Notes on the Construction of the D2-brane from Multiple D0-branes

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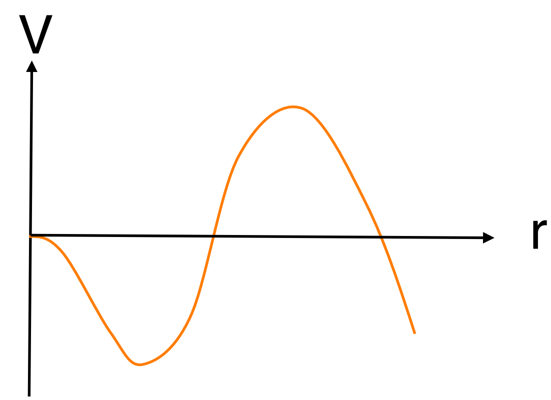
1. Introduction

Dielectric Effect

Let us consider a spherical D2-brane with M units of magnetic flux on its world-volume in the b.g. of constant 4-form R-R flux $G^{(4)}_{0123} = -4h$.

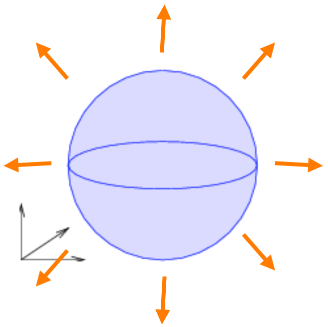
The potential energy is given by

$$V_{D2} = \underbrace{\sqrt{(MT_0)^2 + (4\pi r^2 T_2)^2}}_{\text{BI}} - \underbrace{\frac{16\pi h T_2 r^3}{3}}_{\text{CS}}$$



The b.g. R-R 4-form flux supports the spherical D2-brane configuration against collapse.

→ dielectric effect



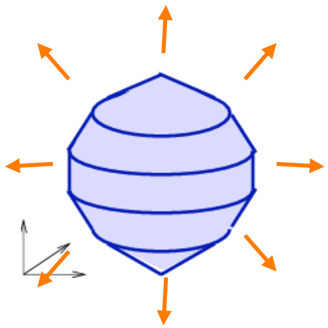
Myers Effect

Let us consider M D0-branes in the b.g. of constant 4-form R-R flux

. $G^{(4)}_{0123} = -4h$

This system becomes stable when D0-branes form a fuzzy sphere.

⟶ Myers effect



Then the potential energy is written as

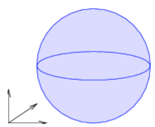
$$V_{\mathrm{D0}} = \underbrace{\sqrt{(MT_0)^2 + (4\pi r^2 T_2)^2 (1 - \frac{1}{M^2})}}_{\text{nBI}} - \underbrace{\frac{16\pi h T_2 r^3}{3}(1 - \frac{1}{M^2})}_{\text{nCS}}$$

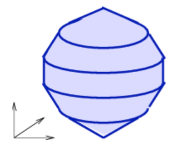
If M is sufficiently large, the potential energy of D0-branes coincides with that of D2-brane.

In the large M limit

Dielectric effect ~ Myers effect

D2 with mag. flux / fuzzy D0s





D2-brane can be constructed from D0-branes via Myers effect.
However, this is the correspondence of classical configurations.

Q. Can we realize gauge and scalar fields on the D2-brane from D0-branes? (nonabelian Born-Infeld $\rightarrow$ abelian Born-Infeld)

cf. operator formulation in the frameworks of BFSS, IKKT, IT, K matrix models etc.

$$[X^1, X^2] = -i\theta \quad \Rightarrow \quad X^1 \sim -i\partial_x,\ X^2 \sim \theta x$$

$$or\ X^1 \sim \theta(-i\partial_x + A_x),\ X^2 \sim \theta(-i\partial_y + A_y),\ F_{xy} = 1/\theta$$

$$or\ X^1 \sim x - \theta A_y,\ X^2 \sim y + \theta A_x,\ with * -product$$

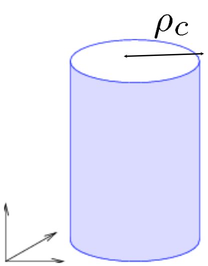
2. Some Comments on Fuzzy Surfaces with Axial Sym.

2.1 The fuzzy cylinder

The matrix equations for the fuzzy cylinder are obtained from those for the smooth cylinder as follows.

smooth cylinder :

$$\begin{cases} -\infty < x^3 < \infty \\ (x^1)^2 + (x^2)^2 = \rho_c^2 \end{cases}$$



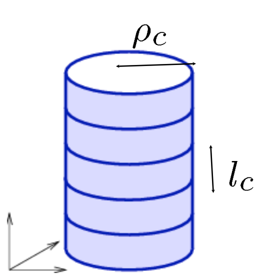
matrix →

$$\begin{cases} -\infty < X^3_{mm} < X^3_{m+1m+1} < \infty \quad (m \in \mathbb{Z}) \\ (X^1)^2 + (X^2)^2 = \rho_c^2 \mathbf{1}_\infty \end{cases}$$

The following matrices satisfy the above matrix equations.

$$X^1_{mn} = \frac{1}{2}\rho_c \delta_{m+1,n} + \frac{1}{2}\rho_c \delta_{m,n+1},$$
$$X^2_{mn} = \frac{i}{2}\rho_c \delta_{m+1,n} - \frac{i}{2}\rho_c \delta_{m,n+1},$$
$$X^3_{mn} = m l_c \delta_{m,n}$$

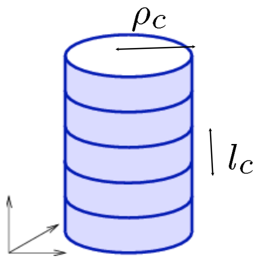
→ fuzzy cylinder



To confirm the image of the fuzzy cylinder, we employ the area formula.

$$A = 2\pi\,\mathrm{Tr}\left(\sqrt{-\frac{1}{2}[X^i, X^j]^2}\right) = \sum_m 2\pi\rho_c l_c, \quad i,j = 1,2,3$$

This shows that the each segment occupies the constant area $2\pi a$.



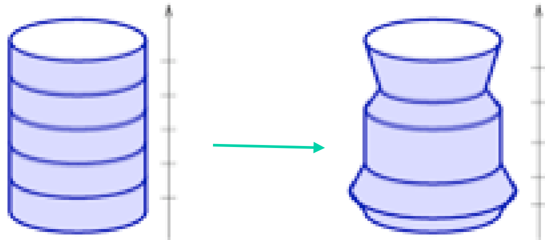
$$2\pi a \equiv 2\pi\rho_c l_c$$

The algebra which represents the fuzzy cylinder is given by

$$[X^1, X^2] = 0, \quad [X^2, X^3] = i l_c X^1, \quad [X^3, X^1] = i l_c X^2$$

2.2 The fuzzy surface with axial symmetry

The fuzzy surface with axial symmetry is obtained by deforming the fuzzy cylinder as follows.



Then matrices are written as

$$X^1_{mn} = \frac{1}{2}\rho_{m+1/2}\delta_{m+1,n} + \frac{1}{2}\rho_{m-1/2}\delta_{m,n+1},$$
$$X^2_{mn} = \frac{i}{2}\rho_{m+1/2}\delta_{m+1,n} - \frac{i}{2}\rho_{m-1/2}\delta_{m,n+1},$$
$$X^3_{mn} = z_m\delta_{m,n}$$

This represents the fuzzy surface with axial symmetry. This is also confirmed by estimating the area formula.

$$A = 2\pi\sum_m\sqrt{\frac{1}{2}\left\{\rho^2_{m+1/2}\left(z_{m+1}-z_m\right)^2+\rho^2_{m-1/2}\left(z_m-z_{m-1}\right)^2\right\}+\frac{1}{4}\left(\rho^2_{m+1/2}-\rho^2_{m-1/2}\right)^2} \sim 2\pi\int dz\,\rho\sqrt{1+\rho'^2}$$

Area for $\rho = \rho(z)$

Let us investigate the axial symmetry of the fuzzy surface.

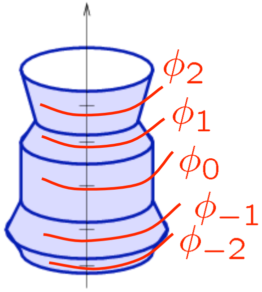
$$\hat{X}^1_{mn} = (\cos\theta X^1 - \sin\theta X^2)_{mn} = \frac{1}{2}\rho_{m+1/2}e^{-i\theta}\delta_{m+1,n} + \frac{1}{2}\rho_{m-1/2}e^{i\theta}\delta_{m,n+1},$$
$$\hat{X}^2_{mn} = (\sin\theta X^1 + \cos\theta X^2)_{mn} = \frac{i}{2}\rho_{m+1/2}e^{-i\theta}\delta_{m+1,n} - \frac{i}{2}\rho_{m-1/2}e^{i\theta}\delta_{m,n+1},$$
$$\hat{X}^3_{mn} = X^3_{mn} = z_m\delta_{m,n}$$

$\hat{X}^i$ represents the same fuzzy surface as X^i because

$$(\hat{X}^1)^2 + (\hat{X}^2)^2 = (X^1)^2 + (X^2)^2, \quad \hat{X}^3 = X^3, \quad \hat{A} = A$$

It is possible to elevate the axial symmetry to `local' rotation symmetry.

$$\hat{X}^1_{mn} = \frac{1}{2}\rho_{m+1/2}e^{-i\theta_{m+1/2}}\delta_{m+1,n} + \frac{1}{2}\rho_{m-1/2}e^{i\theta_{m-1/2}}\delta_{m,n+1},$$
$$\hat{X}^2_{mn} = \frac{i}{2}\rho_{m+1/2}e^{-i\theta_{m+1/2}}\delta_{m+1,n} - \frac{i}{2}\rho_{m-1/2}e^{i\theta_{m-1/2}}\delta_{m,n+1},$$
$$\hat{X}^3_{mn} = z_m\delta_{m,n}$$



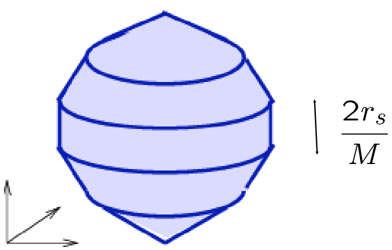
$$\longleftrightarrow \quad \hat{X}^i = UX^iU^\dagger, \quad U_{mn} = e^{i\phi_m}\delta_{m,n}, \quad \phi_{m+1} - \phi_m = \theta_{m+1/2}$$

This means that the local rotation symmetry is identified with $U(1)^\infty$ symmetry.

2.3 The fuzzy sphere

The matrices which represent the fuzzy sphere is given by

$$X^1_{mn} = \frac{r_s}{M}\sqrt{m(M-m)}\delta_{m+1,n} + \frac{r_s}{M}\sqrt{(m-1)(M-m+1)}\delta_{m,n+1},$$
$$X^2_{mn} = \frac{ir_s}{M}\sqrt{m(M-m)}\delta_{m+1,n} - \frac{ir_s}{M}\sqrt{(m-1)(M-m+1)}\delta_{m,n+1},$$
$$X^3_{mn} = \frac{r_s}{M}(2m-M-1)\delta_{m,n}, \quad m,n = 1,\cdots,M$$



This is confirmed by evaluating the area.

$$A = \sum_m 2\pi a = \sum_m \frac{4\pi r_s^2}{M}\sqrt{1-\frac{1}{M^2}}$$

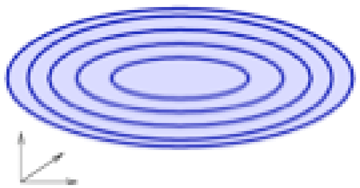
The algebra for the fuzzy sphere is given by

$$[X^i, X^j] = i\epsilon^{ijk}\frac{2r_s}{M}X^k \qquad i,j,k = 1,2,3$$

2.4 The fuzzy plane

The matrices which represent the fuzzy plane is given by

$$X^1_{mn} = \frac{1}{2}\sqrt{2am}\,\delta_{m+1,n} + \frac{1}{2}\sqrt{2a(m-1)}\,\delta_{m,n+1},$$
$$X^2_{mn} = \frac{i}{2}\sqrt{2am}\,\delta_{m+1,n} - \frac{i}{2}\sqrt{2a(m-1)}\,\delta_{m,n+1},$$
$$X^3_{mn} = z_0\delta_{m,n}$$



This is confirmed by evaluating the area.

$$A = 2\pi\sum_m a = 2\pi\sum_m \rho_m l_m \sim 2\pi\int \rho d\rho$$

$$\rho_m = \frac{1}{2}(\rho_{m+1/2} + \rho_{m-1/2}), \quad l_m = \rho_{m+1/2} - \rho_{m-1/2}$$

The algebra for the fuzzy plane is given by

$$[X^1, X^2] = -ia$$

3. Construction of the Cylindrical D2 from Multiple D0s

3.1 Preliminaries

Here we deform the nonabelian Born-Infeld action for D0-branes. The action in the flat space-time background is given by

$$S_{D0} = -T_0 \int dt\, \mathrm{STr}\left(\sqrt{-P[E+E']_{tt} \det Q^i{}_j}\right), \qquad i,j = 1,\cdots,9$$

$$\begin{cases} E_{\mu\nu} = \eta_{\mu\nu} \\ E'_{\mu\nu} = \eta_{\mu i}(Q^{-1i}{}_j - \delta^i{}_j)\eta_{j\nu} \\ Q^i{}_j = \delta^i{}_j + \frac{i}{\lambda}[X^i, X^j] \end{cases} \qquad \begin{cases} P[E]_{tt} = -1 + (D_t X^i)^2 \\ P[E']_{tt} = D_t X^i Q^{-1i}{}_j D_t X^j - (D_t X^i)^2 \end{cases}$$

Since we only consider the case where D0-branes form the fuzzy cylinder in the (x1,x2,x3) space, we set $X^4 = \cdots = X^9 = 0$ and $i,j = 1,2,3$.

Then Q is written as

$$Q^i{}_j = \begin{pmatrix} 1 & \frac{i}{\lambda}[X^1,X^2] & -\frac{i}{\lambda}[X^3,X^1] \\ -\frac{i}{\lambda}[X^1,X^2] & 1 & \frac{i}{\lambda}[X^2,X^3] \\ \frac{i}{\lambda}[X^3,X^1] & -\frac{i}{\lambda}[X^2,X^3] & 1 \end{pmatrix}$$

By using the relations,

$$\begin{cases} \det Q^i{}_j \overset{\mathrm{STr}}{=} 1 - \frac{1}{2\lambda^2}[X^i, X^j]^2 \\ \tilde{Q}_j{}^i = (\det Q) Q^{-1i}{}_j \overset{\mathrm{STr}}{=} \delta^i_j - \frac{i}{\lambda}[X^i, X^j] - \frac{1}{4\lambda^2}\epsilon_{ikl}[X^k, X^l]\epsilon_{jmn}[X^m, X^n] \\ D_t X^i \tilde{Q}_j{}^i D_t X^j \overset{\mathrm{STr}}{=} (D_t X^i)^2 - \frac{1}{4\lambda^2}\left(\epsilon_{ijk} D_t X^i [X^j, X^k]\right)^2, \quad \epsilon_{123} = 1 \end{cases}$$

the nonabelian Born-Infeld action for D0-branes is transformed into the following form,

$$\begin{aligned} S_{\mathrm{D0}} &= -T_0 \int dt \, \mathrm{STr}\left(\sqrt{\det Q^i{}_j - D_t X^i (\det Q) Q^{-1i}{}_j D_t X^j}\right) \\ &= -T_0 \int dt \mathrm{Tr} \sqrt{1 - (D_t X^i)^2 - \frac{1}{2\lambda^2}[X^i, X^j]^2 + \frac{1}{4\lambda^2}\left(\epsilon_{ijk}\left\{D_t X^i, [X^j, X^k]\right\}\right)^2} \end{aligned}$$

Note that we replaced STr with Tr by symmetrizing the interior of the square root.

3.2 The effective action for the fuzzy cylinder

Now we are ready to obtain the effective action for the fuzzy cylinder, if we set fluctuations around that.

Recall that we could add phase factors which corresponds to the rotation around the x^3 axis.

We identify these as gauge degrees of freedom and set fluctuations as follows.

$$X^1_{mn} = \frac{1}{2}(\rho_c + \tilde{\rho})e^{il_c a_z}\Big|_{m+1/2}\delta_{m+1,n} + \frac{1}{2}(\rho_c + \tilde{\rho})e^{-il_c a_z}\Big|_{m-1/2}\delta_{m,n+1},$$

$$X^2_{mn} = \frac{i}{2}(\rho_c + \tilde{\rho})e^{il_c a_z}\Big|_{m+1/2}\delta_{m+1,n} - \frac{i}{2}(\rho_c + \tilde{\rho})e^{-il_c a_z}\Big|_{m-1/2}\delta_{m,n+1},$$

$$X^3_{mn} = \left(ml_c - l_c a_\phi\right)\Big|_m \delta_{m,n}, \qquad A_{t\,mn} = a_t\Big|_m \delta_{m,n}$$

Some calculations

$$(D_tX^1)_{mn} = \frac{1}{2}\Big(\dot{\hat{\rho}} + iaf_{tz} + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big)e^{il_c a_z}\Big|_{m+1/2}\delta_{m+1,n} + \frac{1}{2}\Big(\dot{\hat{\rho}} - iaf_{tz} + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big)e^{-il_c a_z}\Big|_{m-1/2}\delta_{m,n+1},$$

$$(D_tX^2)_{mn} = \frac{i}{2}\Big(\dot{\hat{\rho}} + iaf_{tz} + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big)e^{il_c a_z}\Big|_{m+1/2}\delta_{m+1,n} - \frac{i}{2}\Big(\dot{\hat{\rho}} - iaf_{tz} + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big)e^{-il_c a_z}\Big|_{m-1/2}\delta_{m,n+1},$$

$$(D_tX^3)_{mn} = -l_c\dot{a}_\phi\Big|_m\delta_{m,n}$$

$$\frac{1}{\lambda}[X^1, X^2]_{mn} = \Big(-\frac{ia}{\lambda}\hat{\rho}' + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big)\Big|_m\delta_{m,n},$$

$$\frac{1}{\lambda}[X^2, X^3]_{mn} = \frac{i}{2}\Big\{\frac{a^2}{\lambda\rho_c}\Big(\frac{\rho_c}{a} - a'_\phi\Big) + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big\}e^{il_c a_z}\Big|_{m+1/2}\delta_{m+1,n} + \frac{i}{2}\Big\{\frac{a^2}{\lambda\rho_c}\Big(\frac{\rho_c}{a} - a'_\phi\Big) + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big\}e^{-il_c a_z}\Big|_{m-1/2}\delta_{m,n+1},$$

$$\frac{1}{\lambda}[X^3, X^1]_{mn} = -\frac{1}{2}\Big\{\frac{a^2}{\lambda\rho_c}\Big(\frac{\rho_c}{a} - a'_\phi\Big) + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big\}e^{il_c a_z}\Big|_{m+1/2}\delta_{m+1,n} + \frac{1}{2}\Big\{\frac{a^2}{\lambda\rho_c}\Big(\frac{\rho_c}{a} - a'_\phi\Big) + \mathcal{O}(\tfrac{\hat{\rho}}{\rho_c})\Big\}e^{-il_c a_z}\Big|_{m-1/2}\delta_{m,n+1}$$

Finally we obtain the effective action for the fuzzy cylinder.

$$S_{\mathrm{D0}} = -T_0 \int dt \sum_m \frac{\rho_c l_c}{a}\Big[1+\Big(-\dot{\hat{\rho}}^2+\frac{a^2}{\lambda^2}\hat{\rho}'^2\Big)+a^2\Big\{-\frac{1}{\rho_c^2}\dot{a}_\phi^2+\frac{a^2}{\lambda^2\rho_c^2}\Big(\frac{\rho_c}{a}-a'_\phi\Big)^2-f_{tz}^2\Big\}-\frac{a^4}{\lambda^2\rho_c^2}\Big\{\dot{\hat{\rho}}\Big(\frac{\rho_c}{a}-a'_\phi\Big)+\hat{\rho}'\dot{a}_\phi\Big\}^2+\mathcal{O}\big(\tfrac{\hat{\rho}}{\rho_c}\big)\Big]^{1/2}\Big|_m$$

$$= -\frac{T_0}{a}\int dtdz\,\rho_c\Big[1+\Big(-\dot{\hat{\rho}}^2+\frac{a^2}{\lambda^2}\hat{\rho}'^2\Big)+a^2\Big\{-\frac{1}{\rho_c^2}\dot{a}_\phi^2+\frac{a^2}{\lambda^2\rho_c^2}\Big(\frac{\rho_c}{a}-a'_\phi\Big)^2-f_{tz}^2\Big\}-\frac{a^4}{\lambda^2\rho_c^2}\Big\{\dot{\hat{\rho}}\Big(\frac{\rho_c}{a}-a'_\phi\Big)+\hat{\rho}'\dot{a}_\phi\Big\}^2+\mathcal{O}\big(\tfrac{\hat{\rho}}{\rho_c}\big)\Big]^{1/2}$$

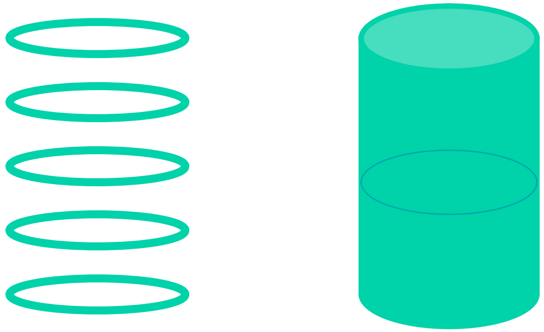
The remaining task is to compare to the D2-brane action with constant magnetic flux.

$$S_{\mathrm{D2}} = -T_2\int dtd\phi dz\,\rho_c\sqrt{1+\partial_\alpha\hat{\rho}\partial^\alpha\hat{\rho}+\frac{\lambda^2}{2}F_{\alpha\beta}F^{\alpha\beta}-\frac{\lambda^2}{4}\big(\epsilon^{\alpha\beta\gamma}\partial_\alpha\hat{\rho}\,F_{\beta\gamma}\big)^2+\mathcal{O}\big(\tfrac{\hat{\rho}}{\rho_c}\big)},$$

$$= -\frac{T_0}{\lambda}\int dtdz\,\rho_c\Big[1+\big(-\dot{\hat{\rho}}^2+\hat{\rho}'^2\big)+\lambda^2\Big\{-\frac{1}{\rho_c^2}\dot{a}_\phi^2+\frac{1}{\rho_c^2}\Big(\frac{\rho_c}{b}-a'_\phi\Big)^2-f_{tz}^2\Big\}-\frac{\lambda^2}{\rho_c^2}\Big\{\dot{\hat{\rho}}\Big(\frac{\rho_c}{b}-a'_\phi\Big)+\hat{\rho}'\dot{a}_\phi\Big\}^2+\mathcal{O}\big(\tfrac{\hat{\rho}}{\rho_c}\big)\Big]^{1/2}$$

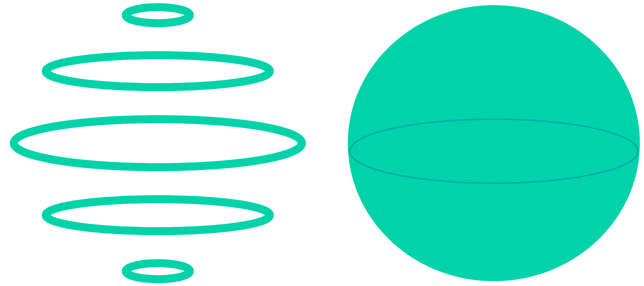
These two actions precisely coincide only in the case of $a = b = \lambda$.

cylinder



$$[X^1, X^2] = 0$$
$$[X^2, X^3] = ilX^1$$
$$[X^3, X^1] = ilX^2$$

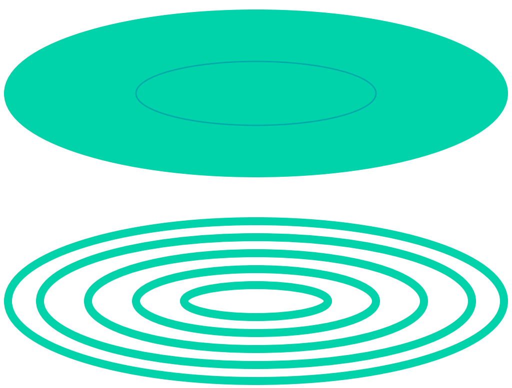
sphere



$$[X^i, X^j] = i\frac{2R}{M}\epsilon^{ijk}X^k$$

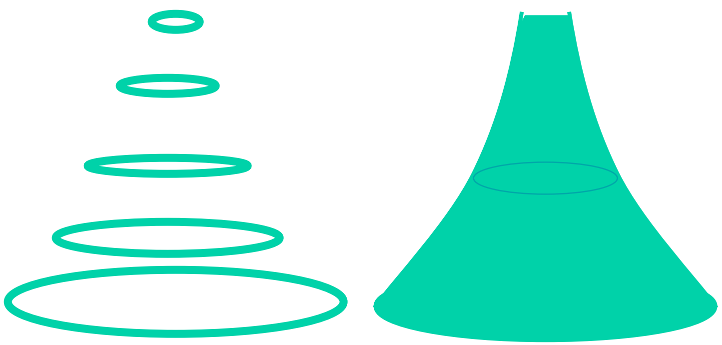
$X^i = \frac{2R}{M}J^i$ J^i : generators of $SU(2)$

plane



$$[X^1, X^2] = -i\lambda$$
$$X^3 = 0$$

BIon



?

4. Fuzzy BIon

4.1 BIon

In this talk:

BIon = F-string ending on D2-D0s bound state   1/4 SUSY

D2-brane action (cylindrical coordinates)

$$S_{D2} = -T_2 \int dt d\rho d\phi \sqrt{X}, \qquad X = \rho^2(1+z'^2) + \lambda^2 F_{\rho\phi}^2 - \rho^2\lambda^2 F_{t\rho}^2$$

Killing spinor eq. (Kappa-symmetry):

$$\left[\lambda F_{t\rho}\rho\Gamma_{11}\Gamma_{\hat\phi}\left(1+\frac{z'}{\lambda F_{t\rho}}\Gamma_{\hat t\hat z}\Gamma_{11}\right) + \sqrt{X}\left(1-\frac{\rho\Gamma_{\hat t\hat\rho\hat\phi} + \lambda F_{\rho\phi}\Gamma_{\hat t}\Gamma_{11}}{\sqrt{X}}\right)\right]\epsilon = 0$$

$$\longrightarrow \quad F = \pm\frac{z'}{\lambda} dt \wedge d\rho \pm \frac{\rho}{b} d\rho \wedge d\phi$$

Electric and magnetic flux exist on the D2-brane

Gauss law constraint:

$$\frac{d}{d\rho}\left(\frac{T_2\lambda^2 F_{t\rho}\rho^2}{\sqrt{X}}\right) = 0 \quad \longrightarrow \quad dz = \frac{L d\rho^2}{2\rho^2} \qquad (z = z_0 + L\ln\rho)$$

4.2 Fuzzy BIon

Here we construct the classical solution of BFSS matrix model which corresponds to the BIon configuration.

The action and EOMs are written as

$$S_{\mathrm{BFSS}} = T_0 \int dt \mathrm{Tr}\Big(\frac{1}{2}(D_t X^i)^2 + \frac{1}{4\lambda^2}[X^i, X^j]^2\Big), \quad i, j = 1, \cdots, 9$$

$$-D_t(D_t X^i) + \frac{1}{\lambda^2}[X^j, [X^i, X^j]] = 0, \qquad [X^i, D_t X^i] = 0$$

Now we substitute the matrices for the fuzzy surface with axial symmetry into the above EOMs.

$$X^1_{mn} = \frac{1}{2}\rho_{m+1/2}\delta_{m+1,n} + \frac{1}{2}\rho_{m-1/2}\delta_{m,n+1},$$

$$X^2_{mn} = \frac{i}{2}\rho_{m+1/2}\delta_{m+1,n} - \frac{i}{2}\rho_{m-1/2}\delta_{m,n+1},$$

$$X^3_{mn} = z_m \delta_{m,n}, \qquad A_{t\,mn} = a_m \delta_{m,n}$$

$$2\lambda^2(a_{m+1} - a_m)^2 - 2(z_{m+1} - z_m)^2 + (\rho^2_{m+3/2} - 2\rho^2_{m+1/2} + \rho^2_{m-1/2}) = 0,$$

$$\rho^2_{m+1/2}(z_{m+1} - z_m) - \rho^2_{m-1/2}(z_m - z_{m-1}) = 0,$$

$$\rho^2_{m+1/2}(a_{m+1} - a_m) - \rho^2_{m-1/2}(a_m - a_{m-1}) = 0$$

Let us solve these equations to obtain the fuzzy BIon. To execute this, note that, in the case of BIon configuraiton, the magnetic flux projected on the (ρ, ϕ)-plane is constant. It is realized in this case by choosing $\rho_{m+1/2}$ as

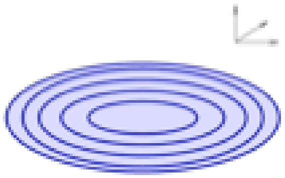
$$\rho^c_{m+1/2} = \sqrt{2am}, \quad m = 1, 2, \cdots$$

The remaining equations are

$$z_{m+1} - z_m = \pm\lambda(a_{m+1} - a_m),$$
$$m(z_{m+1} - z_m) - (m-1)(z_m - z_{m-1}) = 0$$

and easily solved like

$$z^c_m = \pm\lambda a^c_m = z^c_1 + \frac{L}{2}\sum_{i=1}^{m-1}\frac{1}{i}$$



The commutation relations for the fuzzy BIon become

$$[X^1, X^2]_{mn} = -ia\delta_{m,n},$$
$$[X^2, X^3]_{mn} = \frac{i}{2}aL\rho^{c\,-1}_{m+1/2}\delta_{m+1,n} + \frac{i}{2}aL\rho^{c\,-1}_{m-1/2}\delta_{m,n+1},$$
$$[X^3, X^1]_{mn} = -\frac{1}{2}aL\rho^{c\,-1}_{m+1/2}\delta_{m+1,n} + \frac{1}{2}aL\rho^{c\,-1}_{m-1/2}\delta_{m,n+1}$$

1/4 SUSY:

$$0 = \left(D_t X^i \Gamma^{0i} + \frac{i}{2\lambda}[X^i, X^j]\Gamma^{ij}\right)P_+\epsilon + P_+\epsilon'$$
$$= \pm\frac{i}{\lambda}\left([X^3, X^1]\Gamma^{01} + [X^3, X^2]\Gamma^{02}\right)P_+\left(1 \mp \Gamma^{03}\Gamma_{11}\right)\epsilon + \frac{i}{\lambda}[X^1, X^2]P_+\Gamma^{12}\epsilon + P_+\epsilon'$$

$$P_+ = (1 + \Gamma_{11})/2$$

$$\longrightarrow \qquad \epsilon' = \frac{a}{\lambda}\Gamma^0\epsilon, \qquad \epsilon = \frac{1 \pm \Gamma^{03}\Gamma_{11}}{2}\frac{1 + \Gamma^{012}}{2}\epsilon_0$$

Corresponding equation:

$$z^c_{m+1} - z^c_m = \frac{L(\rho^{c\,2}_{m+1} - \rho^{c\,2}_m)}{2\rho^{c\,2}_{m+1/2}}$$

Finally let us consider effective action for the fuzzy BIon by evaluating nonabelian Born-Infeld action.

Fluctuations should be added like

$$X^1_{mn} = \frac{1}{2}(\rho^c + l a_\phi) e^{i l a_\rho}\Big|_{m+1/2} \delta_{m+1,n} + \frac{1}{2}(\rho^c + l a_\phi) e^{-i l a_\rho}\Big|_{m-1/2} \delta_{m,n+1},$$
$$X^2_{mn} = \frac{i}{2}(\rho^c + l a_\phi) e^{i l a_\rho}\Big|_{m+1/2} \delta_{m+1,n} - \frac{i}{2}(\rho^c + l a_\phi) e^{-i l a_\rho}\Big|_{m-1/2} \delta_{m,n+1},$$
$$X^3_{mn} = \left(z^c + \tilde{z}\right)\Big|_m \delta_{m,n}, \qquad A_{t\,mn} = \left(a^c + a_t\right)\Big|_m \delta_{m,n}$$

And after some calculations, we obtain

$$\begin{aligned} S_{D0} \cong -T_0 \int dt \sum_m \frac{l\rho^c}{a} \Bigg[1 + \Big\{ -\dot{\tilde{z}}^2 + \frac{a^2}{\lambda^2}\Big(\frac{L}{\rho^c} + \tilde{z}'\Big)^2 \Big\} \\ + \lambda^2 \Big\{ -\frac{a^2}{\lambda^2}\Big(\mp \frac{L}{\lambda\rho^c} + f_{t\rho}\Big)^2 + \frac{a^4}{\lambda^4 \rho^{c\,2}}\Big(\frac{\rho^c}{a} + a'_\phi\Big)^2 - \frac{a^2}{\lambda^2\rho^{c\,2}} \dot{a}^2_\phi \Big\} \\ - \lambda^2 \Big\{ \frac{a^2}{\lambda^2 \rho^c} \dot{\tilde{z}} \Big(\frac{\rho^c}{a} + a'_\phi\Big) - \frac{a^2}{\lambda^2\rho^c}\Big(\frac{L}{\rho^c} + \tilde{z}'\Big)\dot{a}_\phi \Big\}^2 \Bigg]^{1/2} \Bigg|_m \end{aligned}$$

In the continuous limit, this coincides with the effective action for the BIon in case $a = b = \lambda$.

5. Conclusions and Discussions

We give the formulation to construct effective actions for the fuzzy geometries by keeping the explicit matrix forms.

fuzzy cylinder, sphere, plane, BIon

These actions are coincident with those of corresponding D2-branes only in case $a = b = \lambda = 2\pi\ell_s^2$.

Scalar fluctuations along the fuzzy geometry $\Longleftrightarrow$ gauge fields
Scalar fluctuations transverse the fuzzy geometry $\Longleftrightarrow$ scalar fields

interpretation

$a = b$: D0 charge = magnetic flux on the D2-brane

$a = \lambda$: note that $(2\pi\ell_s)^2 T_2 = T_0$.

This means that the mass of the D2-brane with the area $(2\pi\ell_s)^2$ is equal to that of the D0-brane. Namely the D0-brane can transform into the D2-brane with the area $(2\pi\ell_s)^2$ in view of the energy conservation.