「トポロジカル量子現象」第５回集中連携研究会
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超流動^{3}He-A相の エッジ流による角運動量

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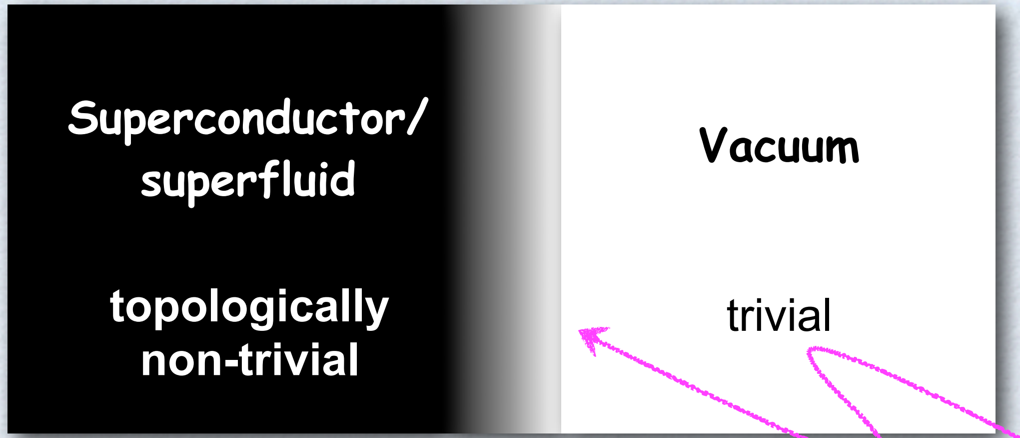
Topological Superfluid



Topological # defined in momentum space

$$\nu = \mathbb{Z} \qquad \textit{2D p+ip}$$

$$\nu = \mathbb{Z}_2 \qquad \textit{2D BW}$$

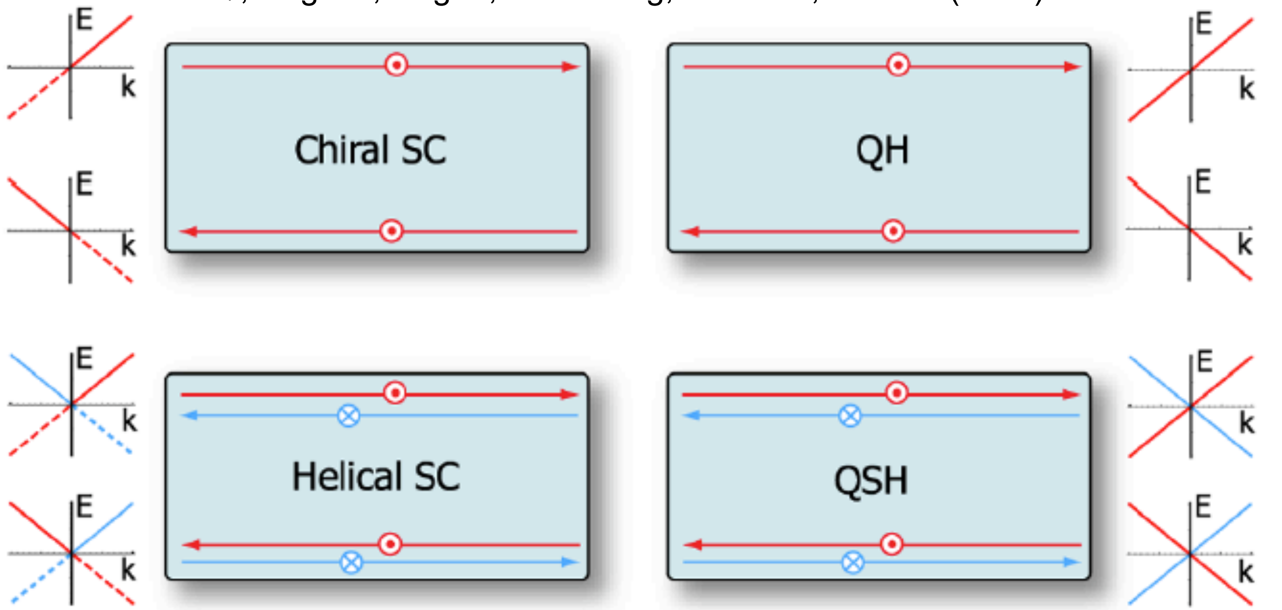
Read and Green, PRB **61**, 10267 (2000)
Schnyder *et al.*, PRB **78**, 195125 (2008)

Surface Andreev bound states
~ topological phase transition

Qi, Hughes, Raghu, S.C.Zhang, PRL **102**, 187001 (2009)

A-phase in 2D

B-phase in 2D



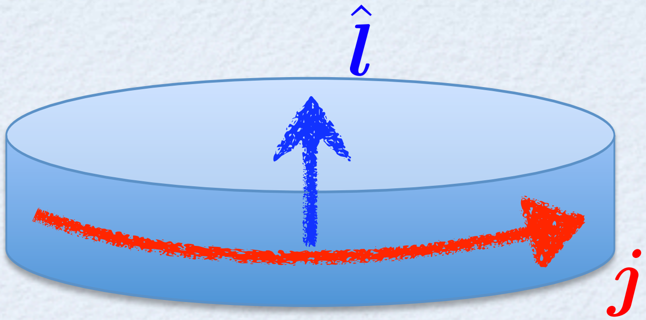
Broken time-reversal symmetry

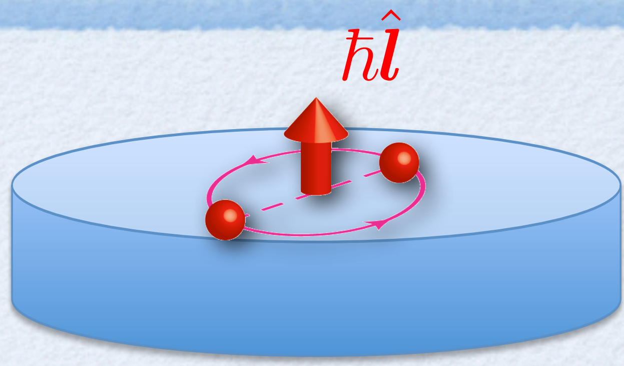
Time-reversal invariant

Majorana field

Dirac field

Contents





表面Andreev束縛状態に伴う
エッジカレントの角運動量

$N\hbar/2 \quad (T \to 0)$

$\stackrel{?}{=}$ Intrinsic Angular Momentum

$N\hbar/2$

エッジカレントの性質

- 角運動量へのbound state, continuumの寄与
- 角運動量の温度変化

超流動密度の温度変化との比較

Quasi-Classical Theory

Riccati方程式

$$\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta} - \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)\hat{\Delta}^\dagger \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{a}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

$$-\hbar \boldsymbol{v}_F \cdot \boldsymbol{\nabla} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) = \hat{\Delta}^\dagger - \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)\hat{\Delta} \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n) - 2\omega_n \hat{b}(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)$$

$$\begin{pmatrix} \hat{g} & i\hat{f} \\ -i\underline{\hat{f}} & -\underline{\hat{g}} \end{pmatrix} = \begin{pmatrix} (\hat{1}+\hat{a}\hat{b})^{-1} & 0 \\ 0 & (\hat{1}+\hat{b}\hat{a})^{-1} \end{pmatrix} \begin{pmatrix} \hat{1}-\hat{a}\hat{b} & 2i\hat{a} \\ -2i\hat{b} & -(\hat{1}-\hat{b}\hat{a}) \end{pmatrix}$$

$\hat{\Delta}$

ギャップ方程式

$$\hat{\Delta}(\boldsymbol{k}_F, \boldsymbol{r}) = N_0 \pi k_B T \sum_{-\omega_c \le \omega_n \le \omega_c} \left\langle V(\boldsymbol{k}_F, \boldsymbol{k}'_F) \hat{f}(\boldsymbol{k}'_F, \boldsymbol{r}, \omega_n) \right\rangle_{\boldsymbol{k}'_F}$$

pair potential : $V(\boldsymbol{k}_F, \boldsymbol{k}'_F) = 3g_1 \boldsymbol{k}_F \cdot \boldsymbol{k}'_F$

弱結合（低圧極限）
フェルミ液体補正なし

Mass Current and LDOS

$$\hat{g} = \begin{pmatrix} g_0 + g_z & g_x - i g_y \\ g_x + i g_y & g_0 - g_z \end{pmatrix}$$

- **Mass current**

$$\boldsymbol{j}(\boldsymbol{r}) = m N_0 \pi k_B T \sum_{-\omega_c \le \omega_n \le \omega_c} \langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)] \rangle_{\boldsymbol{k}_F}$$

$$\boldsymbol{j}(\boldsymbol{r}, E) = m N_0 \, \langle \boldsymbol{v}_F \mathrm{Re}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

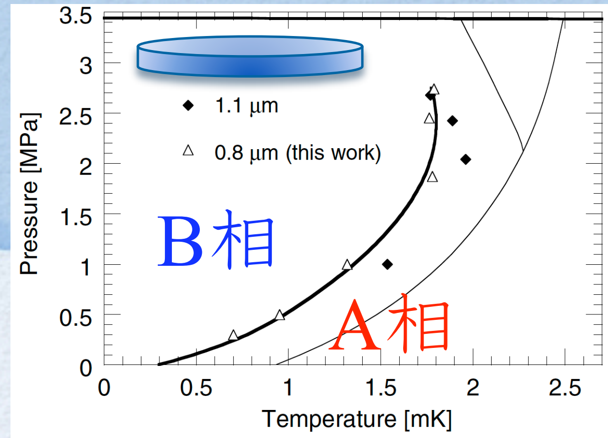
- **Local density of states (LDOS)**

$$N(\boldsymbol{r}, E) = \langle N(\boldsymbol{k}_F, \boldsymbol{r}, E) \rangle_{\boldsymbol{k}_F} = N_0 \, \langle \mathrm{Re}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)|_{i\omega_n \to E + i\eta}] \rangle_{\boldsymbol{k}_F}$$

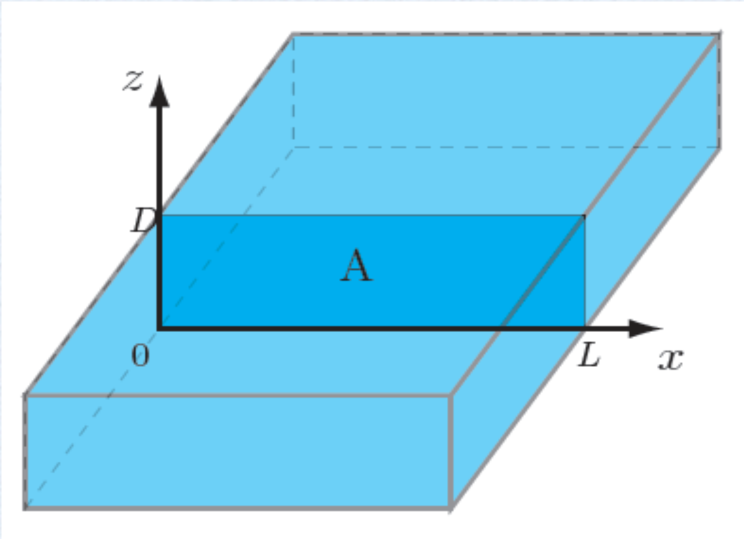
Angle-resolved LDOS

ピークから分散関係

System



Kawasaki *et al.*, PRL **93**, 105301 (2004).



断面A

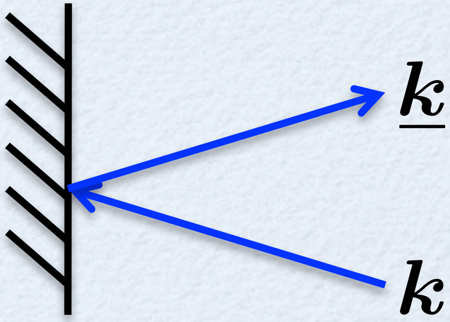
- z方向へ厚さD
- x方向へ巨視的な長さ
- y方向へは一様

$$L = 40\xi_0, T = 0.2T_c$$

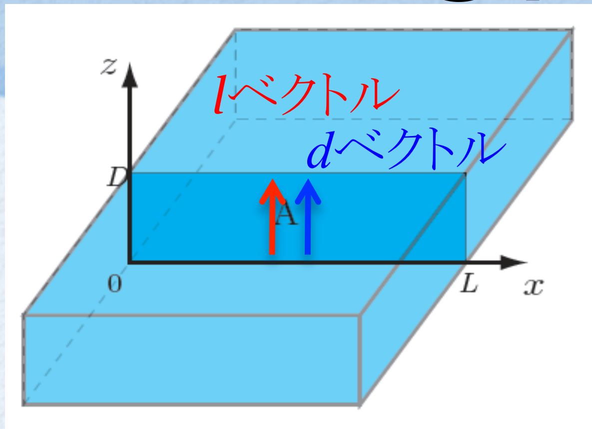
境界条件

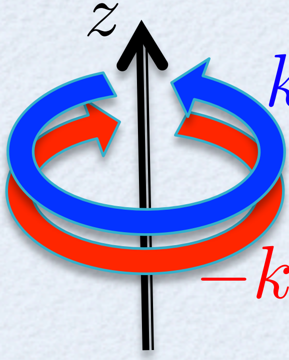
- specular surface

$$\hat{g}(\boldsymbol{k}, \boldsymbol{R}_{\mathrm{surf}}, \omega_n) = \hat{g}(\underline{\boldsymbol{k}}, \boldsymbol{R}_{\mathrm{surf}}, \omega_n)$$



Order Parameter



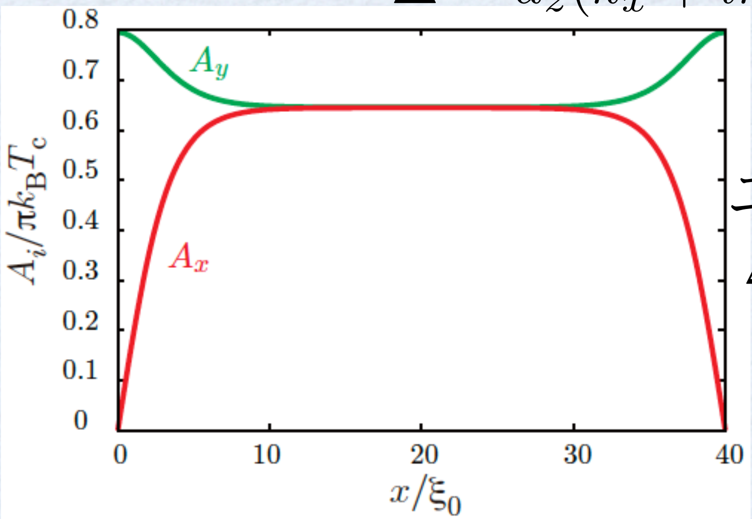


$k_x + ik_y$

$-k_x + ik_y$

$D = 8\xi_0$ バルク

$\Delta = d_z(k_x + ik_y)$

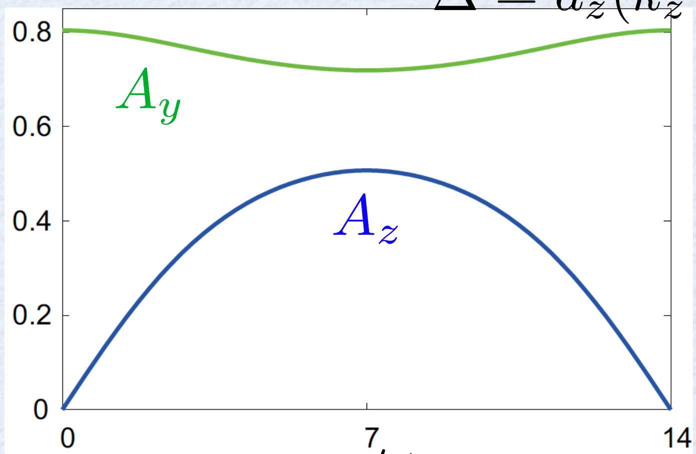


エッジ

$\Delta = d_z k_y$

$D = 14\xi_0$ エッジ

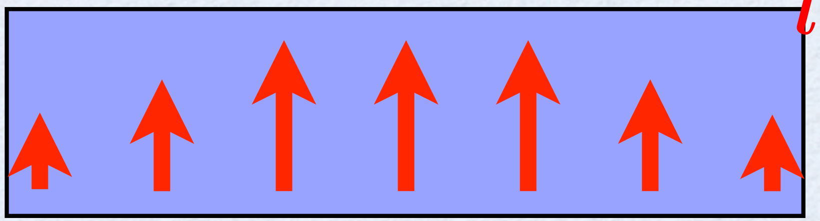
$\Delta = d_z(k_z + ik_y)$

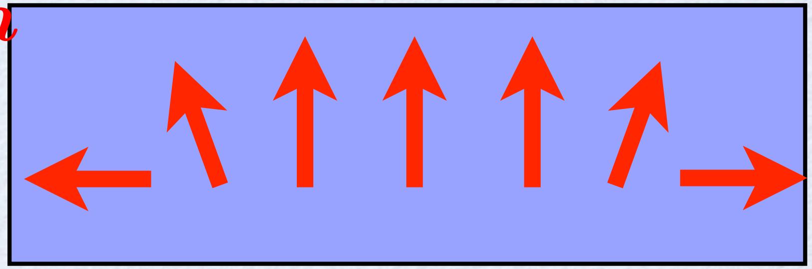


$A_i = (\tilde{\boldsymbol{m}} + i\tilde{\boldsymbol{n}})_i$

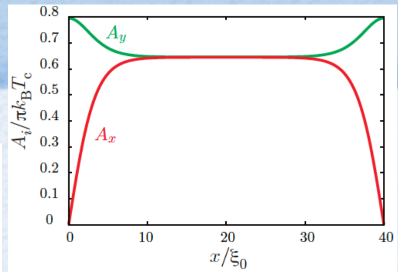
z/ξ_0

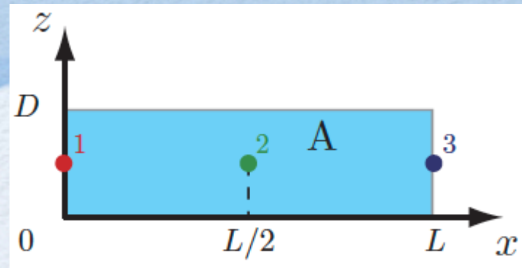
$\tilde{\boldsymbol{l}} = \tilde{\boldsymbol{m}} \times \tilde{\boldsymbol{n}}$



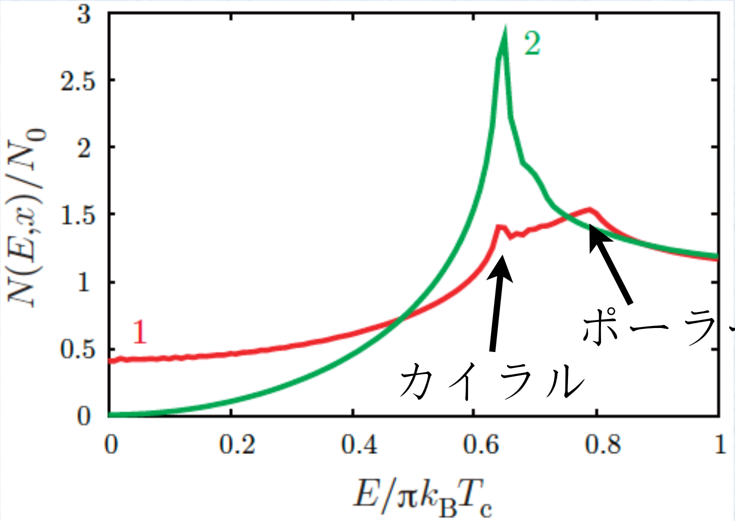


Surface Andreev Bound State





- **LDOS**



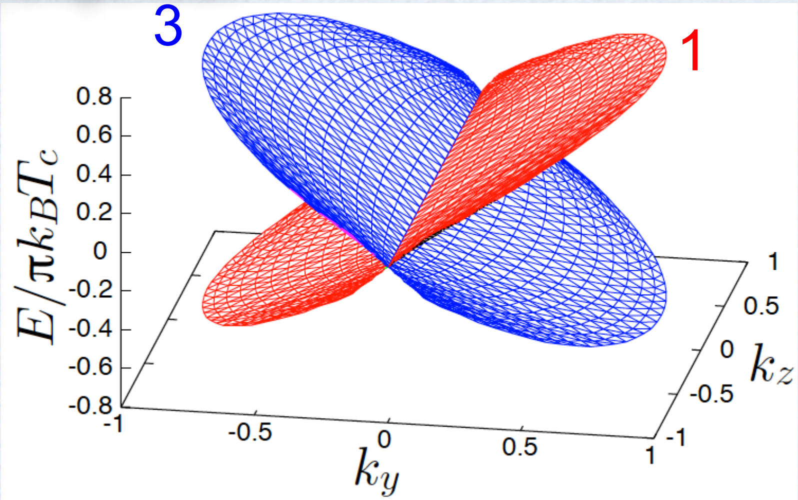
バルク：ポイントノード

$$N(E, x = L/2) \propto E^2$$

エッジ：ゼロエネルギー状態

$$N(E, x = 0) = N(E = 0) + \alpha E^2$$

- **Dispersion**



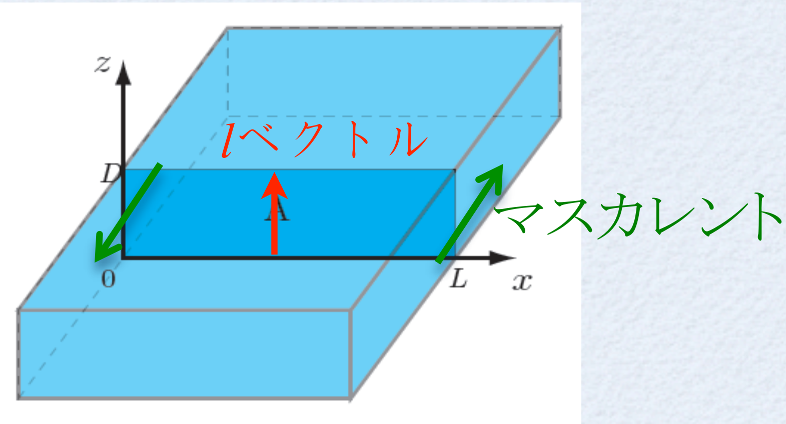
$k_y = k_z = 0, (k_x = \pm 1)$: エッジに垂直

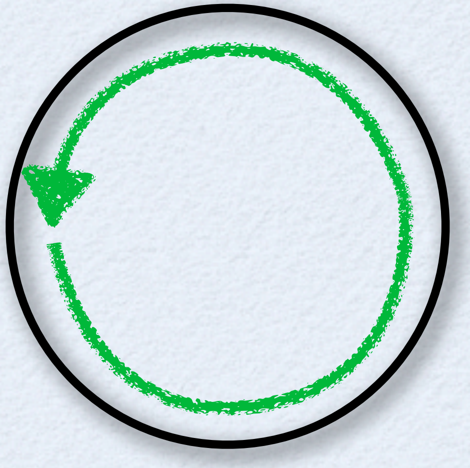
ゼロエネルギー

- k_z方向（ポイントノード）には分散なし
- k_y方向（一様）には線形な分散
- エッジごとに分散の傾きが異なる

Edge Current

- **A-phase: mass current**



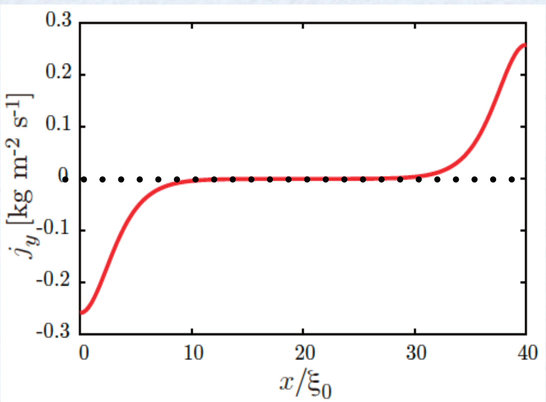


$$L_z \approx 0.42 N\hbar \; (T = 0.2T_c)$$

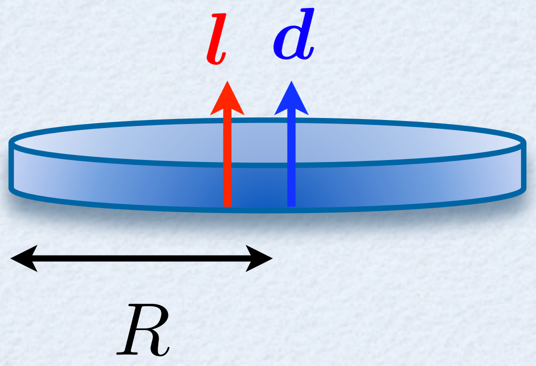
N：スラブ中の^{3}He原子数

$$L_z = \frac{1}{2} N\hbar \; (T = 0)$$

Intrinsic angular momentum



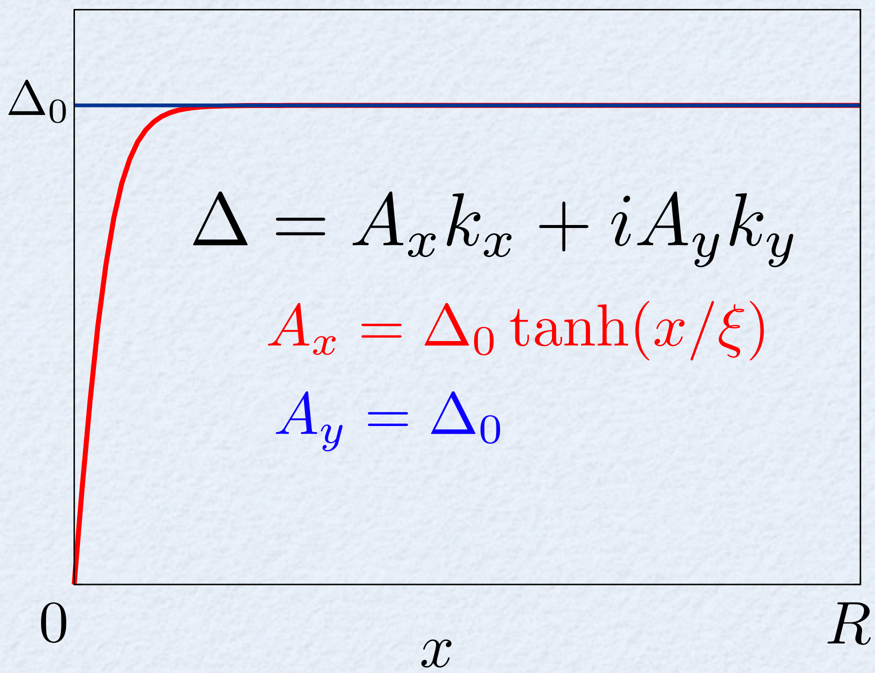
Low Temperature Limit



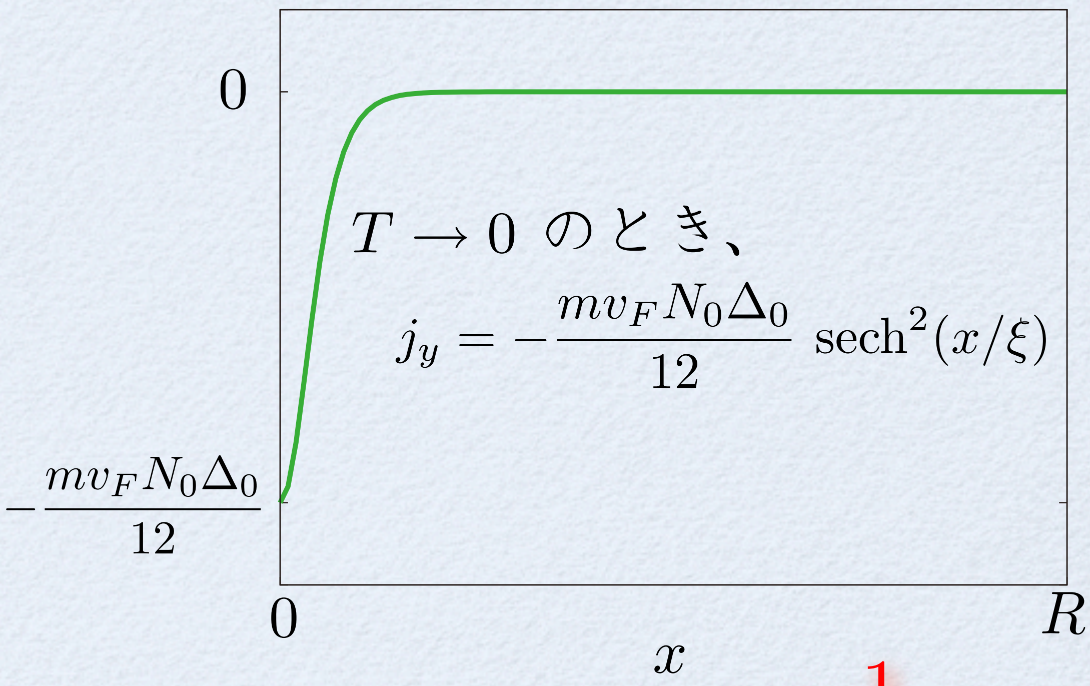
- analytic solution

$$g_0 = \frac{1}{\sqrt{\omega_n^2 + \Delta_0^2 \sin^2\theta}} \left[\omega_n + \frac{\Delta_0^2 \sin^2\theta \cos^2\phi}{2(\omega_n + i\Delta_0 \sin\theta \sin\phi)} \operatorname{sech}^2(x/\xi) \right]$$

$$\boldsymbol{j}(\boldsymbol{r}) = mN_0 \pi k_B T \sum_{\omega_n} \langle \boldsymbol{v}_F \mathrm{Im}[g_0(\boldsymbol{k}_F, \boldsymbol{r}, \omega_n)] \rangle_{\boldsymbol{k}_F}$$



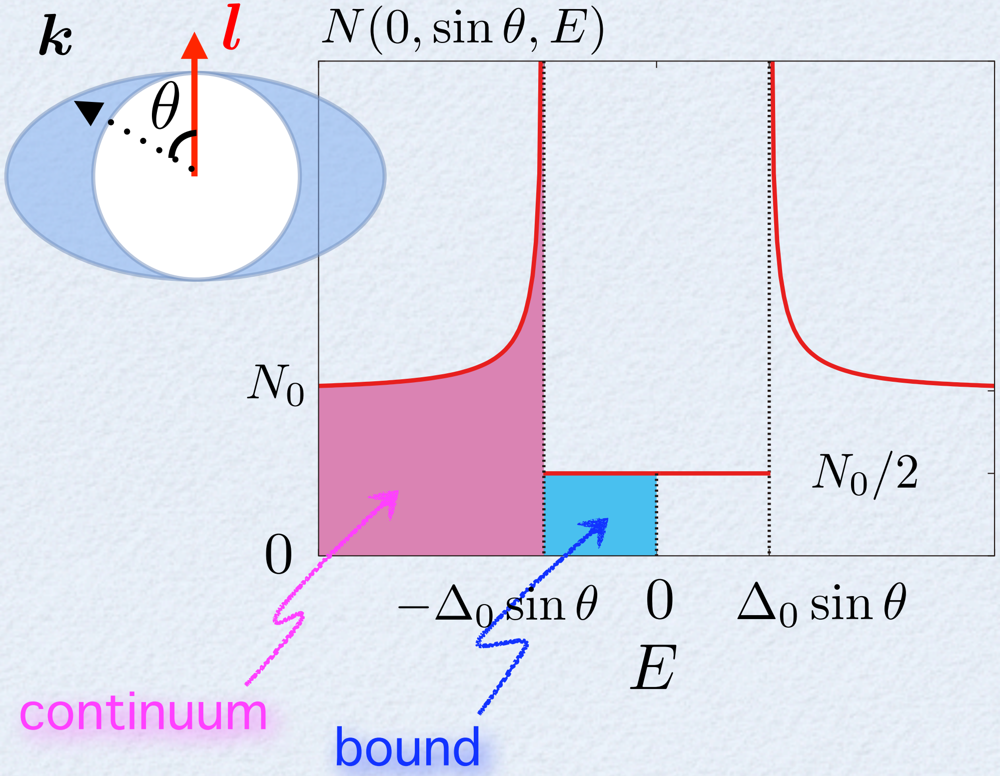
$(k_x = \cos\phi \sin\theta, k_y = \sin\phi \sin\theta)$



$\xi \ll R$ のとき、$L_z = \frac{1}{2} N\hbar$

Bound and Continuum States

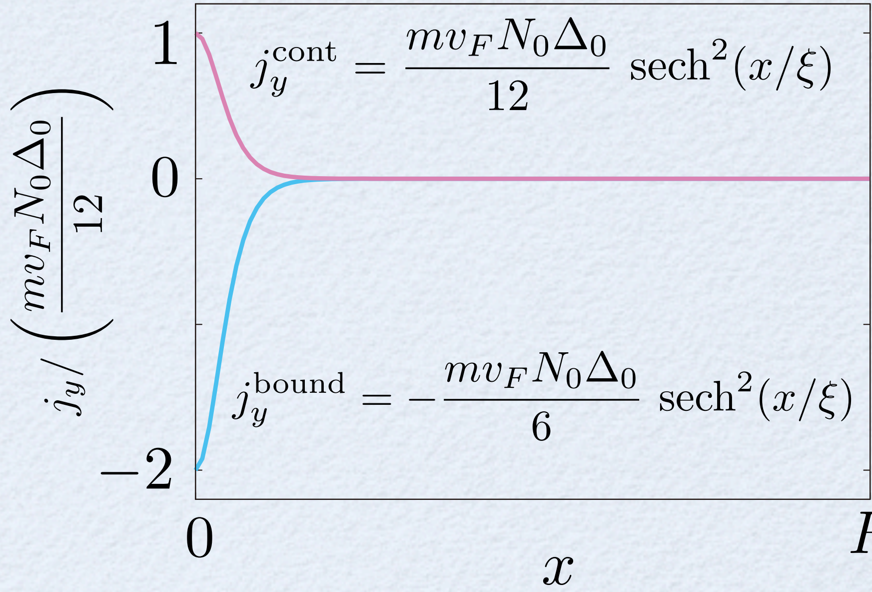
$$N(x, \sin\theta, E) = N_0 \int_0^{2\pi} \frac{d\phi}{2\pi} \mathrm{Re}\left[g_0(x, \boldsymbol{k}, \omega_n)|_{i\omega_n \to E + i\eta} \right]$$



$$\boldsymbol{j} = \boldsymbol{j}^{\mathrm{bound}} + \boldsymbol{j}^{\mathrm{cont}}$$

$$\boldsymbol{j}^{\mathrm{bound}}(x) = mN_0 \left\langle \int_{-\Delta_0 \sin\theta}^{0} dE \boldsymbol{v}_F \mathrm{Re}[g_0^R(x, \boldsymbol{k}_F, E)] \right\rangle_{\boldsymbol{k}_F}$$

$$\boldsymbol{j}^{\mathrm{cont}}(x) = mN_0 \left\langle \int_{-\infty}^{-\Delta_0 \sin\theta} dE \boldsymbol{v}_F \mathrm{Re}[g_0^R(x, \boldsymbol{k}_F, E)] \right\rangle_{\boldsymbol{k}_F}$$

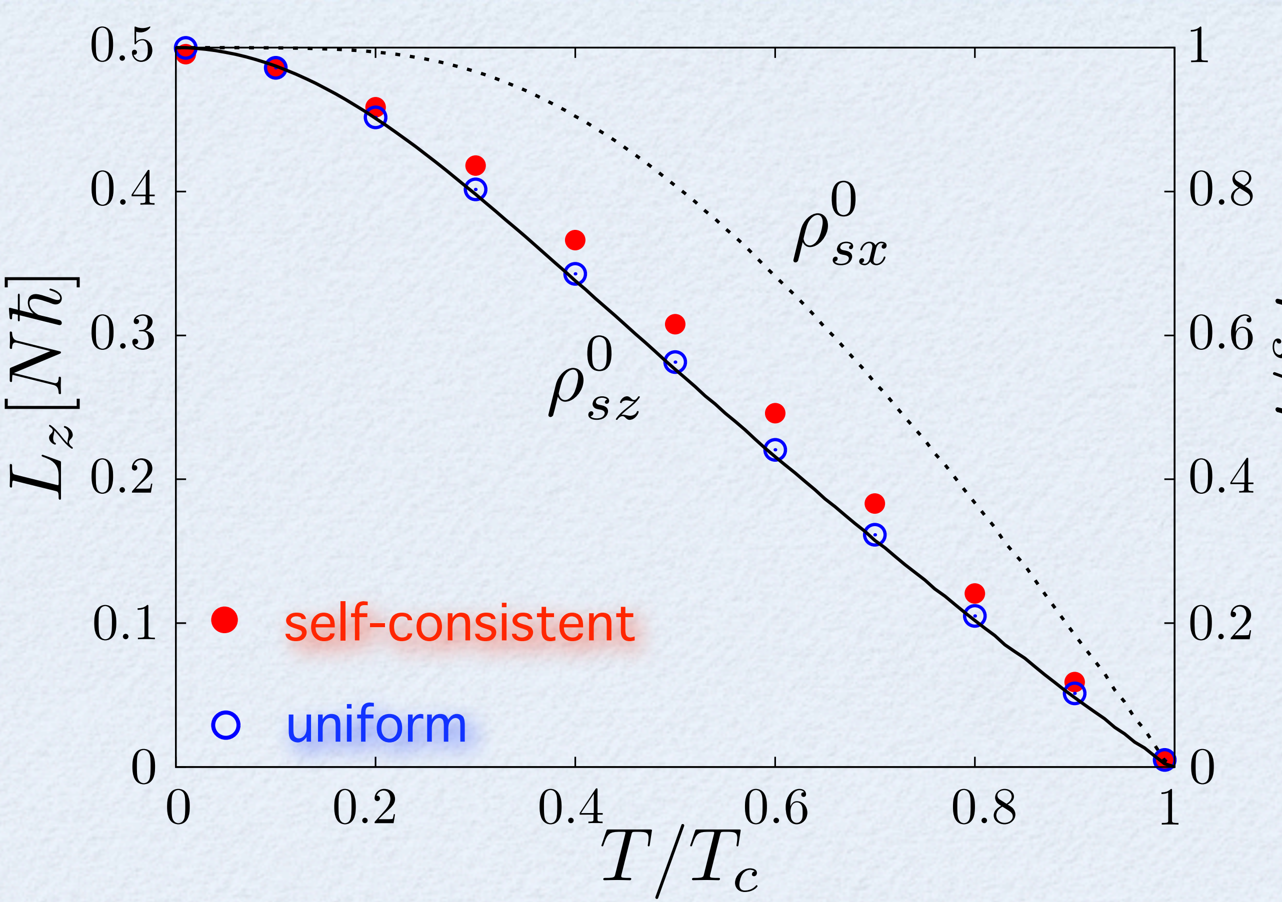


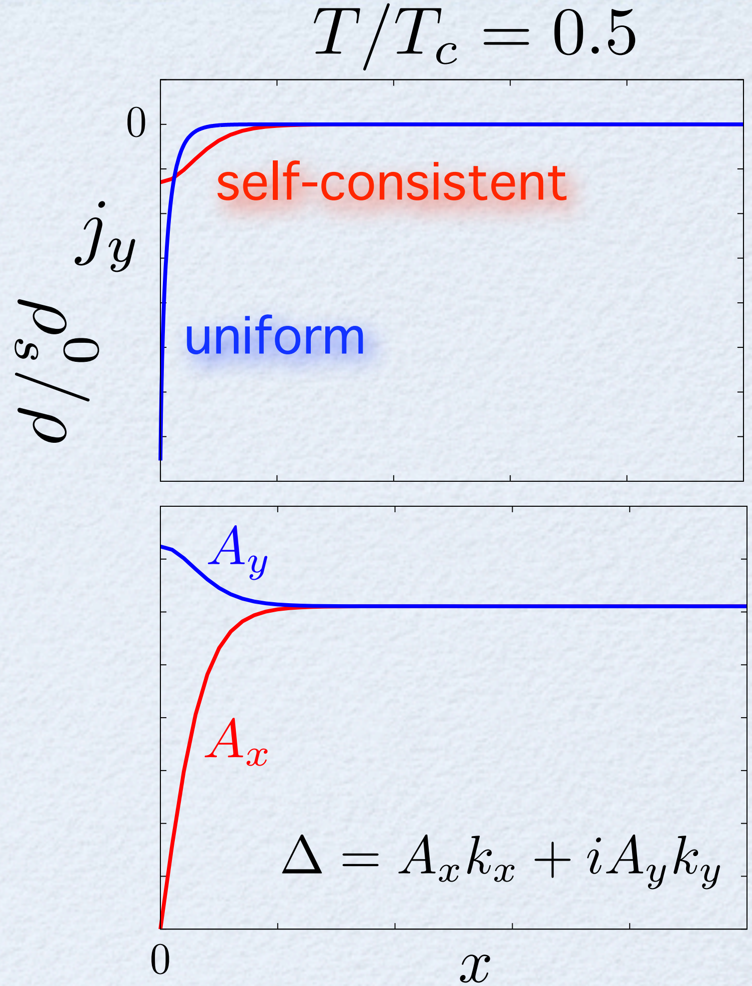
$$L_z^{\mathrm{bound}} = N\hbar$$

$$L_z^{\mathrm{cont}} = -\frac{1}{2} N\hbar$$

$$L_z = \frac{1}{2} N\hbar$$

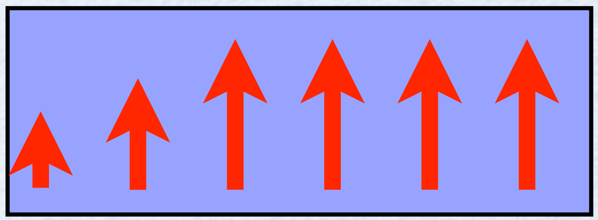
Temperature Dependence





$$\rho_{si}^0 = 3\rho \left\langle k_i^2 \left(1 - \int_0^\infty \frac{d\omega}{2k_BT} \mathrm{sech}^2 \frac{\sqrt{\omega^2 + |\Delta(\boldsymbol{k})|^2}}{2k_BT}\right)\right\rangle_{\boldsymbol{k}}$$ Cross, JLTP **21**, 525 (1975).

$\tilde{l}$ ベクトルが表面で短くなるが、
角運動量は増加



Summary

- 表面Andreev束縛状態に伴うエッジカレントの角運動量

  $T \to 0$ で、 $L_z^{\mathrm{bound}} = N\hbar$

  $L_z^{\mathrm{cont}} = -\frac{1}{2}N\hbar$

  → $L_z = \frac{1}{2}N\hbar$

  Intrinsic Angular Momentum
  の値と一致

  温度上昇によって、T_c で $L_z = 0$ へ単調減少

  $L_z(T)/(N\hbar/2) \geq \rho^0_{sz}(T)/\rho$

  クーパー対による角運動量では理解できない

- Future plan
  - 温度変化

    $$L_z^{\mathrm{cont}} = -\frac{1}{2}L_z^{\mathrm{bound}} \ ?$$

  - フェルミ液体補正