PB24

Edge State and Edge Current in Superfluid ^{3}He

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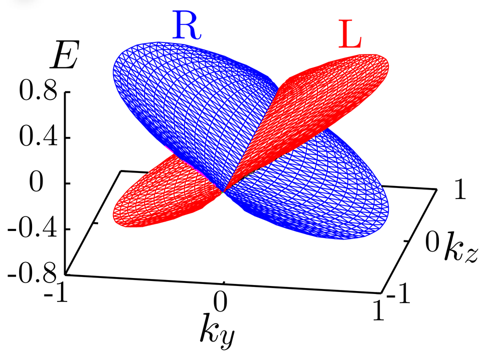
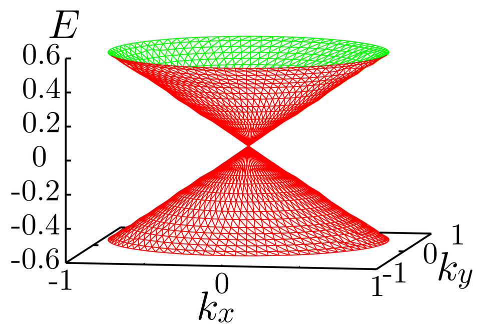
Introduction

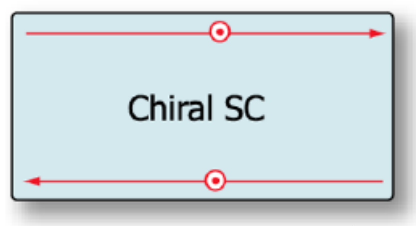
	A-phase	B-phase
Order parameter	$(k_x+ik_y)d_z$	$k_xd_x+k_yd_y+k_zd_z$
Time reversal symmetry	broken	unbroken
Andreev bound state (Majorana fermion)		



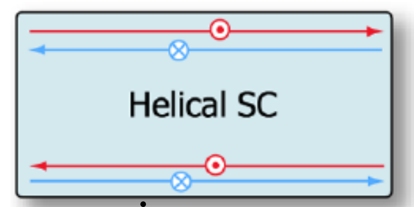


Y. Tsutsumi, *et al.*, PRB **83**, 094510 (2011).

- Edge current



mass current



spin current

X.-L. Qi, *et al.*, PRL **102**, 187001 (2009).

We investigate how to reflect differences of nature between the A- and B-phases on LDOS, and temperature dependence and energy spectrum of edge current.

Quasi-Classical Theory

Eilenberger equation

$$-i\hbar \boldsymbol{v}_F\cdot\boldsymbol{\nabla}\widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)=\left[\begin{pmatrix} i\omega_n\hat{1} & -\hat{\Delta}(\boldsymbol{k}_F,\boldsymbol{r}) \\ \hat{\Delta}^\dagger(\boldsymbol{k}_F,\boldsymbol{r}) & -i\omega_n\hat{1}\end{pmatrix},\widehat{g}(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)\right]$$

$$\widehat{g}=-i\pi\begin{pmatrix}\hat{g} & i\hat{f}\\ -i\underline{\hat{f}} & -\underline{\hat{g}}\end{pmatrix}$$

- Temperature dependence

$$\hat{g}=\begin{pmatrix} g_0+g_z & g_x-ig_y\\ g_x+ig_y & g_0-g_z\end{pmatrix}$$

Mass current $\quad \boldsymbol{j}(\boldsymbol{r},T)=mN_0\pi k_BT\sum_{\omega_n}\langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)]\rangle_{\boldsymbol{k}_F}$

Spin current $\quad \boldsymbol{j}_s^\mu(\boldsymbol{r},T)=\frac{\hbar}{2}N_0\pi k_BT\sum_{\omega_n}\langle \boldsymbol{v}_F \mathrm{Im}[g_\mu(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)]\rangle_{\boldsymbol{k}_F}$

- Energy spectrum

Mass current

$$\boldsymbol{j}(\boldsymbol{r},E)=\langle \boldsymbol{j}(\boldsymbol{k}_F,\boldsymbol{r},E)\rangle_{\boldsymbol{k}_F}=mN_0\langle \boldsymbol{v}_F\mathrm{Re}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)|_{i\omega_n\to E+i\eta}]\rangle_{\boldsymbol{k}_F}$$

Spin current

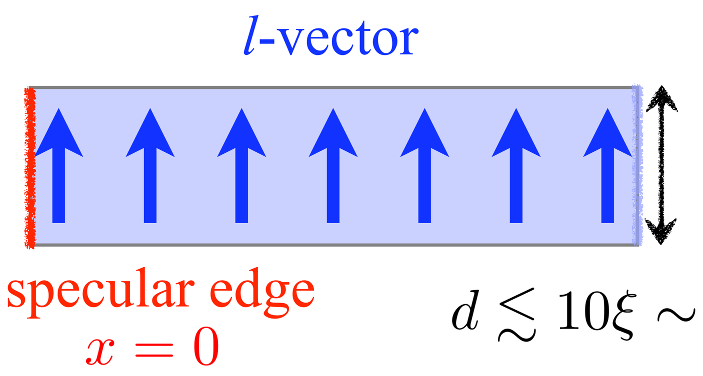
$$\boldsymbol{j}_s^\mu(\boldsymbol{r},E)=\langle \boldsymbol{j}_s^\mu(\boldsymbol{k}_F,\boldsymbol{r},E)\rangle_{\boldsymbol{k}_F}=\frac{\hbar}{2}N_0\langle \boldsymbol{v}_F\mathrm{Re}[g_\mu(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)|_{i\omega_n\to E+i\eta}]\rangle_{\boldsymbol{k}_F}$$

Local density of states (LDOS)

$$N(\boldsymbol{r},E)=\langle N(\boldsymbol{k}_F,\boldsymbol{r},E)\rangle_{\boldsymbol{k}_F}=N_0\langle \mathrm{Re}[g_0(\boldsymbol{k}_F,\boldsymbol{r},\omega_n)|_{i\omega_n\to E+i\eta}]\rangle_{\boldsymbol{k}_F}$$

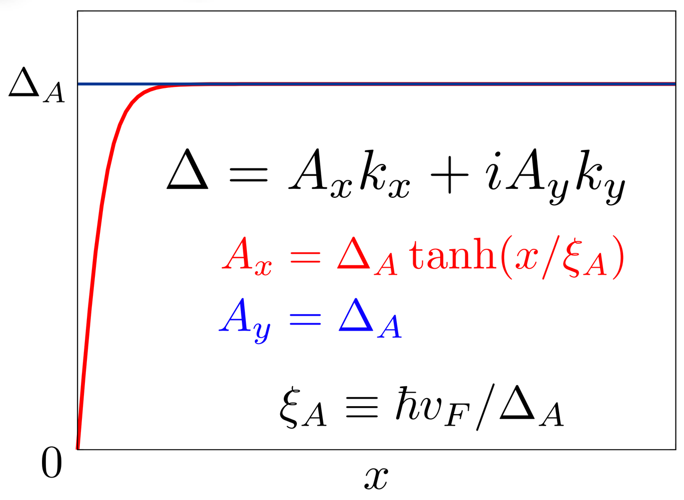
A-phase

- System



$d\lesssim 10\xi\sim 1\mu$m

- Order parameter



$$\Delta=A_xk_x+iA_yk_y$$

$A_x=\Delta_A\tanh(x/\xi_A)$, $A_y=\Delta_A$, $\xi_A\equiv\hbar v_F/\Delta_A$

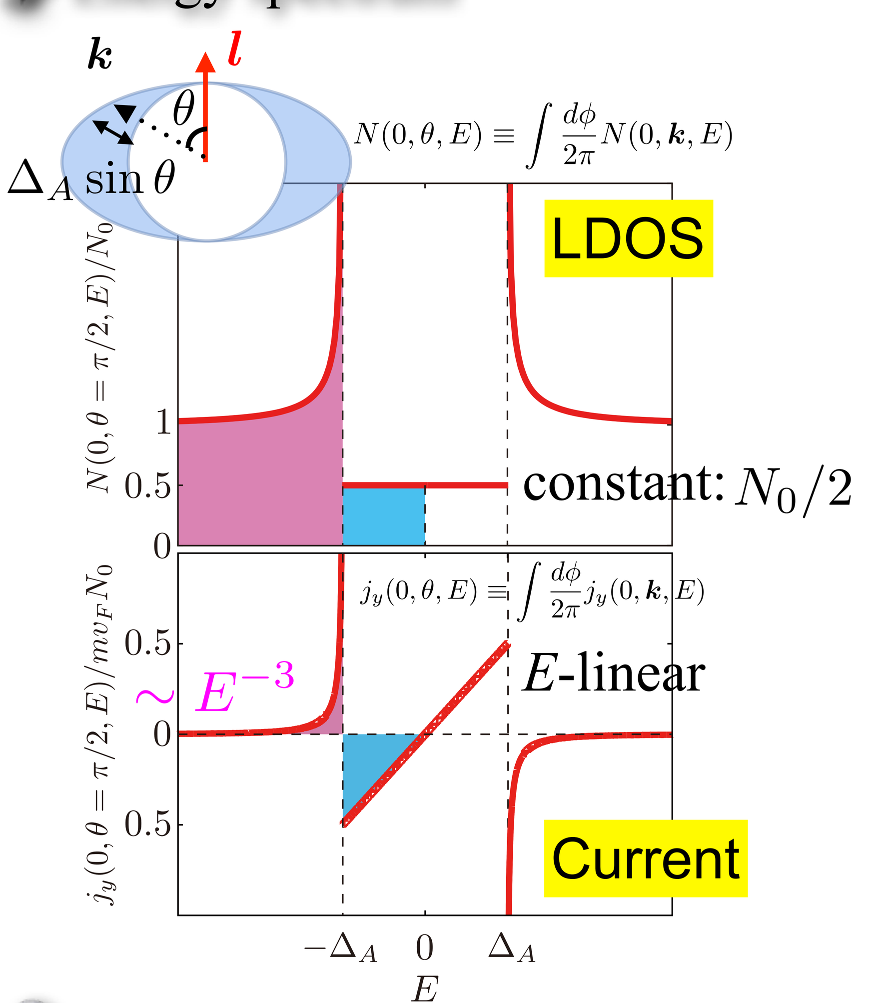
Analytic solution

$$g_0=\frac{1}{\sqrt{\omega_n^2+\Delta_A^2\sin^2\theta}}\left[\omega_n+\frac{\Delta_A^2\sin^2\theta\cos^2\phi}{2(\omega_n+i\Delta_A\sin\theta\sin\phi)}\mathrm{sech}^2\left(\frac{x}{\xi_A}\right)\right]$$

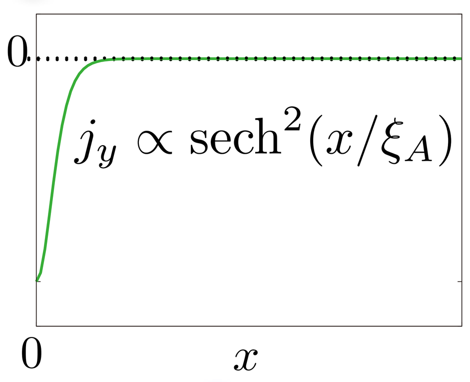
bulk; edge state

$(k_x=\cos\phi\sin\theta, k_y=\sin\phi\sin\theta)$

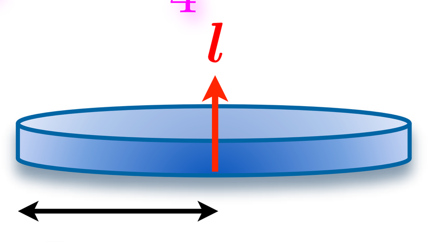
- Energy spectrum



- Angular momentum



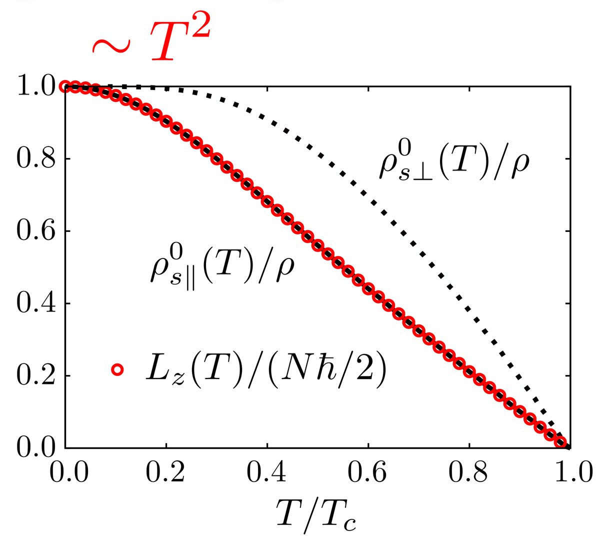
$J_y^{\mathrm{bound}}=-\frac{n\hbar}{2}$, $J_y^{\mathrm{cont}}=\frac{n\hbar}{4}$ $\Rightarrow$ $J_y=-\frac{n\hbar}{4}$



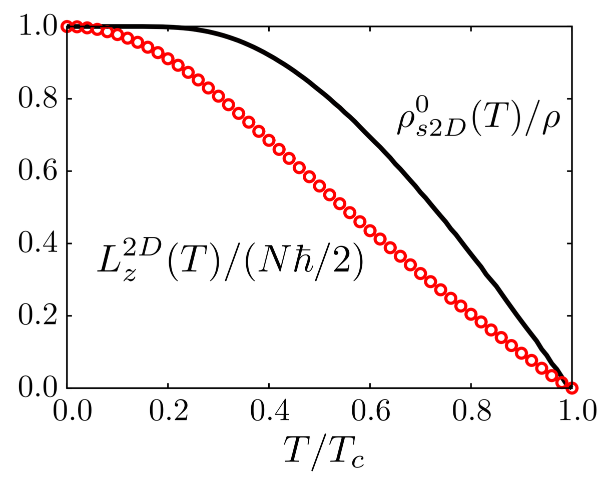
$L_z=\frac{N\hbar}{2}$

$R\gg\xi_A$

- Temperature dependence



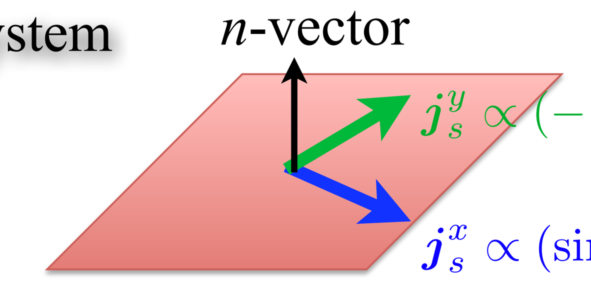
Cf. 2D Fermi surface (Sr_2RuO_4)



Y. Nagato, *et al.*, JPSJ **80**, 113706 (2011).
J.A. Sauls, PRB **84**, 214509 (2011).

B-phase

- System



$$\boldsymbol{\Delta}=\Delta_BR(z,\theta_L)(k_x\boldsymbol{x}+k_y\boldsymbol{y}+k_z\boldsymbol{z})$$

$\theta_L=\cos^{-1}(-1/4)\approx 104°$

$j_s^y\propto(-\cos\theta_L\boldsymbol{x}+\sin\theta_L\boldsymbol{y})$

$j_s^x\propto(\sin\theta_L\boldsymbol{x}+\cos\theta_L\boldsymbol{y})$

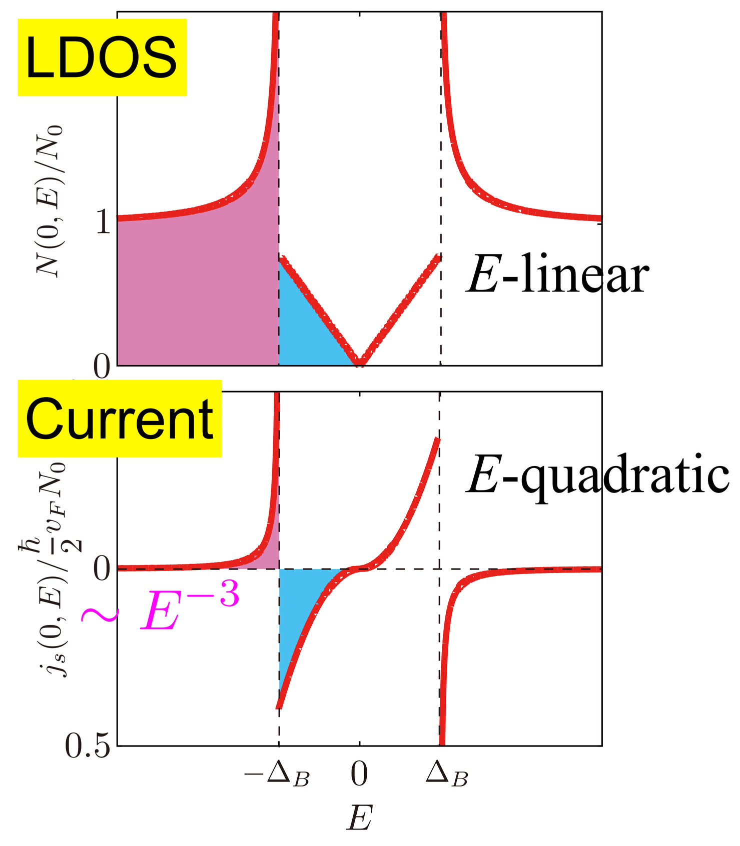
Analytic solution

$$g_0=\frac{1}{\omega_n^2+\Delta_B^2}\left[\omega_n+\frac{\Delta_B^2\cos^2\theta}{4}\left(\frac{1}{\omega_n+i\Delta_B\sin\theta}+\frac{1}{\omega_n-i\Delta_B\sin\theta}\right)\mathrm{sech}^2\left(\frac{z}{\xi_B}\right)\right]$$

$$g_x=\frac{\sin(\phi+\theta_L)}{\omega_n^2+\Delta_B^2}\frac{\Delta_B^2\cos^2\theta}{4}\left(\frac{1}{\omega_n+i\Delta_B\sin\theta}-\frac{1}{\omega_n-i\Delta_B\sin\theta}\right)\mathrm{sech}^2\left(\frac{z}{\xi_B}\right)$$

$$g_y=-\frac{\cos(\phi+\theta_L)}{\omega_n^2+\Delta_B^2}\frac{\Delta_B^2\cos^2\theta}{4}\left(\frac{1}{\omega_n+i\Delta_B\sin\theta}-\frac{1}{\omega_n-i\Delta_B\sin\theta}\right)\mathrm{sech}^2\left(\frac{z}{\xi_B}\right)$$

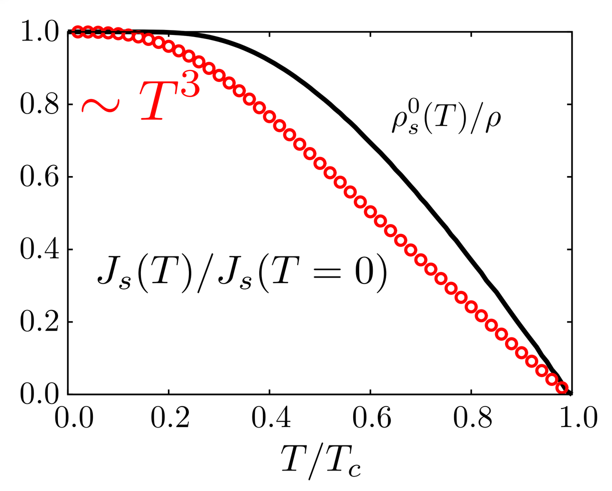
- Energy spectrum



- Total spin current

$$|J_s|=\frac{\hbar}{2m}\frac{n\hbar}{6}$$

- Temperature dependence



Summary

Differences of nature between the A- and B-phases are reflected on the following:

- LDOS in Andreev bound state
- Energy spectrum of edge current in bound state
- Amplitude of total edge current
- Low temperature depletion of edge current

Y. Tsutsumi and K. Machida, PRB **85**, 100506 (2012).
Y. Tsutsumi and K. Machida, arXiv:1203.2722.