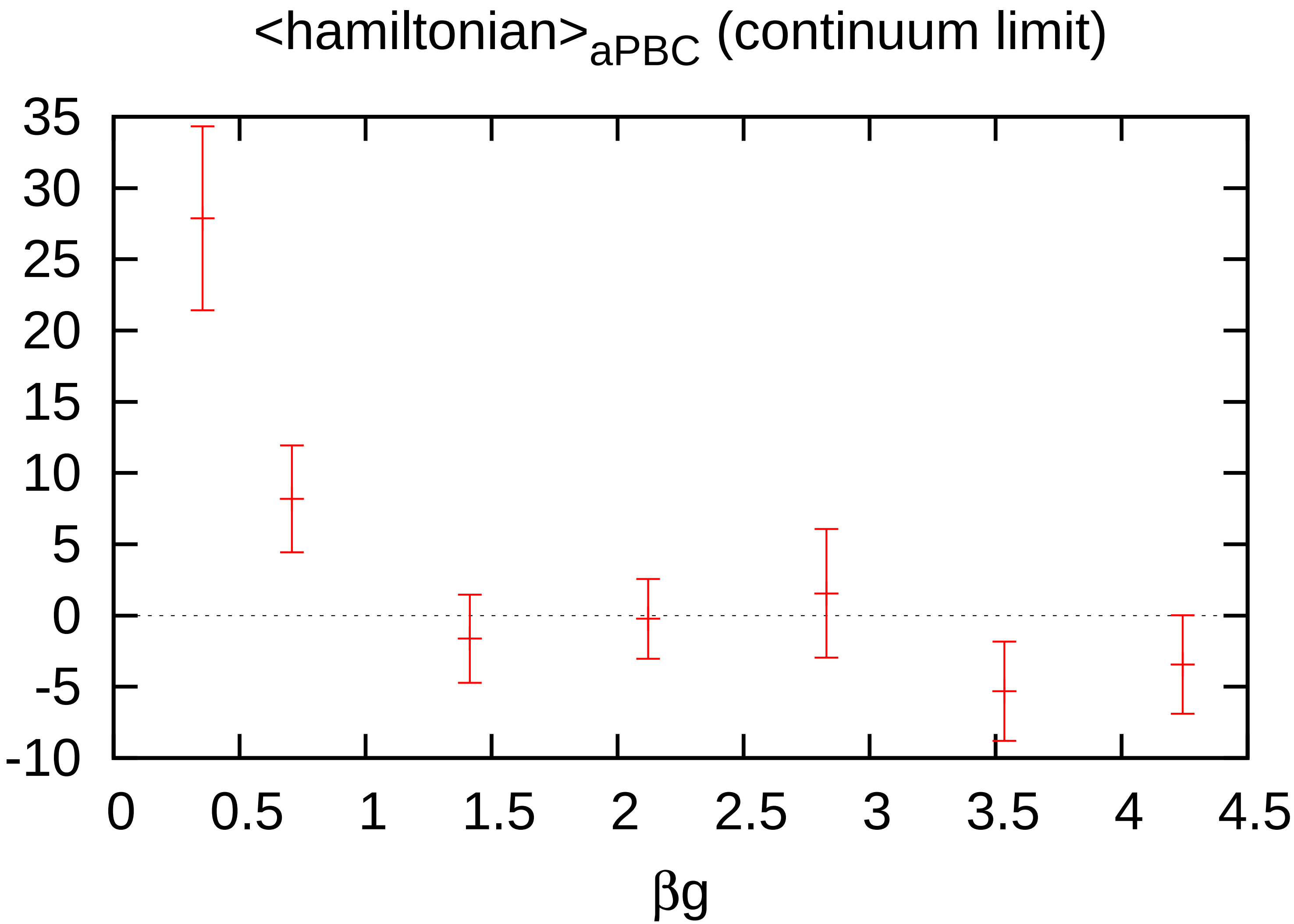
$$
\boxed{\mathcal{H}(x) = Q\mathcal{J}_0^0/2}
$$

Monte Carlo simulation

$\langle\mathcal{H}\rangle$ under *anti*-periodic boundary condition

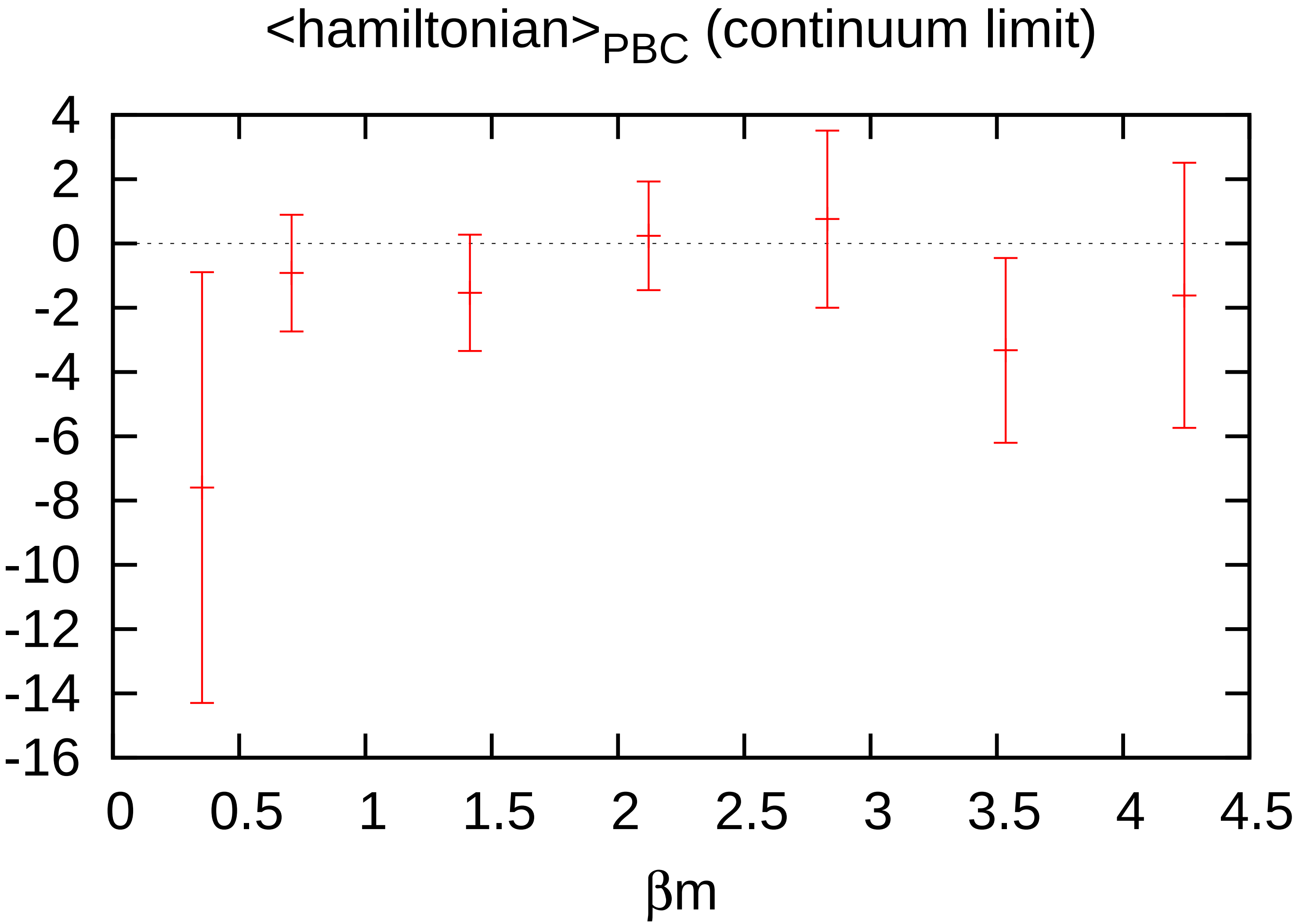
- quench + reweight: $S = S_b + S_f$, $\langle\mathcal{O}\rangle_S = \dfrac{\langle\mathcal{O}\mathrm{Pf}(D)\rangle_{S_b}}{\langle\mathrm{Pf}(D)\rangle_{S_b}}$
$$Z = \int \mathcal{D}f\,\mathcal{D}b\,e^{-S_b-S_f} = \int \mathcal{D}b\,\mathrm{Pf}(D)e^{-S_b}$$
- gauge grope: $SU(2)$
- fixed spatial physical length: $gL = 1.414$
- lattice size: 3×6 – 36×12
- lattice spacing: ag = 0.2357 – 0.0707
- 9900–99900 independent configurations for each parameter
- Computer: RIKEN Super Combined Cluster

Result



$\langle \mathcal{H} \rangle_{\mathrm{aPBC}} \longrightarrow 0$, SUSY is not spontaneously broken

Result

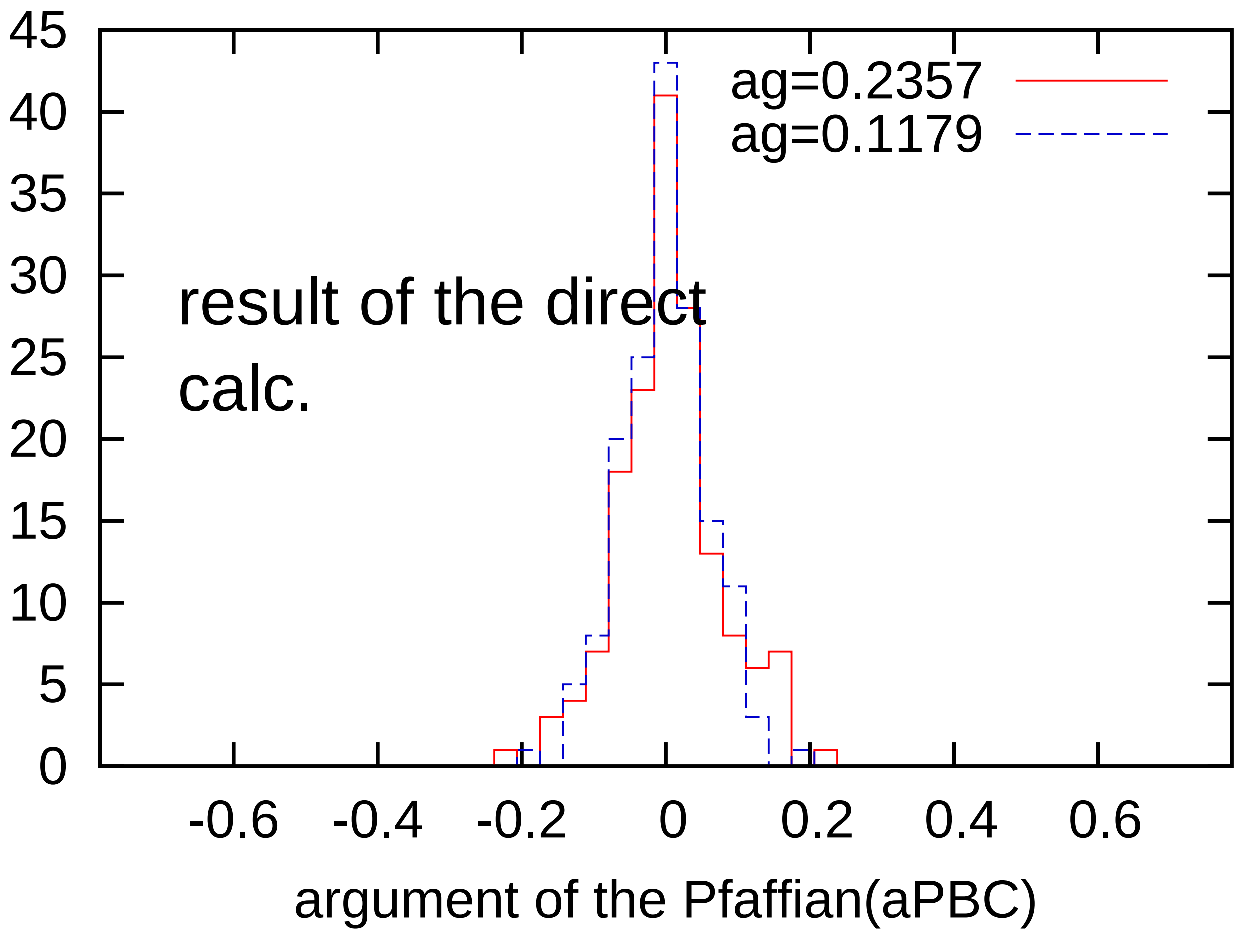


$\langle \mathcal{H} \rangle_{\mathrm{PBC}} = 0$ within error.

$\Rightarrow$consistent with $\langle \mathcal{H} \rangle_{\mathrm{PBC}} \propto \frac{\partial}{\partial \beta}$(Witten index)

Details: more about reweighting of Pfaffian

The Pfaffian is real positive in the continuum: argument $= 0$



$\Rightarrow$ practically,
$\mathrm{Pf} = +\sqrt{\mathrm{Det}}$
$(-\pi/2 < \theta < \pi/2)$,
less computation time

$$Z_{\mathrm{aPBC}} = N \int_{\mathrm{aPBC}} \mathcal{D}b\,\mathcal{D}f\,e^{-S_b - S_f} = N \int_{\mathrm{aPBC}} \mathcal{D}b\,\mathrm{Pf}\,e^{-S_b} \neq 0$$

Details: effect of the boundary condition (SQM)

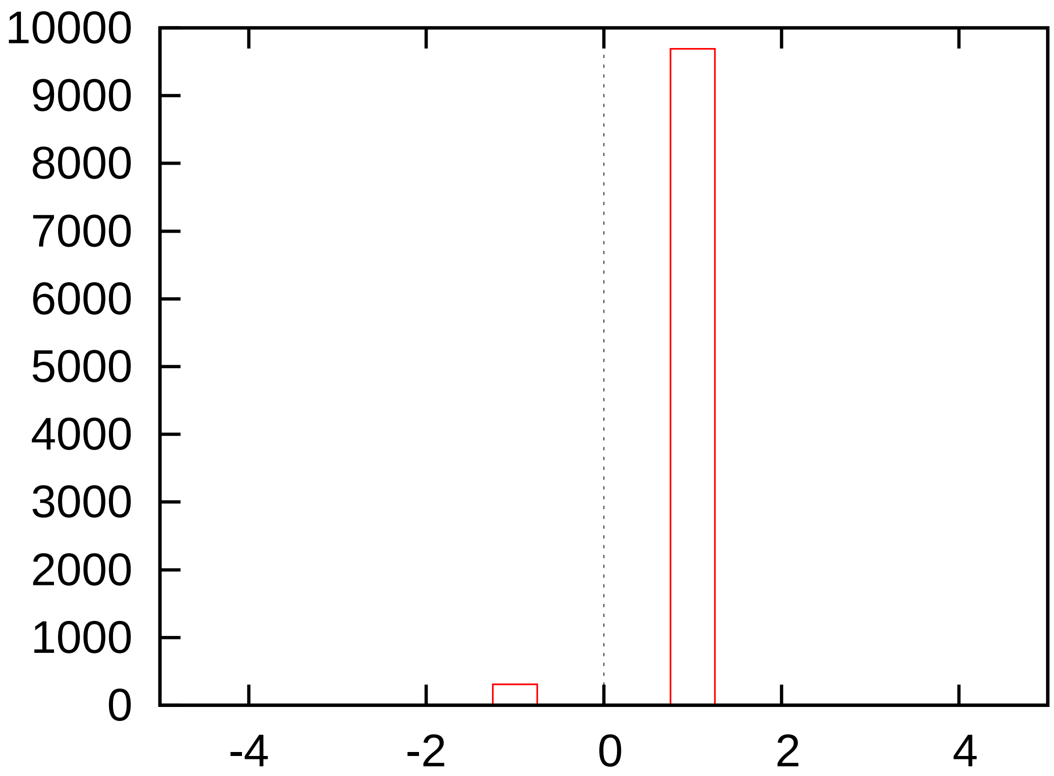
$$Z = N \int \mathcal{D}b\,\mathcal{D}f\,e^{-S_b - S_f}$$

$$= N \int \mathcal{D}b\,\mathrm{Det}\,e^{-S_b} = N \int \mathcal{D}b\,\mathrm{sign}(\mathrm{Det})\,e^{-S_b - \ln|\mathrm{Det}|}$$

Details: effect of the boundary condition (SQM)

$$Z = N\int \mathcal{D}b\,\mathcal{D}f\, e^{-S_b - S_f}$$

$$= N\int \mathcal{D}b\,\mathrm{Det}\, e^{-S_b} = N\int \mathcal{D}b\,\mathrm{sign}(\mathrm{Det})\, e^{-S_b - \ln|\mathrm{Det}|}$$



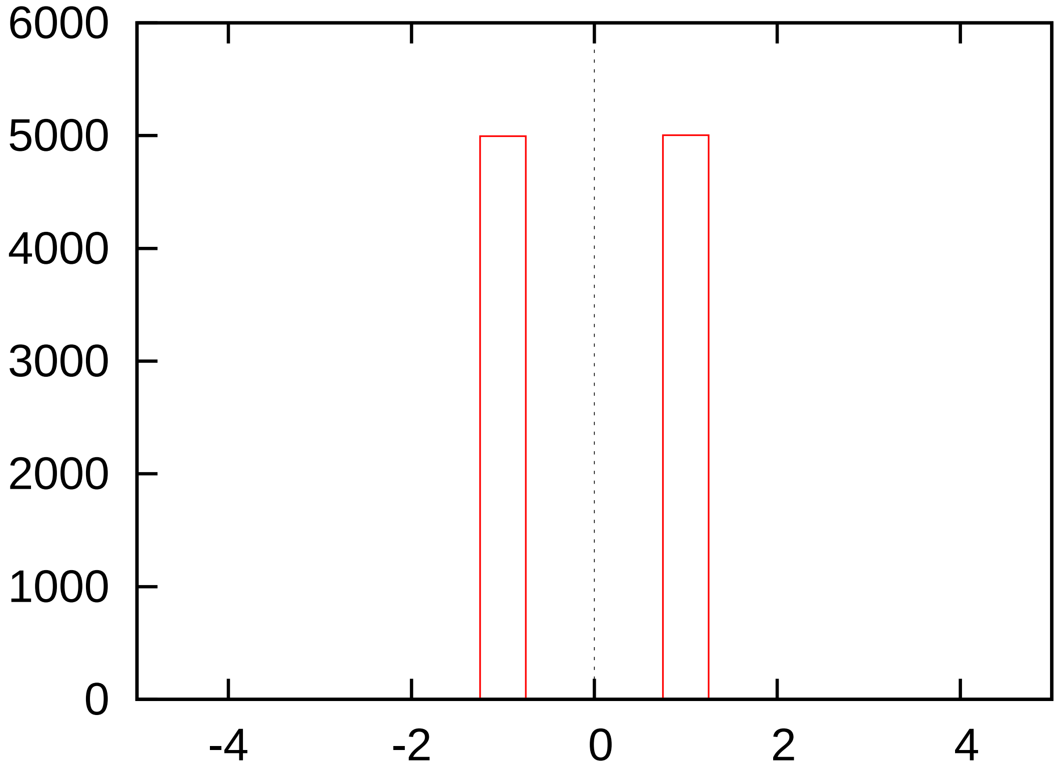
Histogram of sing(Det)

SUSY breaking potential
anti-PBC case

Details: effect of the boundary condition (SQM)

$$Z = N \int \mathcal{D}b\,\mathcal{D}f\, e^{-S_b - S_f}$$

$$= N \int \mathcal{D}b\,\mathrm{Det}\, e^{-S_b} = N \int \mathcal{D}b\,\mathrm{sign}(\mathrm{Det})\, e^{-S_b - \ln|\mathrm{Det}|}$$



Histogram of sing(Det)

SUSY breaking potential

PBC case

$Z_{\mathrm{PBC}} = 0$

$\Downarrow$

numerically, expectation value is ill-defined

5 Conclusion

We have developed a method to observe spontaneous SUSY breaking using lattice simulation

- Lattice model with one exactly kept nilpotent Q
- Algebraic construction of the Hamiltonian($\Rightarrow \int_{\mathrm{PBC}} d\mu\, He^{-S} = 0$)
- Measure H at finite temperature (anti-PBC), then take $\beta \to \infty$

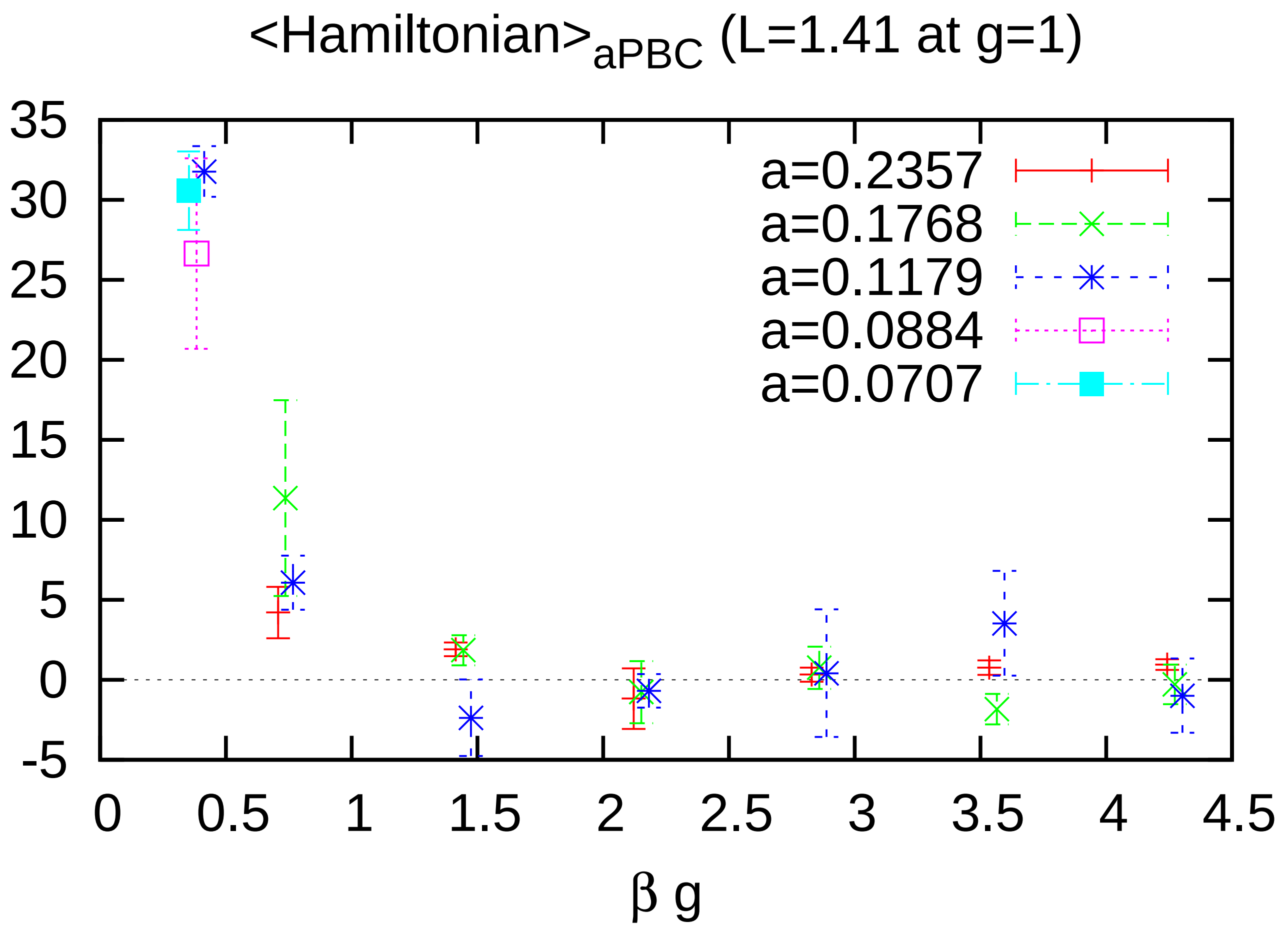
Two-dimensional $N = (2, 2)$ pure SYM: SUSY is *not* broken

First physical result with recent development of lattice SUSY

Future works

- Simulation with reweighting⇒pseudo fermion (less error)
- Other models: Two-dimensional $N = (4, 4)$ SYM, SYM with matter,...

Taking the continuum limit



Taking the continuum limit