Symmetry Breaking and Phase Transition in Nuclei

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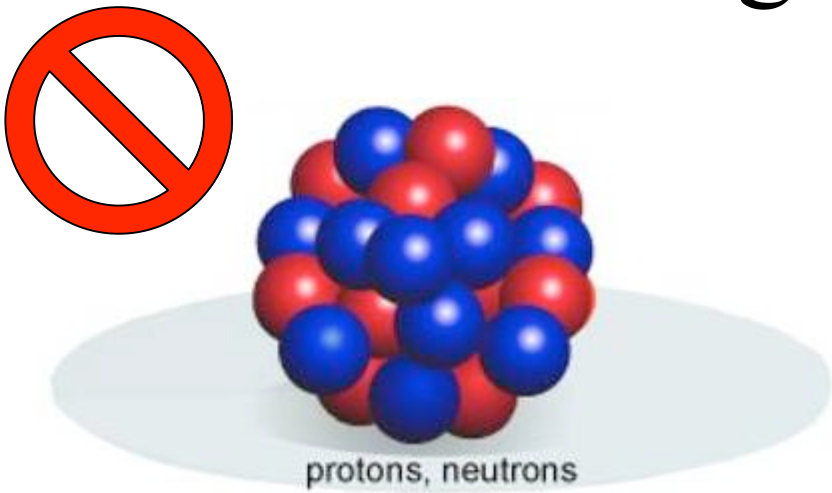
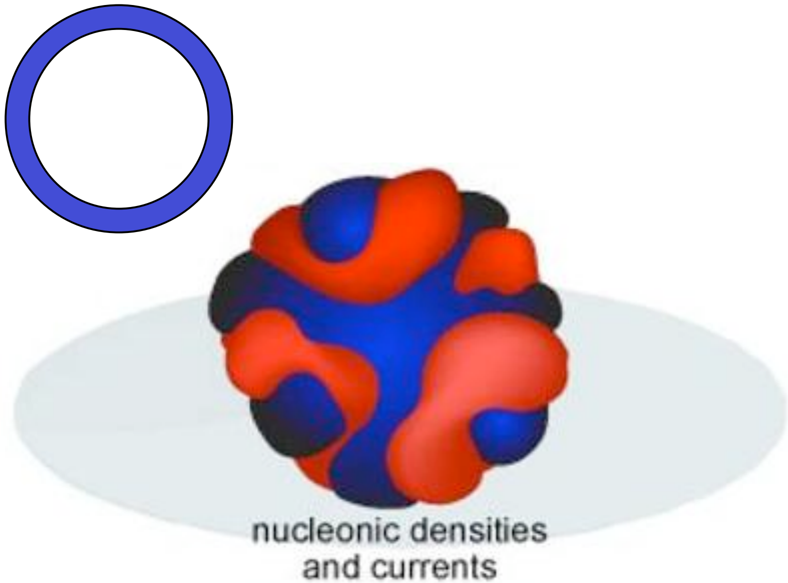
Basic properties of nucleus
Nucleus in different time scales
Spontaneous Symmetry Breaking in finite time
New type of SSB in superdeformed nuclei
Opportunities by RIKEN RI Beam Factory

RIKEN Theory Colloquium, April 15, 2009

Image of nucleus





Nucleus as a liquid drop

Bethe-Weizsäcker mass formula

$$B(N,Z) = a_V A - a_S A^{2/3} - a_{sym}\frac{(N-Z)^2}{A} - a_C\frac{Z^2}{A^{1/3}} + \delta(A)$$

$$A = N + Z$$

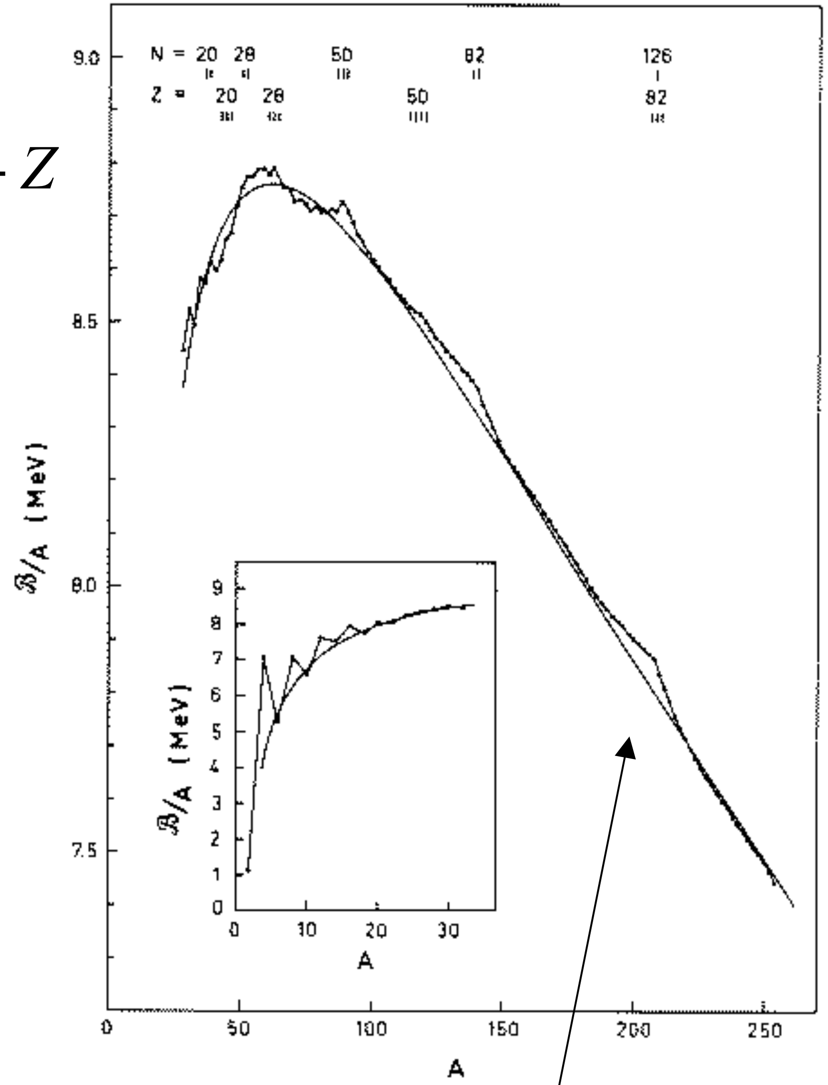
Binding energy per nucleon

B/A ≈ 8 MeV

Density

$\rho \approx 0.14\ \mathrm{fm}^{-3}$

Nuclear Saturation



These tiny differences lead to SSB

Nucleus as a quantum liquid

- Classical vs Quantum
  - Strength of interaction vs Zero-point kinetic energy

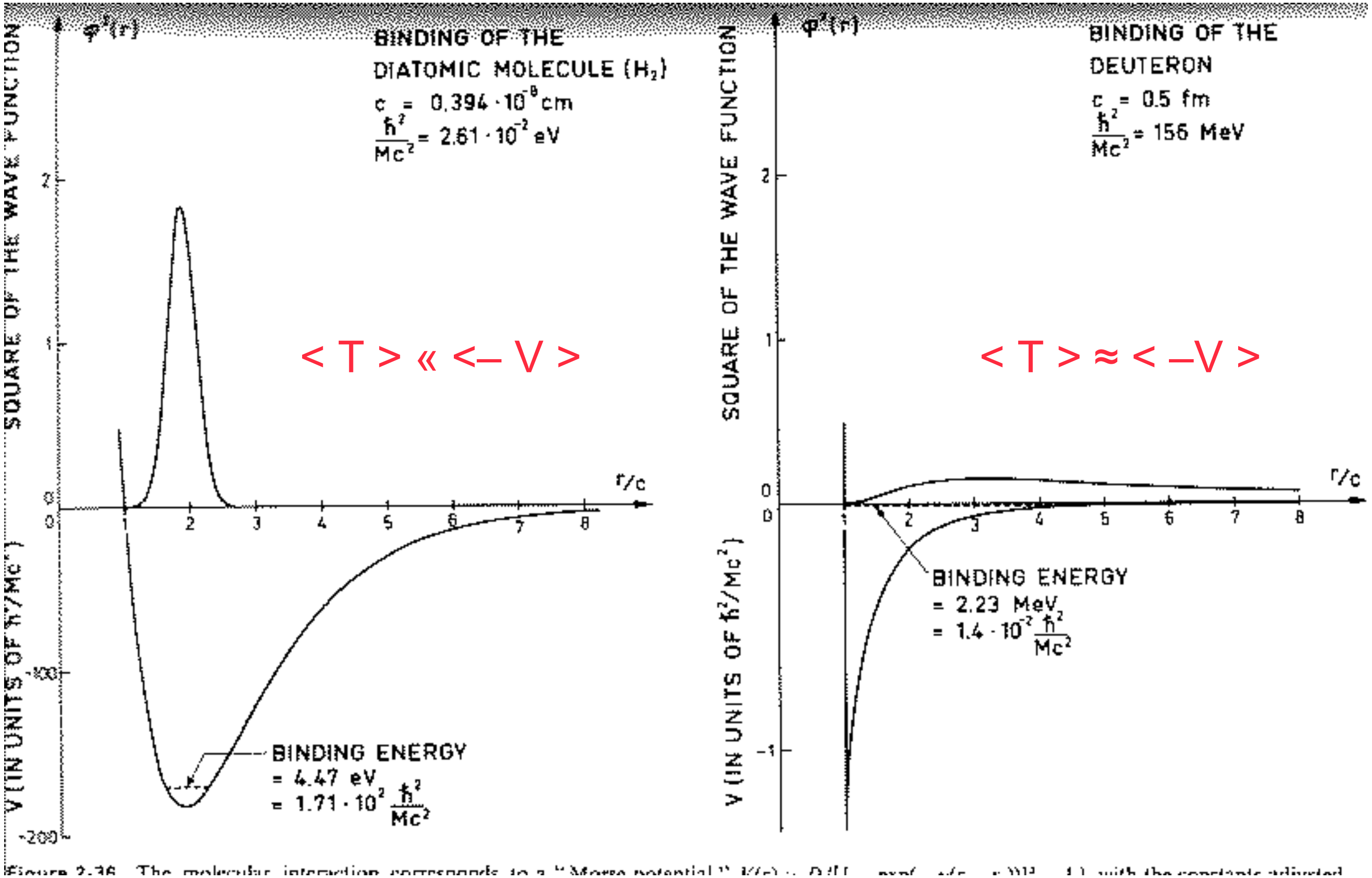
$$V_0 \quad \text{vs} \quad \hbar^2 / 2Mc^2$$

$$c: \text{ Length scale of the interaction}$$

$$V_0: \text{Energy scale of the interaction}$$

Nuclear force vs molecular force

Bohr, Mottelson, Nucl. Str. Vol.1



Crystallized at low temperature

Classical → MD

Liquid at low temperature

Quantum

Spontaneous symmetry breaking (SSB) of the rotational and gauge symmetry

- Hamiltonian is rotationally invariant

$$\left[H, \vec{J}\right] = 0$$

  - **But many nuclei are deformed !**

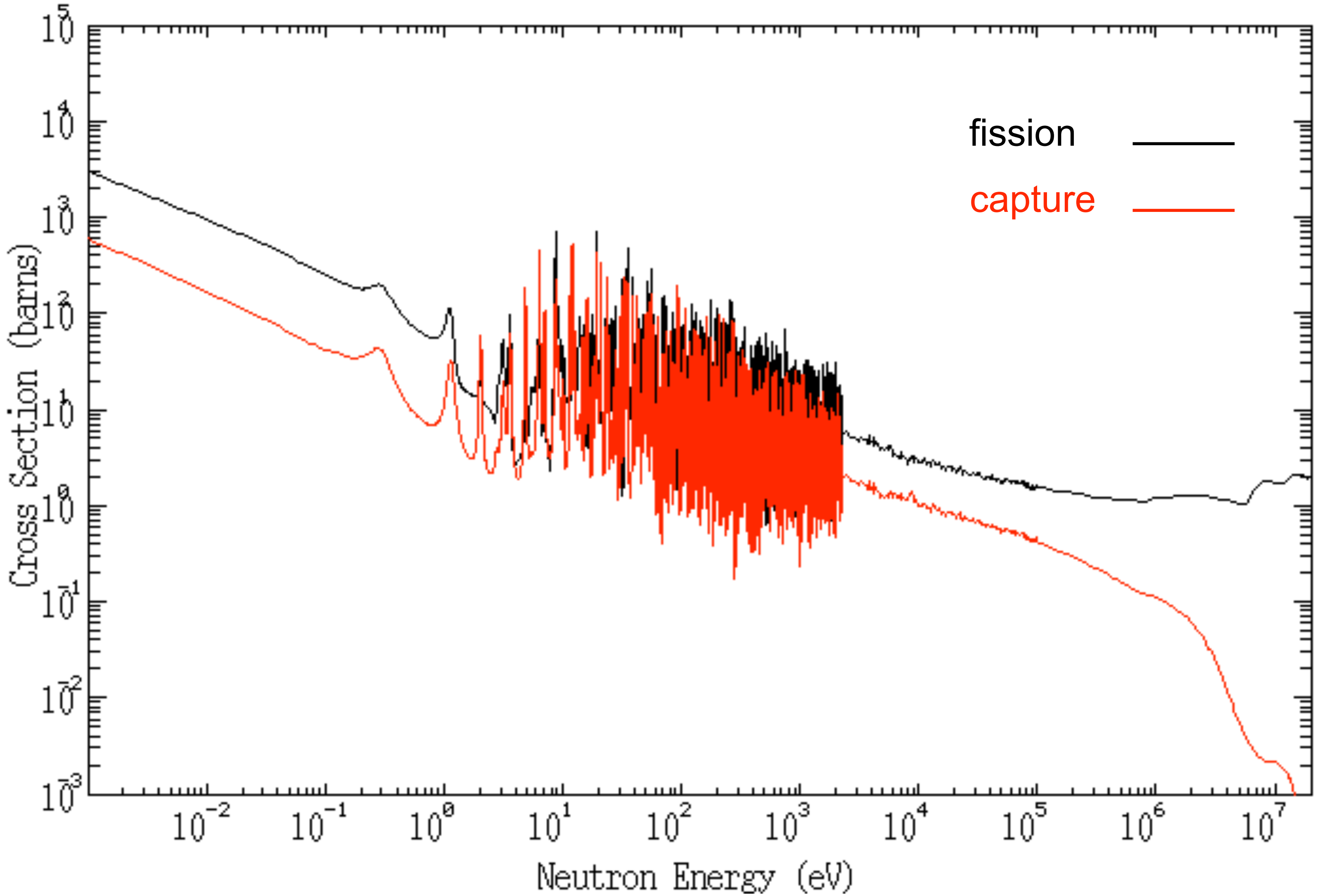
- Hamiltonian is rotationally invariant in the gauge space

$$\left[H, N\right] = 0$$

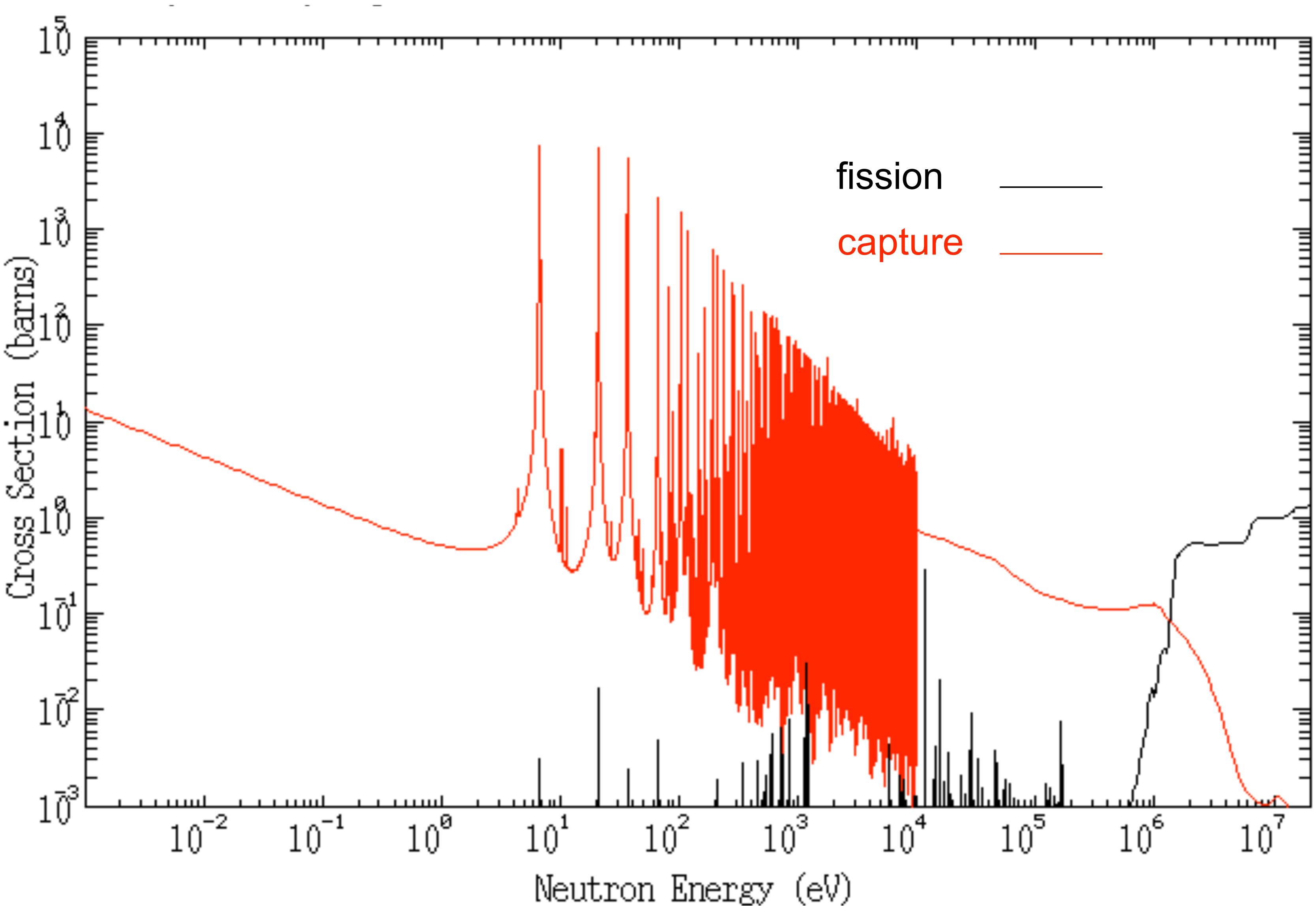
  - **But many (heavy) nuclei are superfluid !**

- e. g., Uranium nucleus (235,238) are believed to be deformed and in the superfluid phase.

Neutron cross section of ^{235}U



Neutron cross section of ^{238}U



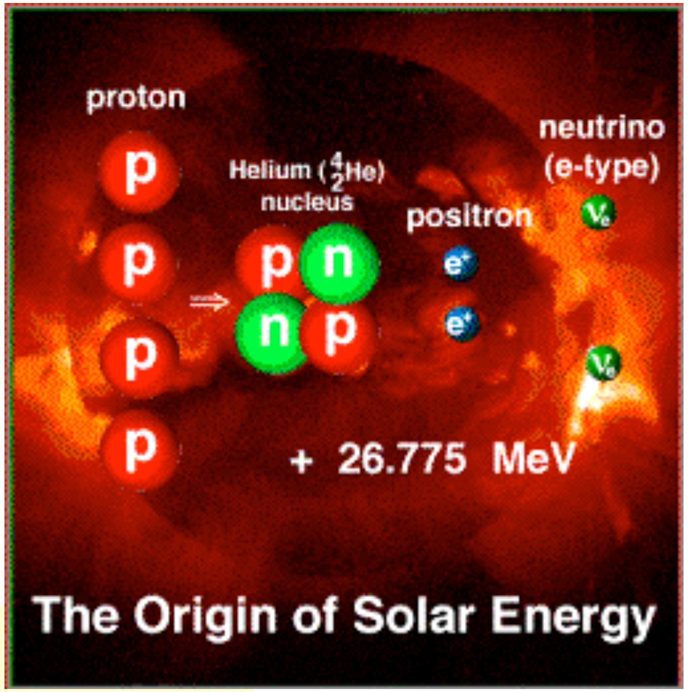
Different reaction rate

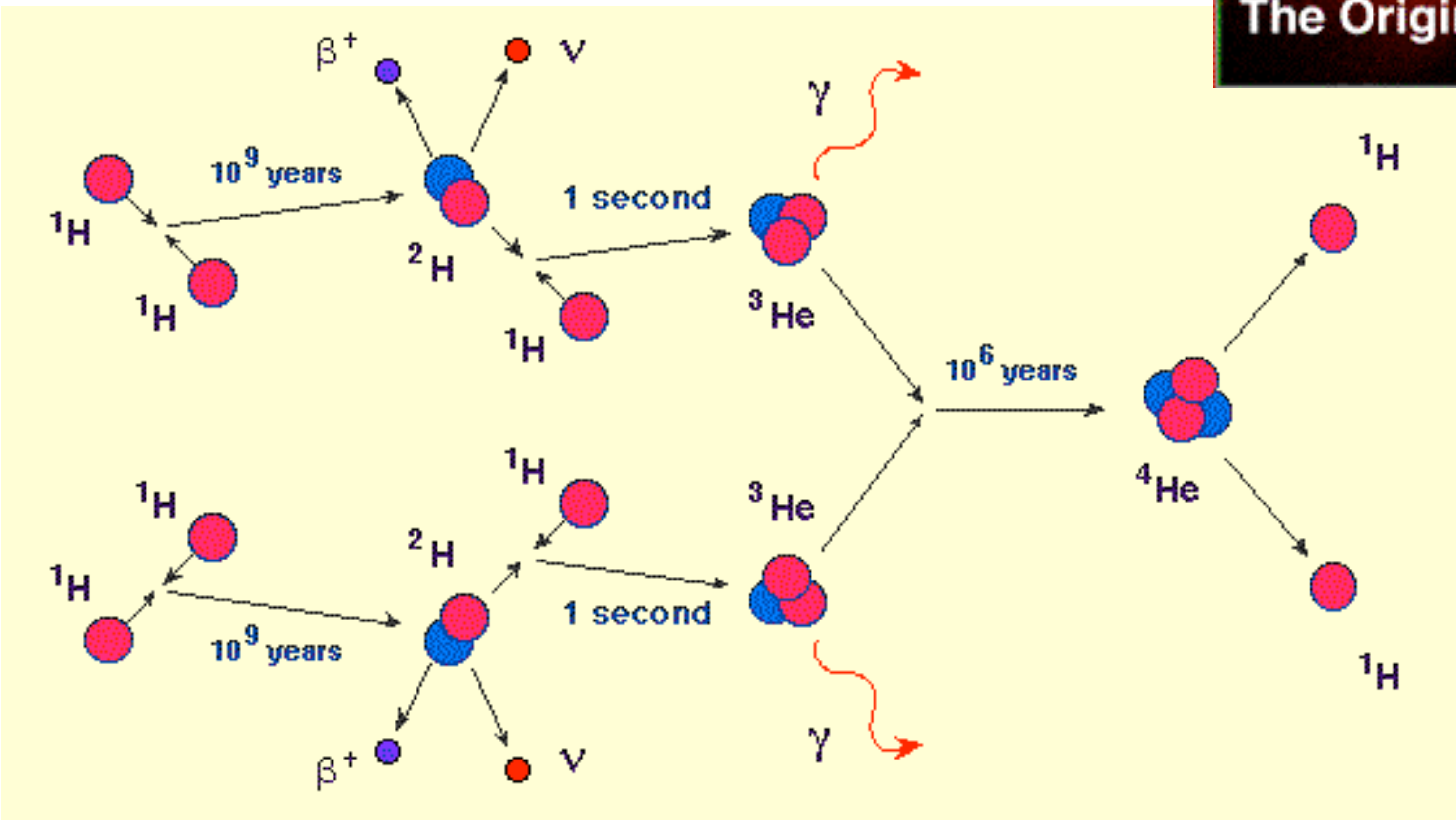
- Transfer reaction (Strong interaction)
  - $^{15}N(p,\alpha)^{12}C$
  - $\sigma \sim 0.5$ b (E=2 MeV)
- Capture reaction (Electromagnetic interaction)
  - $^{3}He(\alpha,\gamma)^{7}Be$
  - $\sigma \sim 10^{-6}$ b (E=2 MeV)
- Weak process (Weak interaction)
  - $p(p,e^{+}\nu)d$
  - $\sigma \sim 10^{-20}$ b (E=2 MeV)

→ *Different time scale*

pp chain (I)

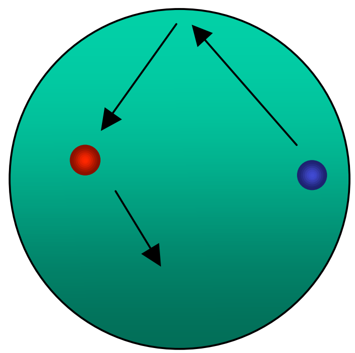
- $p(p,e^{+}\nu)d$ reaction determines the lifetime of the sun



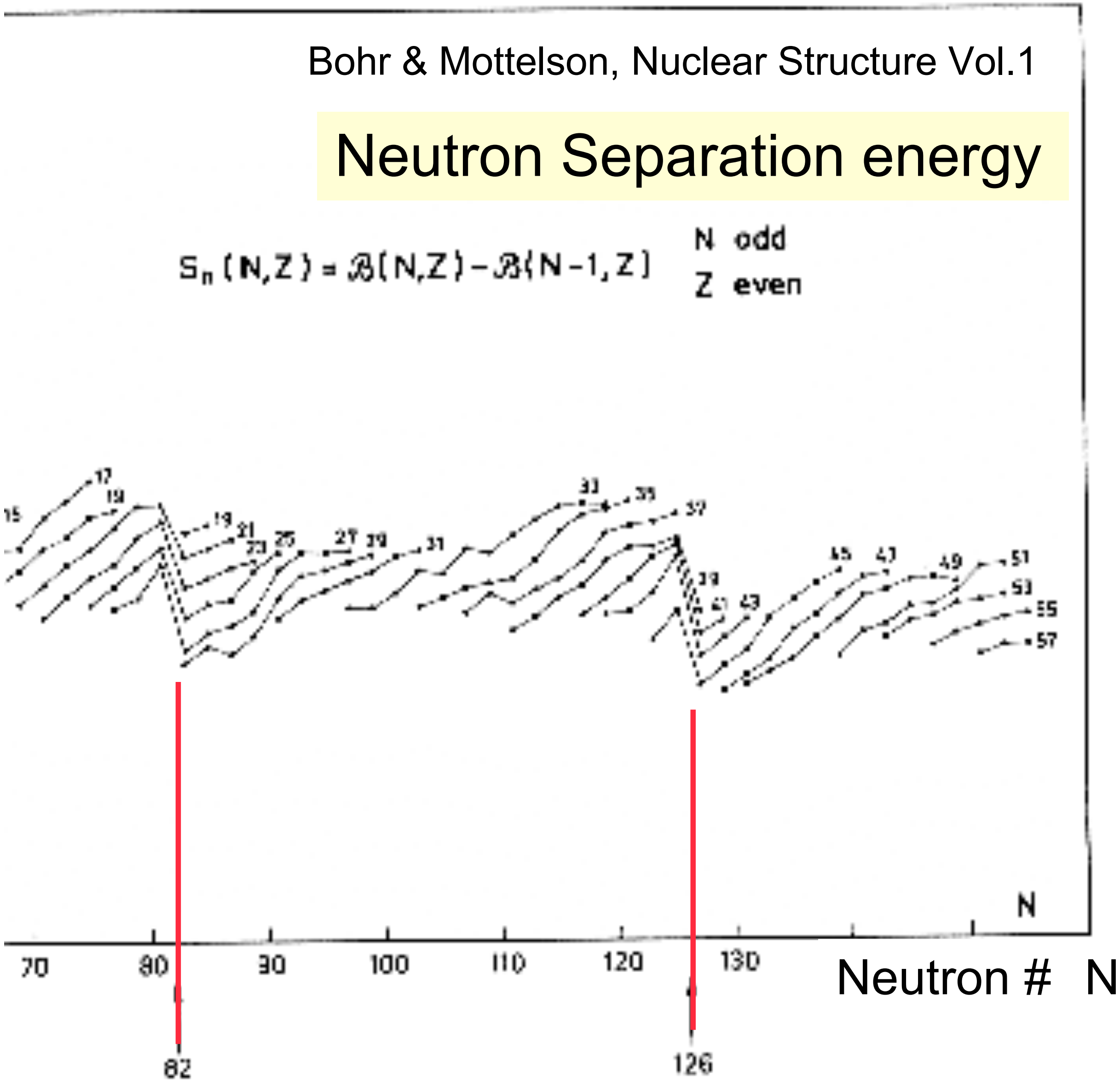


Nucleus with Different Time Scales

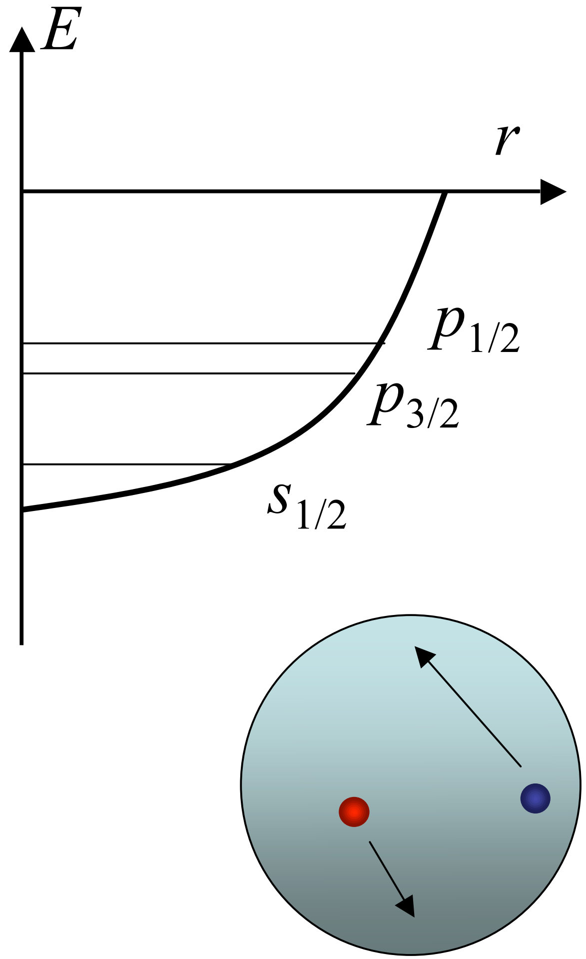
- Time period of nucleonic Fermi motion
  - $\tau_F \sim R/v_F \sim 10^{-22}$ sec
- Collision time
  - $\tau_c >> \tau_F$ (Nucleon near Fermi energy)
  - $\tau_c \geq \tau_F$ (Thermal neutron)



Nucleonic single-particle motion in nucleus

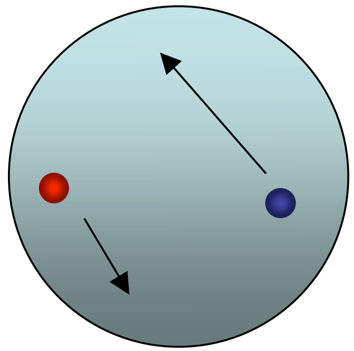


Shell Model
(Mayer-Jensen)



SSB in the Nuclear Time Scale

- Time period of nucleonic Fermi motion
  - $\tau_F \sim R/v_F \sim 10^{-22}$ sec



- Collision time
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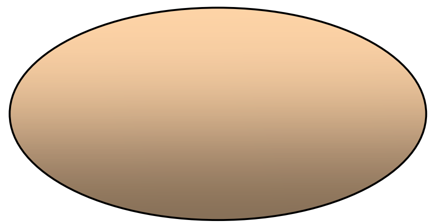
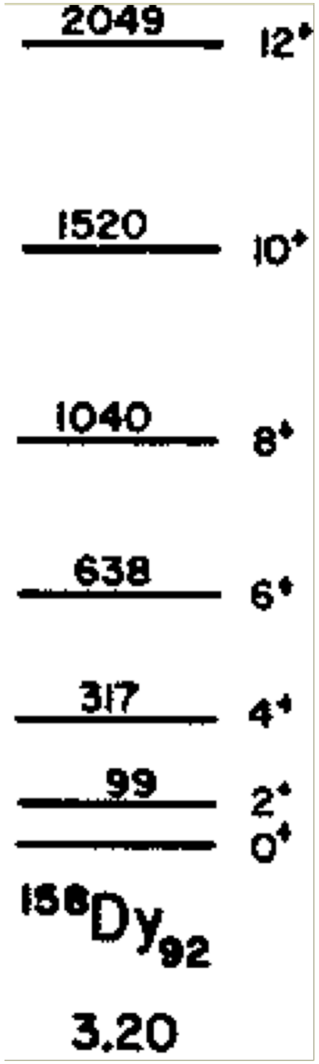


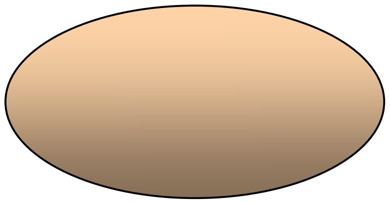
- Time scale of symmetry breaking
  - $\tau_{SB} >> \tau_F$ (10^{-20} sec)
  - Broken symmetry is restored by Nambu-Goldstone modes in time of τ_{SB}

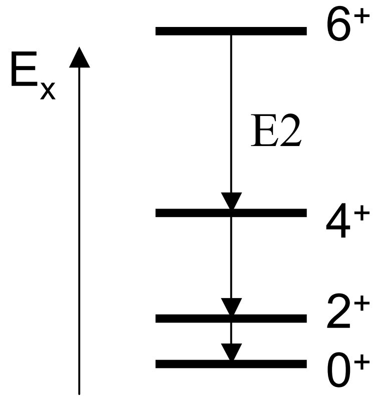
Nuclear Deformation



- Rotational excitation spectra
- Strong transition strength

$$E_x = \frac{I(I+1)}{2\Im}$$





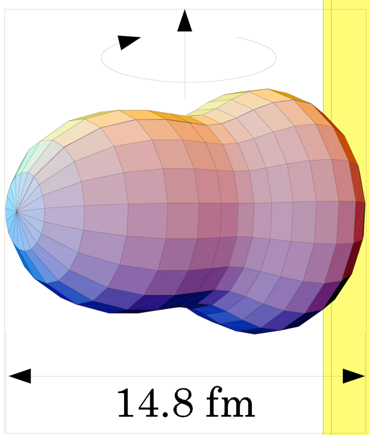
$$B(E2; I \rightarrow I-2) \equiv \frac{1}{2I+1} \left| \langle I \| M(E2) \| I-2 \rangle \right|^2$$

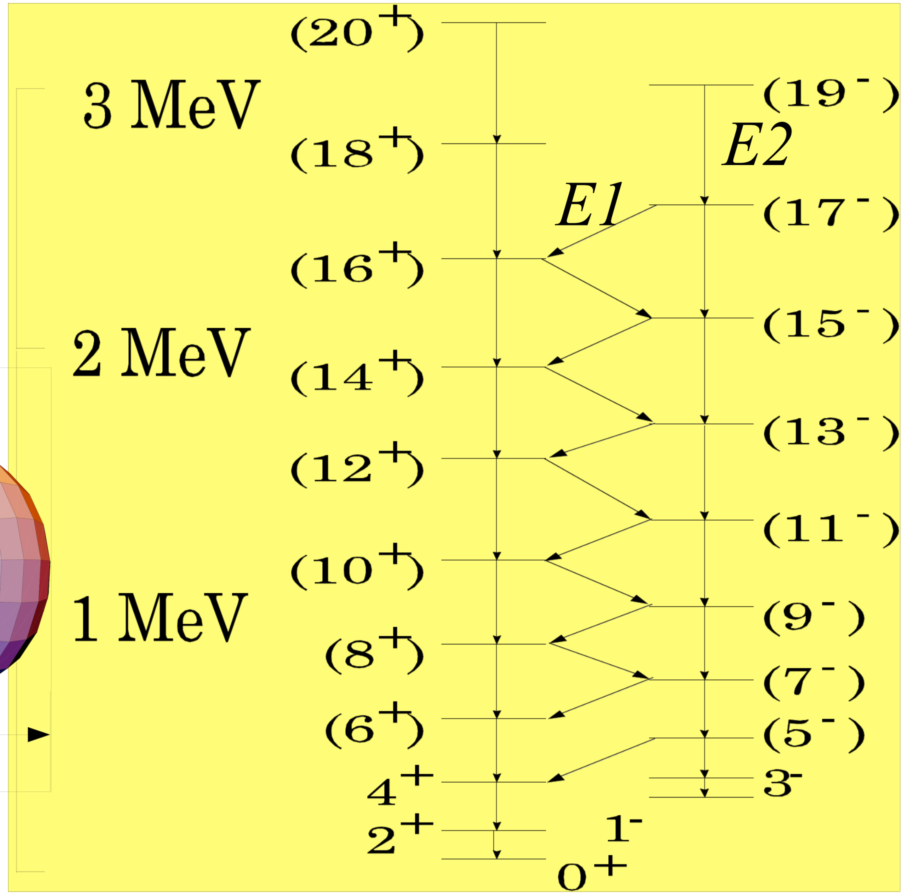
More than hundred times larger than typical single-particle values

Nuclear Deformation

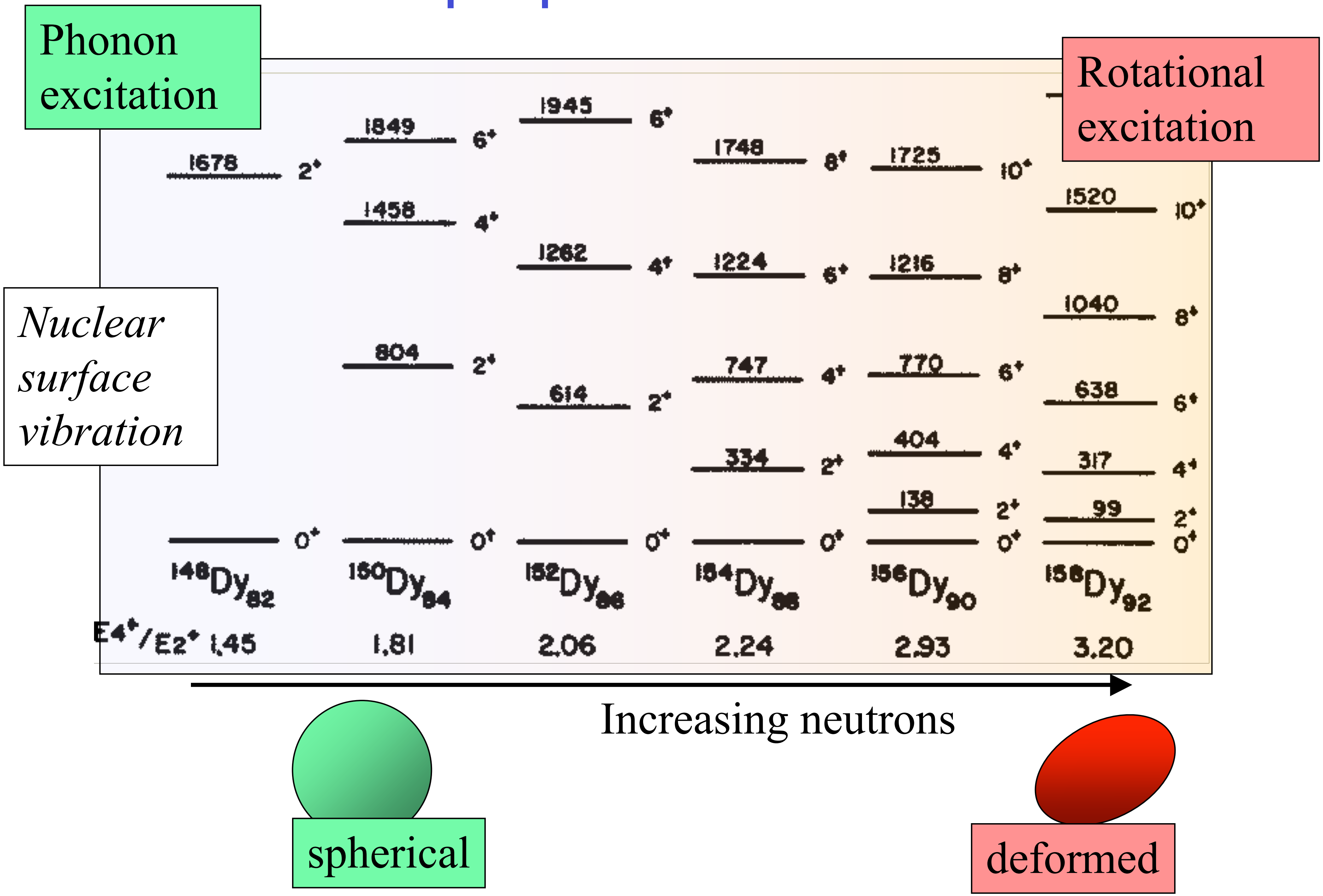
- Characteristic spectra associated with different symmetry breaking

$$E_x = \frac{I(I+1)}{2\Im}$$





Shape phase transition



Nuclear deformation as phonon condensation

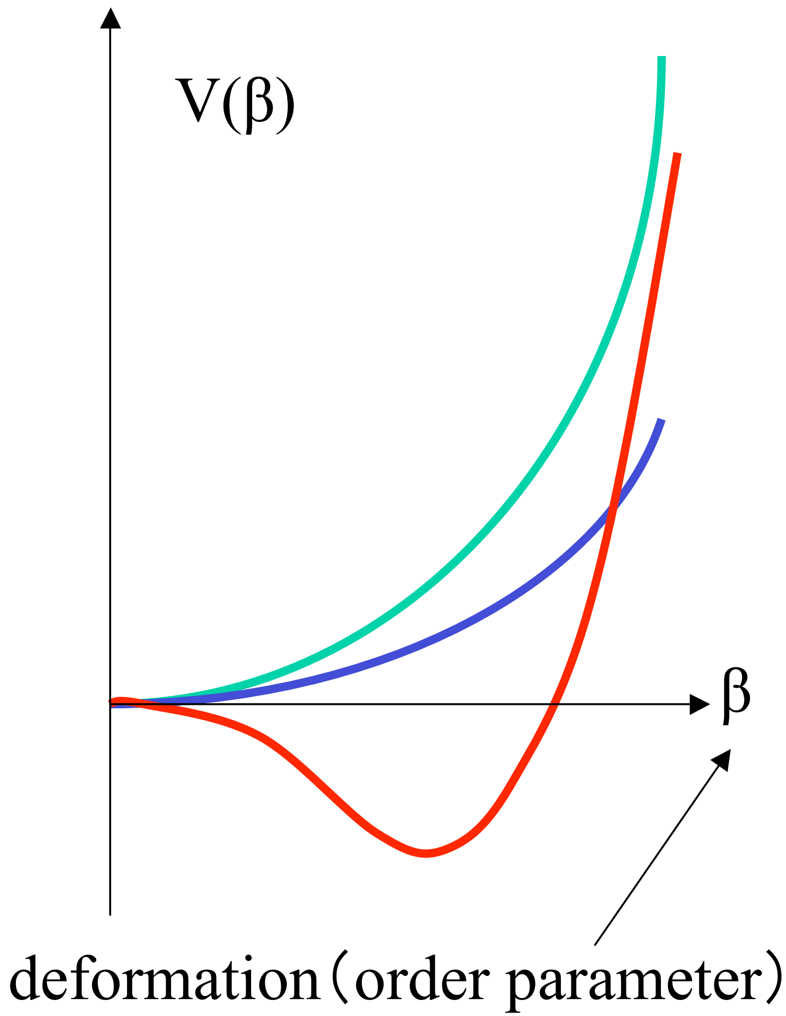
$|\Phi_0\rangle$ Ground state

$B^+_{2^+}|\Phi_0\rangle$ Excited state (2^+)

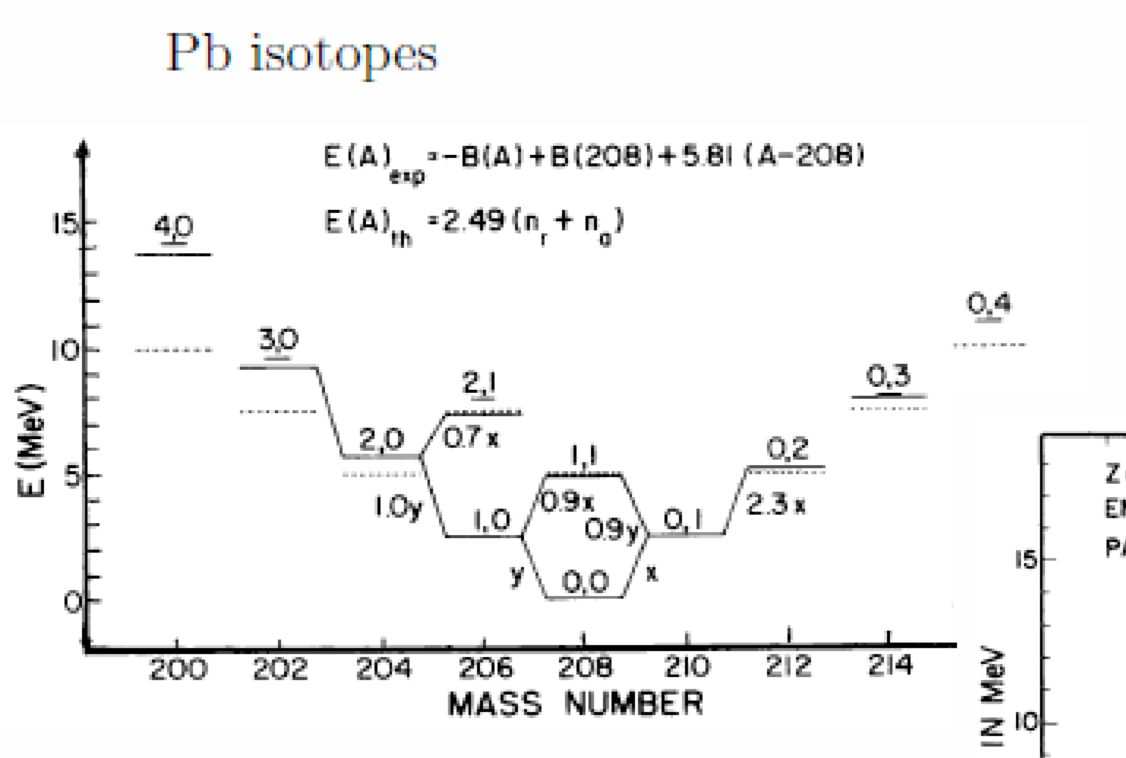
Away from the closed-shell

$$|\Phi_{\rm def}\rangle = \exp(cB^+_{2^+})|\Phi_0\rangle$$

$$\beta \equiv \langle\Phi_{\rm def}|r^2 Y_{20}(\hat{r})|\Phi_{\rm def}\rangle$$



Pairing vibration to Pairing rotation

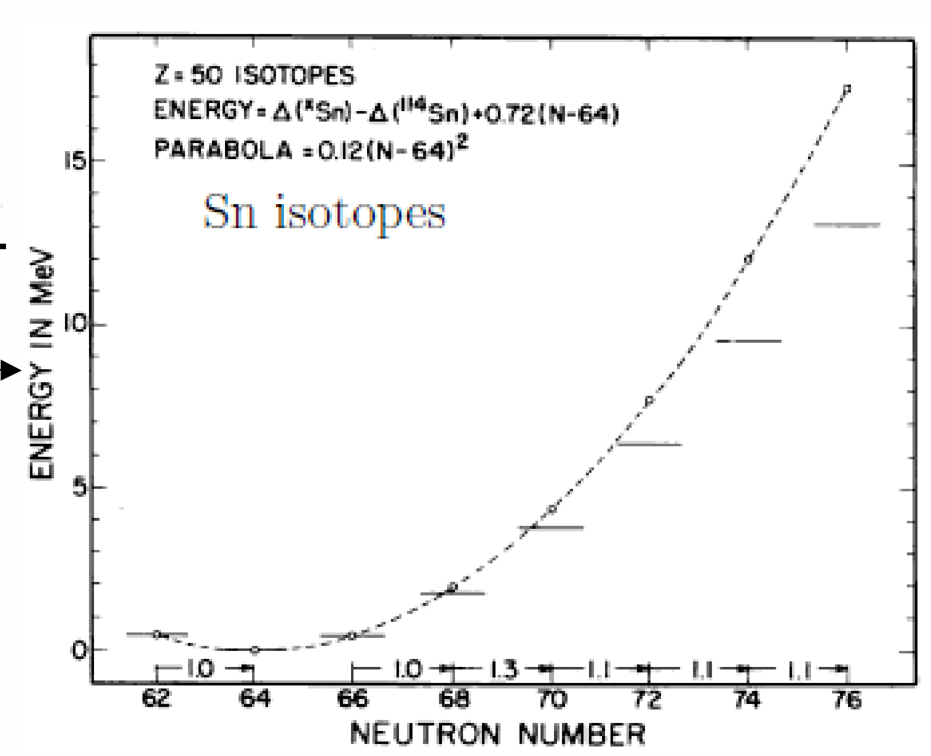


$$E_N = \frac{(N - N_0)^2}{2\Im}$$

Pair removal

Pair addition

$$E_N = \hbar\omega \left| N - N_0 \right|$$



Nuclear superfluidity as pair condensation

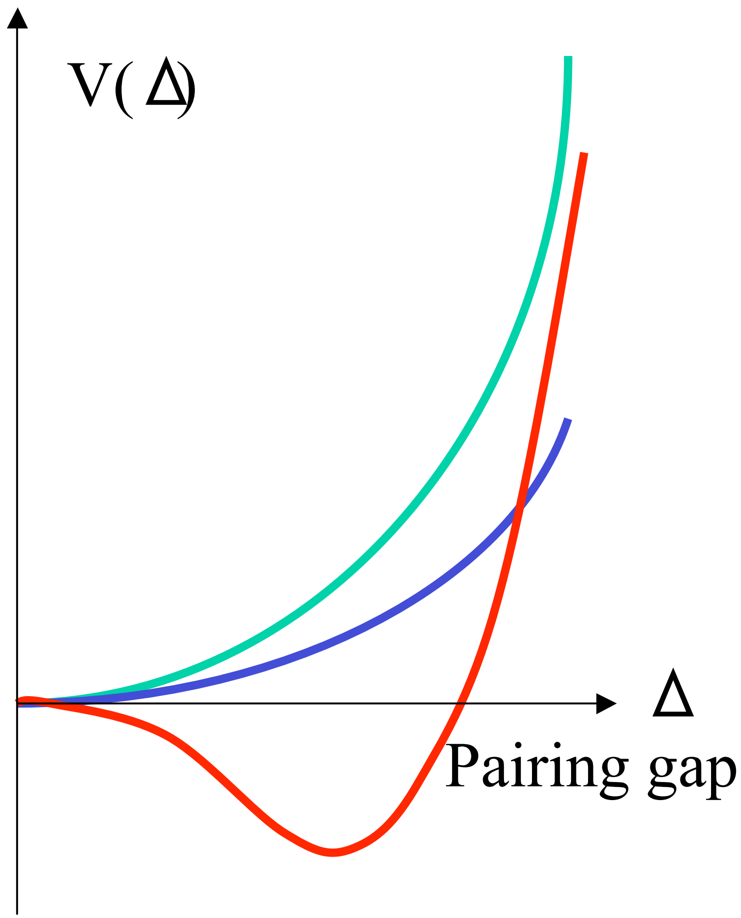
$|\Phi_0\rangle$ Ground state

$P^+_{0^+}|\Phi_0\rangle$ Pairing vibration

Away from the closed-shell

$$|\Phi_{\text{super}}\rangle = \exp(cP^+_{0^+})|\Phi_0\rangle$$

$$\Delta \propto \langle \Phi_{\text{super}}|(\hat{\psi}\hat{\psi})_{0^+}|\Phi_{\text{super}}\rangle$$



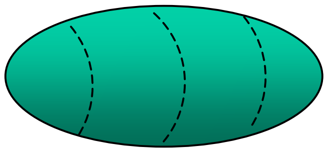
Symmetry breaking in nuclei

$$e^{i\phi J}\left|\Psi\right\rangle \neq \left|\Psi\right\rangle$$

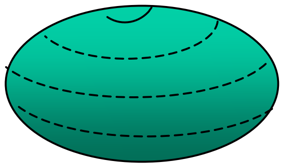
$$e^{i\phi N}\left|\Psi\right\rangle \neq \left|\Psi\right\rangle$$

Quadrupole deformation

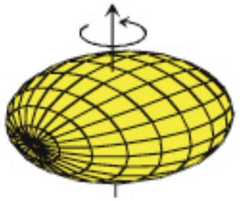
$$\beta_{2\mu} = \left\langle\Psi\right|r^2 Y_{2\mu}\left|\Psi\right\rangle$$



prolate



oblate

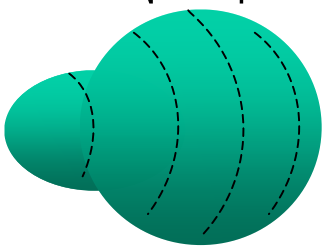


triaxial

Octupole deformation

$$\beta_{30} = \left\langle\Psi\right|r^3 Y_{30}\left|\Psi\right\rangle$$

$$\hat{P}\left|\Psi\right\rangle \neq \pm\left|\Psi\right\rangle$$



Pear shape (μ=0)

Pairing deformation (superfluidity)

$$\Delta = \left\langle\Psi\right|\hat{\psi}^{+}\hat{\psi}\left|\Psi\right\rangle$$

Deformation in the gauge space

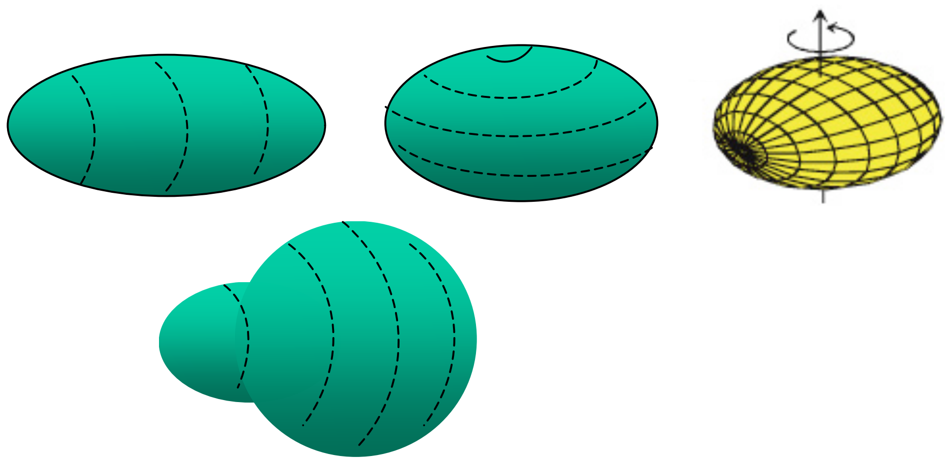
Nuclear Superconductivity
Nuclear Superfluidity

Symmetry breaking in nuclei

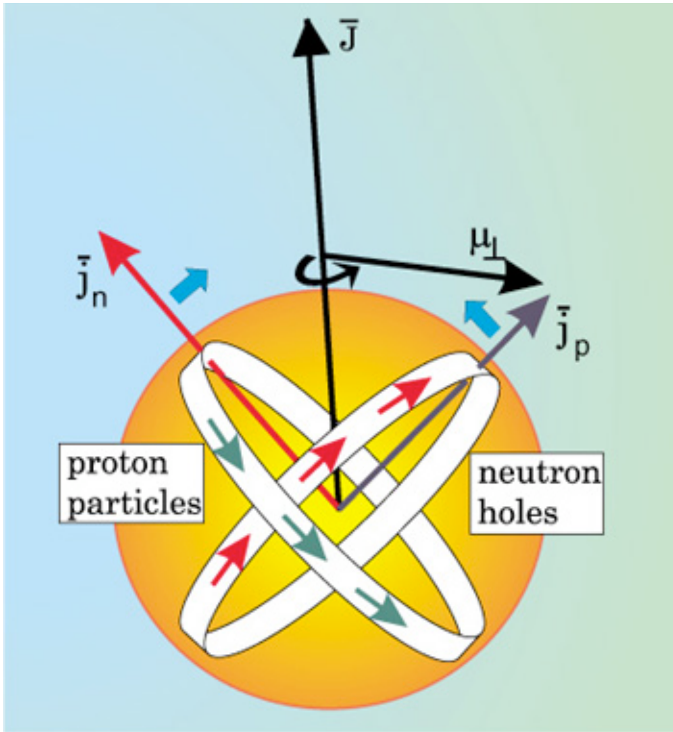
$$e^{i\phi J}|\Psi\rangle \neq |\Psi\rangle$$

S. Frauendorf

Spatial deformation
(Electric deformation)



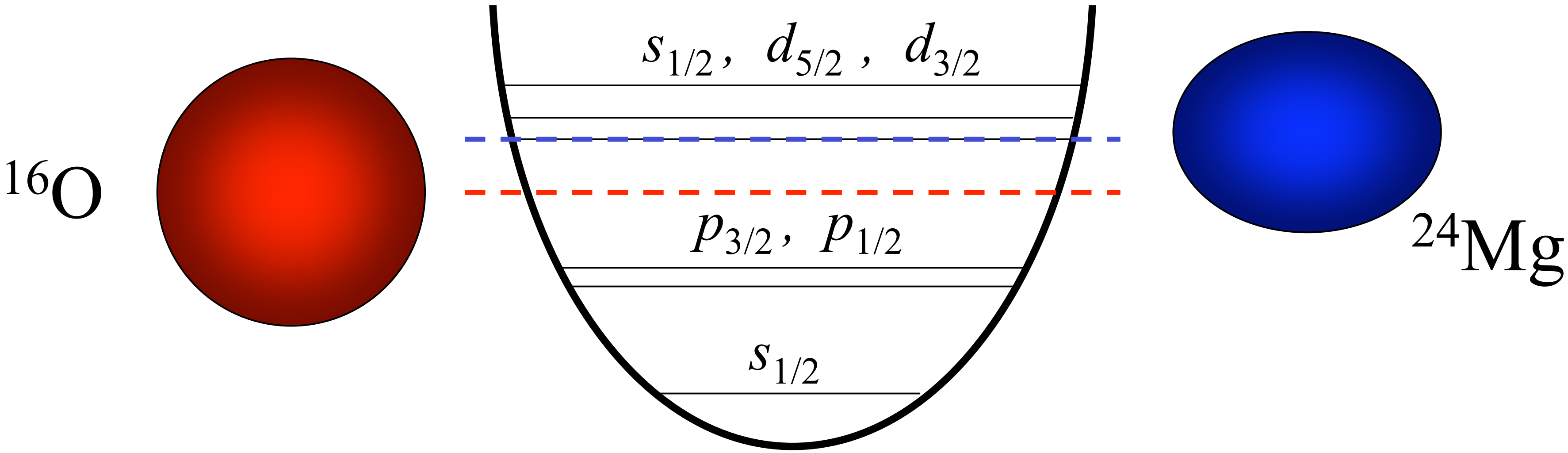
Magnetic deformation ?



Shell Structure

Approximate degeneracy of single-particle levels

Stability and Instability of spherical shape



Driving force of SSB

Jahn-Teller Mechanism

Shell Structure

Bohr, Mottelson, Nucl. Str. Vol.2

Single-particle energies in a spherical potential can be specified by the quantum numbers (n, l)

$$\varepsilon(n,l) \approx \varepsilon(n_0,l_0) + \left.\frac{\partial \varepsilon}{\partial n}\right|_0 (n-n_0) + \left.\frac{\partial \varepsilon}{\partial l}\right|_0 (l-l_0)$$

$$\left.\frac{\partial \varepsilon}{\partial n}\right|_0 : \left.\frac{\partial \varepsilon}{\partial l}\right|_0 = a:b$$

Integer a:b → Degeneracy

(2,0), (1,1), (0,3)

(1,1), (0,3)

(1,0), (0,2)

(0,1)

(n, l)=(0,0)

Nuclear shell structure

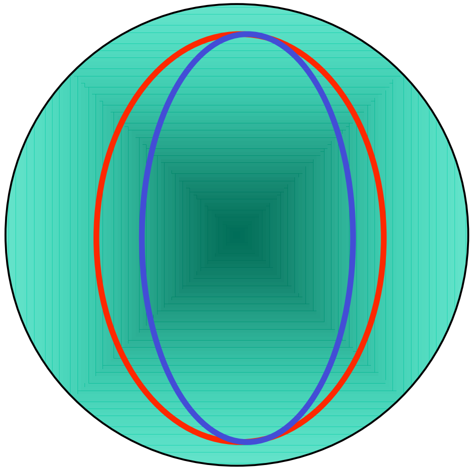
$$a:b=2:1$$

Classical correspondence

Action-angle variables: (l, φ) $\qquad \frac{\partial \varepsilon}{\partial l} = \varphi$

$$\left.\frac{\partial \varepsilon}{\partial n}\right|_0 : \left.\frac{\partial \varepsilon}{\partial l}\right|_0 = \varphi_r : \varphi_l = a : b$$

Closed periodic orbital



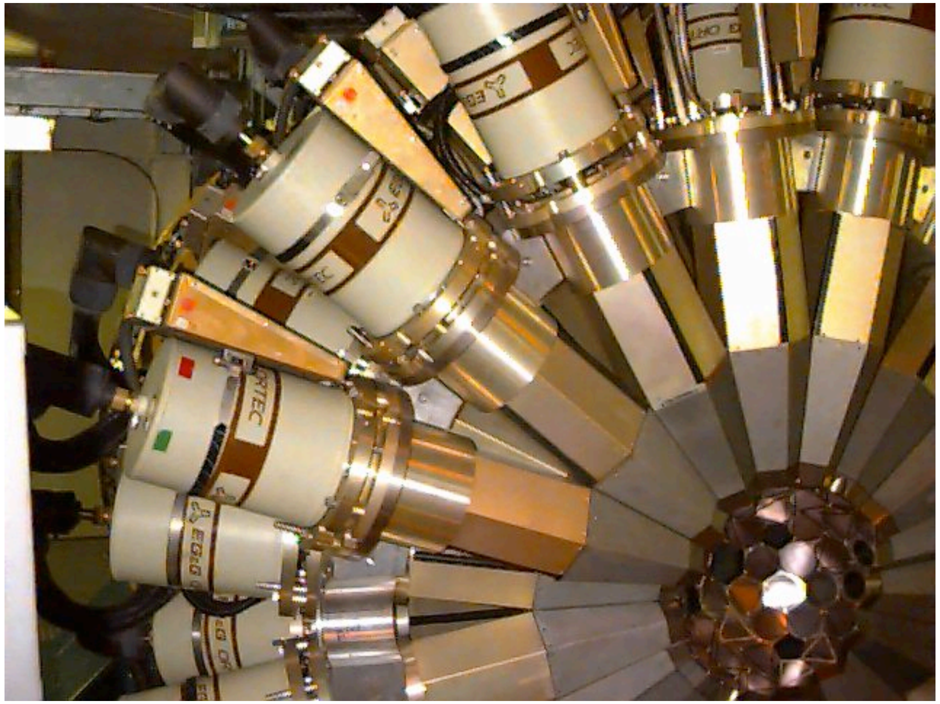
$a : b = 2 : 1$ → Elliptic orbits

→ SSB to ellipsoidal shape in open-shell nuclei

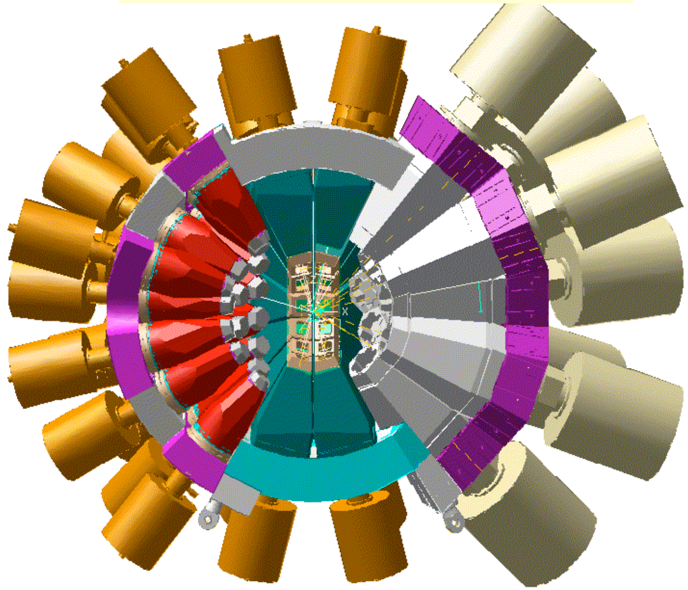
Spatial shape of w.f. created by superposing degenerate s.p. states.

Germanium gamma-ray detector arrays developed in 1980'-90's

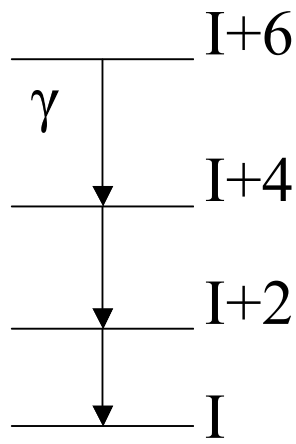
GAMMASPHERE



EUROBALL



Discovery of Superdeformed band



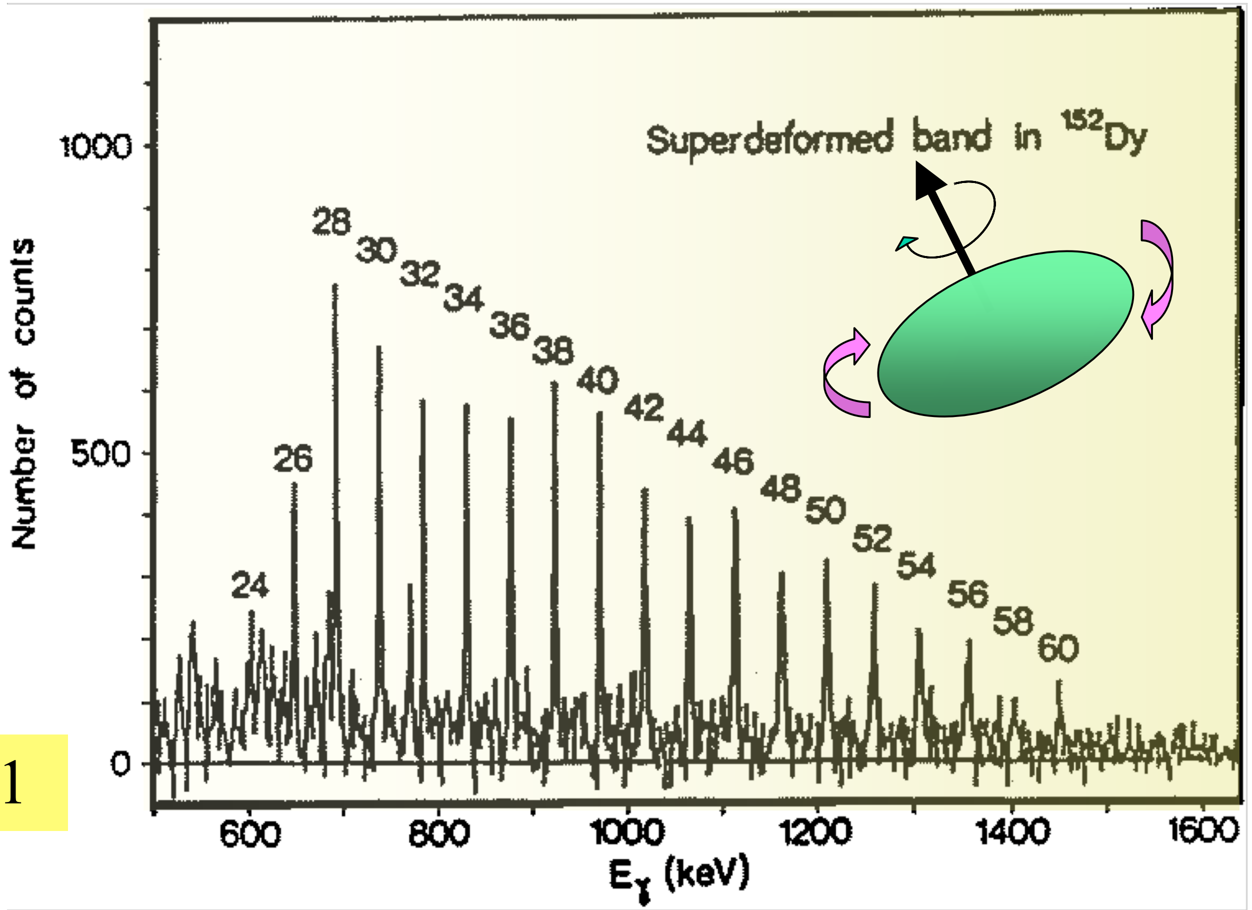
1986 @Daresbury, UK

P.J. Twin et al, PRL **57** (1986) 811

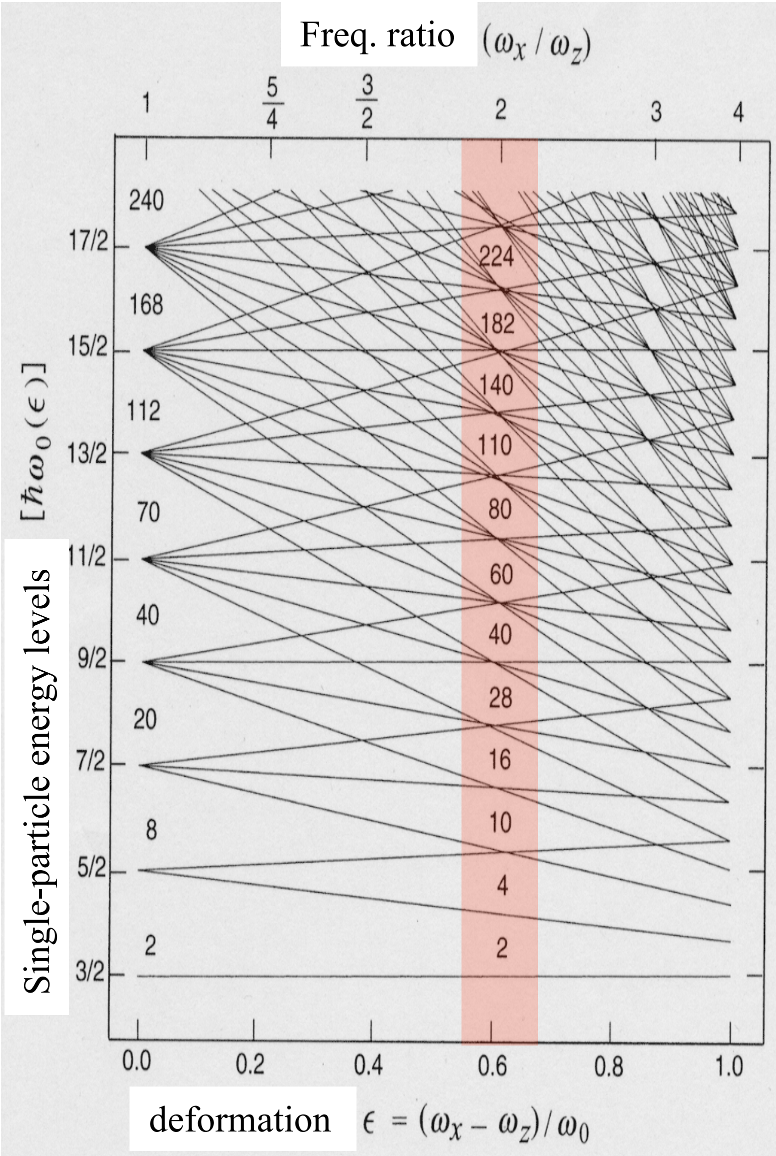
J.D. Garret et al, Nature **323** (1986) 395.

- Large moment of inertia
- Large intraband B(E2)
  B(E2)≈2000 W.u.

Major : Minor axes ~ 2:1



Mechanism of stabilizing superdeformation



3D Harmonic Oscillator Potential

Degeneracy at spherical shape

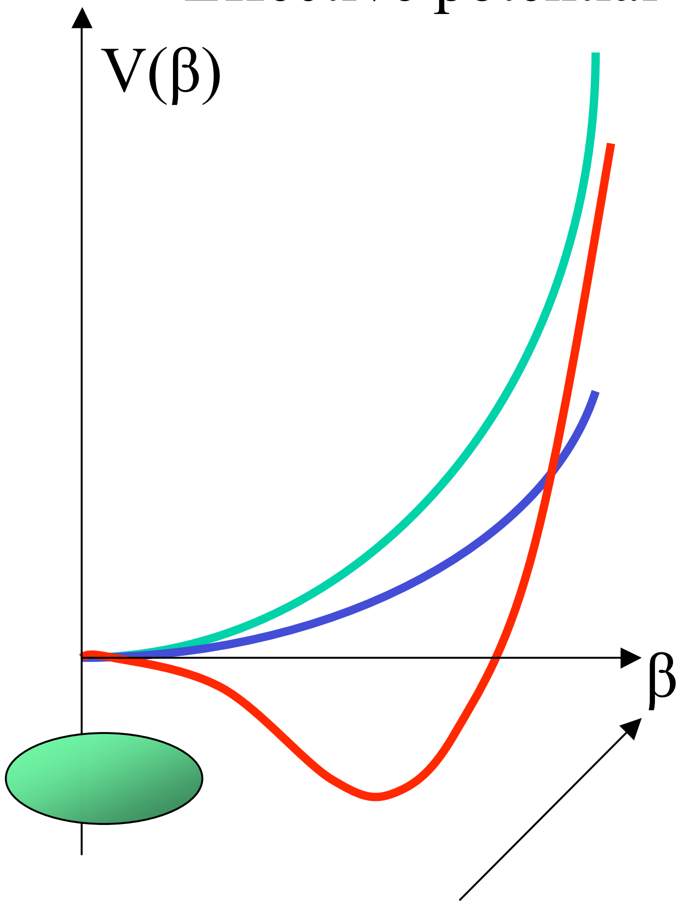
Magic numbers: 2, 8, 20, …

Degeneracy at 2:1 deformation

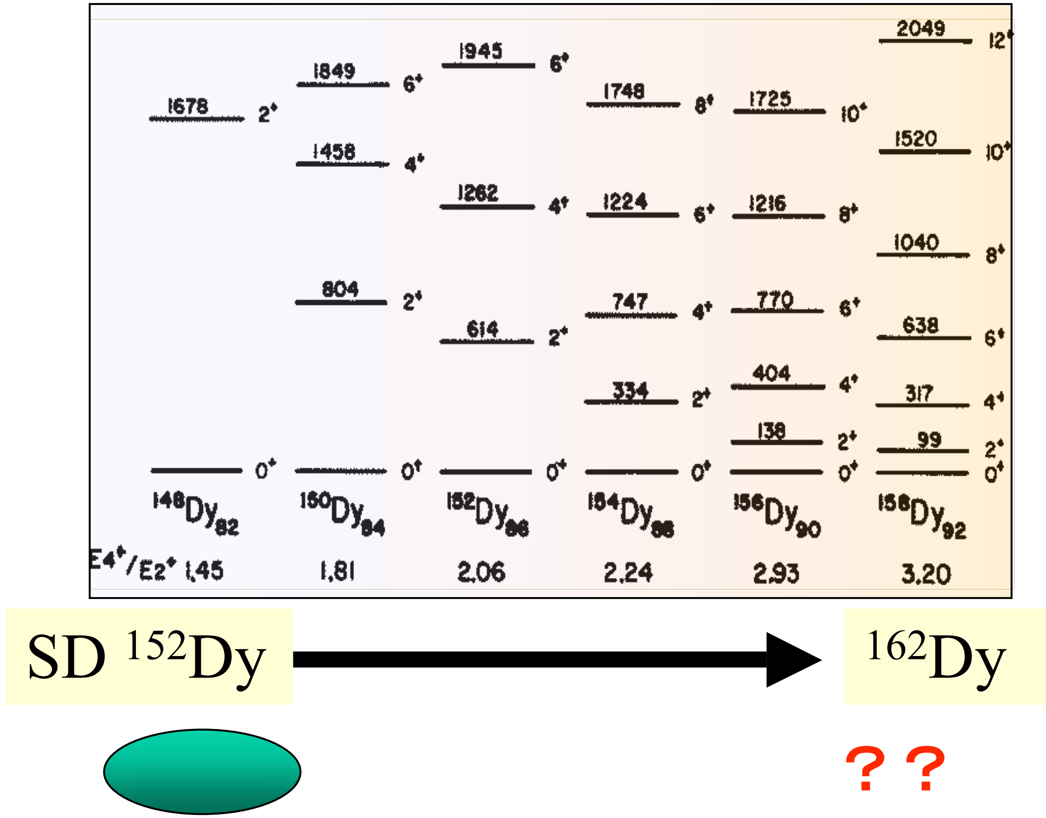
Superdeformed magic number: 2, 4, 10, 16, …

What happens if we add neutrons to "closed-shell" superdeformed nuclei?

Effective potential



What kind of parameter?



Typical classical periodic orbits in superdeformed potential

Deformed HO potential

$$(n,l) \Rightarrow (n_x, n_y, n_z)$$

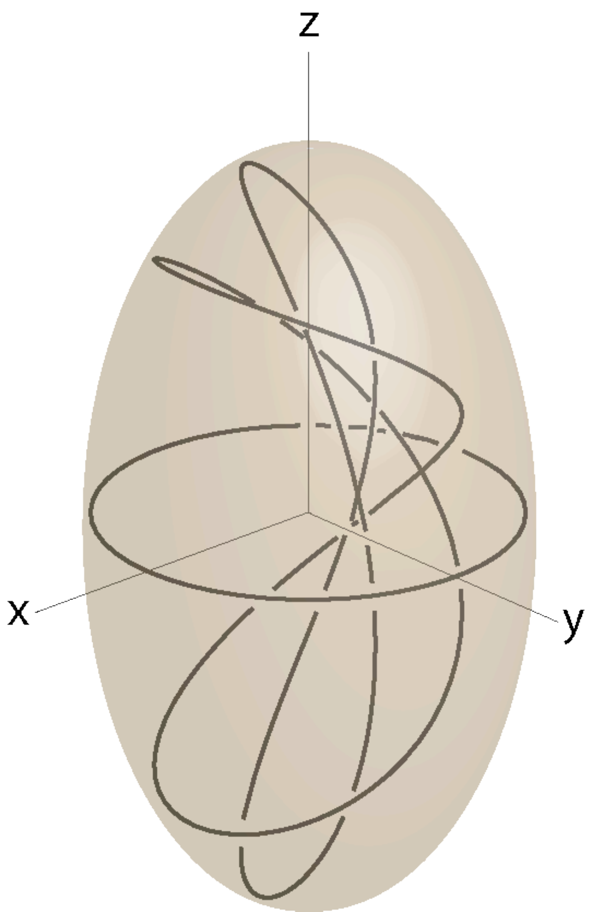
$$\left.\frac{\partial \varepsilon}{\partial n_x}\right|_0 : \left.\frac{\partial \varepsilon}{\partial n_y}\right|_0 : \left.\frac{\partial \varepsilon}{\partial n_z}\right|_0 = a:b:c$$

Superdeformation (Axis ratio 2:1)

$$a:b:c = 2:2:1$$

Bending figure of eight

SSB to Bending (Banana) Shape



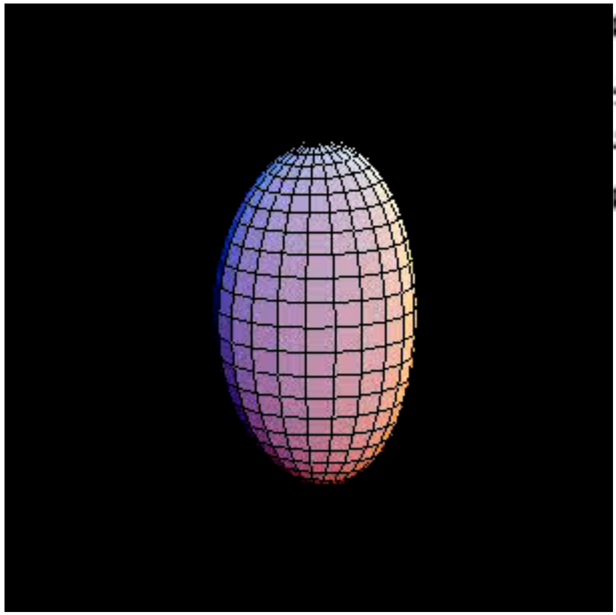
Bending-shape phase transition in open-shell SD states

T.N., S.M., K.M., Prog. Theor. Phys. **87** (1992) 607.

Increase of valence neutrons

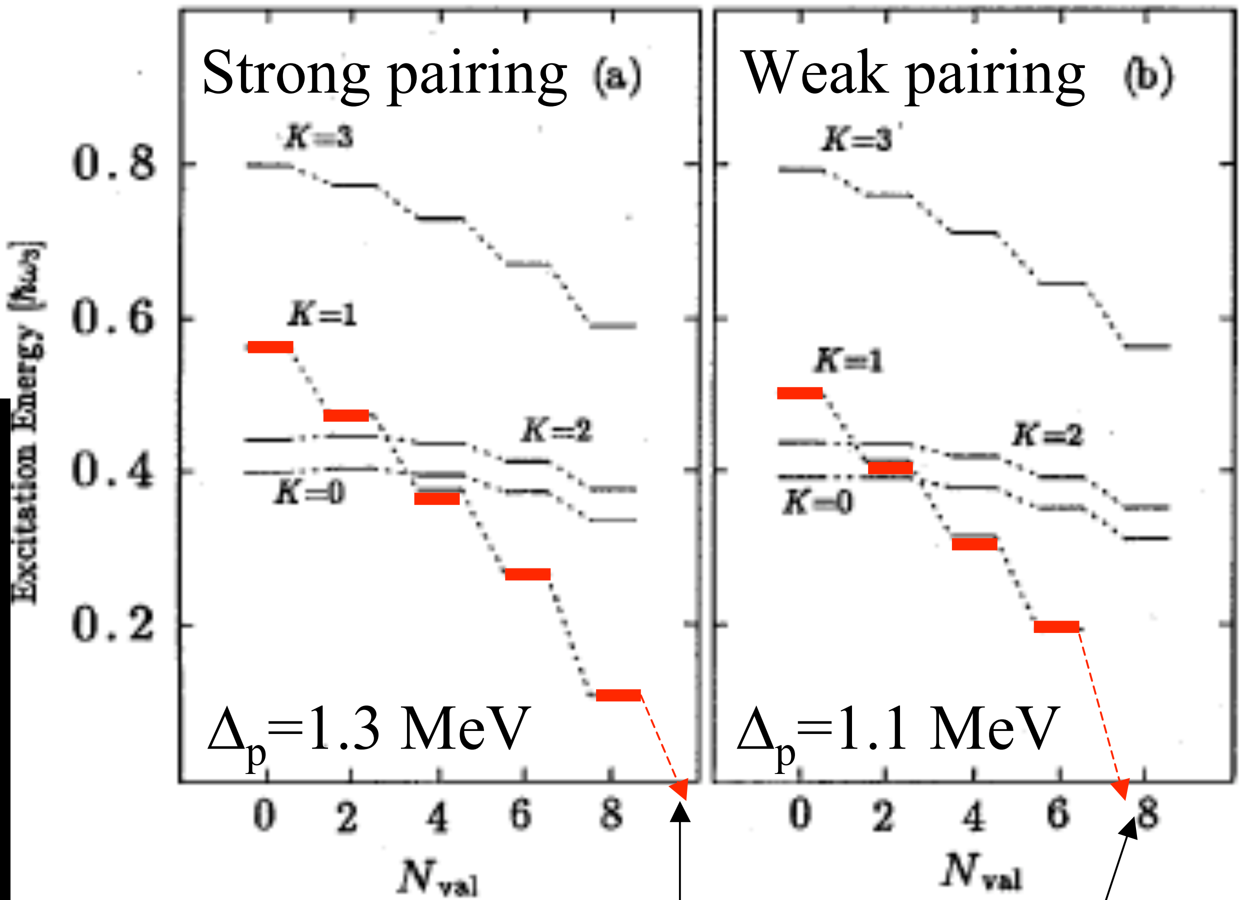
⇓

Banana-super-deformation



$Z=80+2$, $N=80+N_{val}$

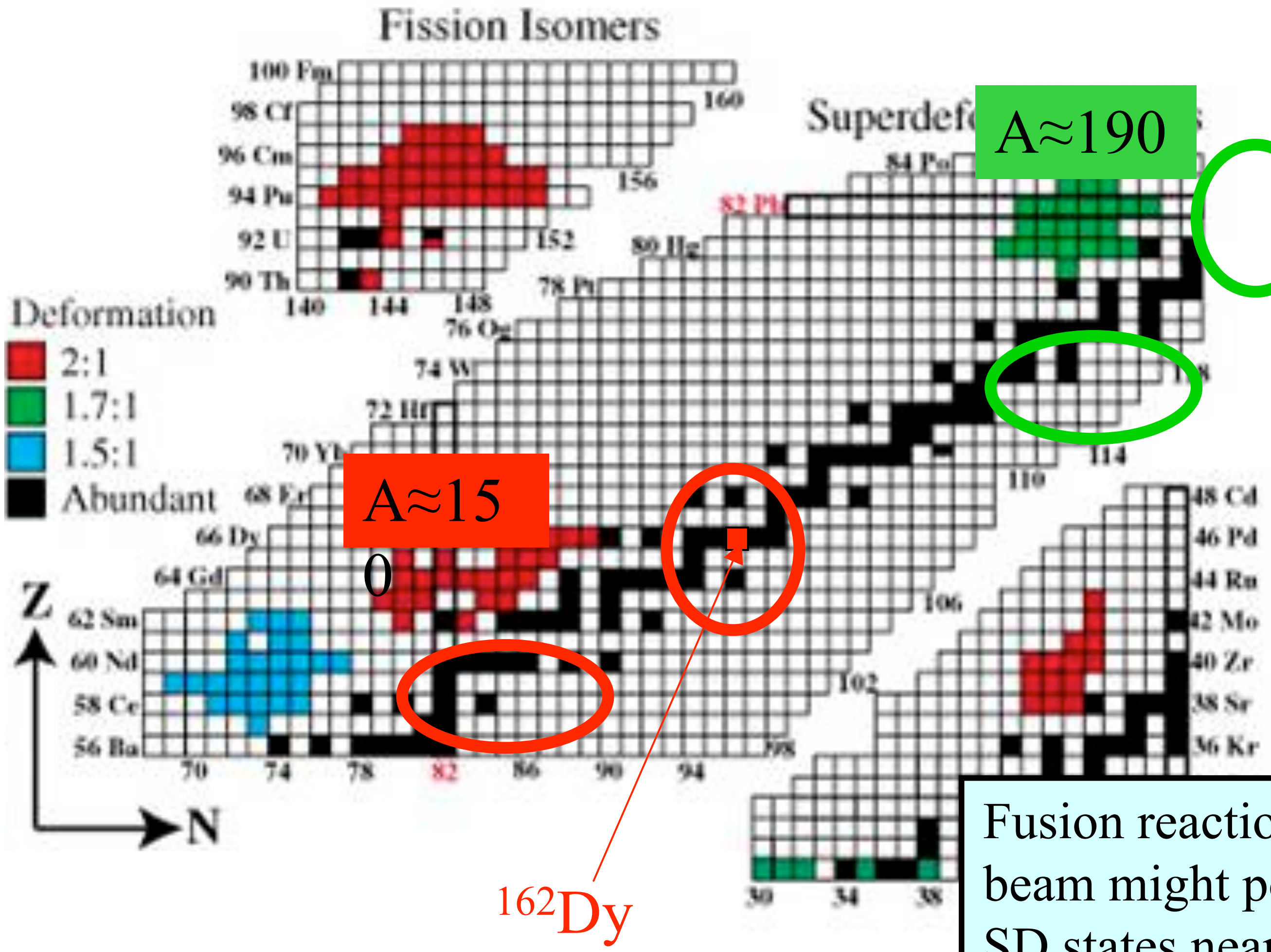
HO+QRPA



SSB towards banana shape

Where are they?

Increasing (decreasing) valence neutrons (protons) by 8-10 leads to regions near beta-stable line



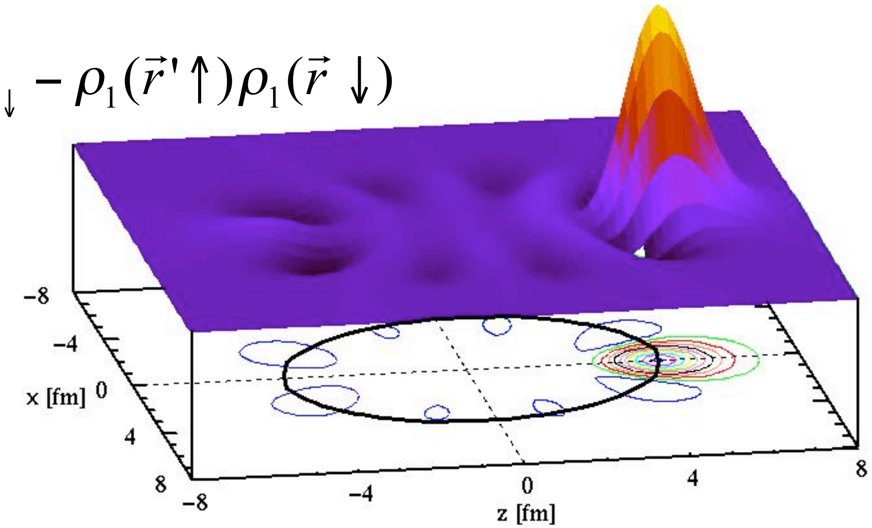
Fusion reaction with radioactive beam might populate these high-spin SD states near beta-stable line.

From BCS to BEC ?

- Correlation becomes prominent at low density.
  - In normal nuclear density, the size of nucleonic Cooper pair is larger than the size of nucleus. (~ 10 fm or larger)
  - At low density, the pair wave function may be much more compact (~ 2 fm)

$$\rho_2^{corr}(\vec{r}'\uparrow;\vec{r}\downarrow) = \sum_{i\neq j}\delta(\vec{r}-\vec{r}_i)\delta_{\sigma_i\uparrow}\delta(\vec{r}'-\vec{r}_j)\delta_{\sigma_j\downarrow} - \rho_1(\vec{r}'\uparrow)\rho_1(\vec{r}\downarrow)$$

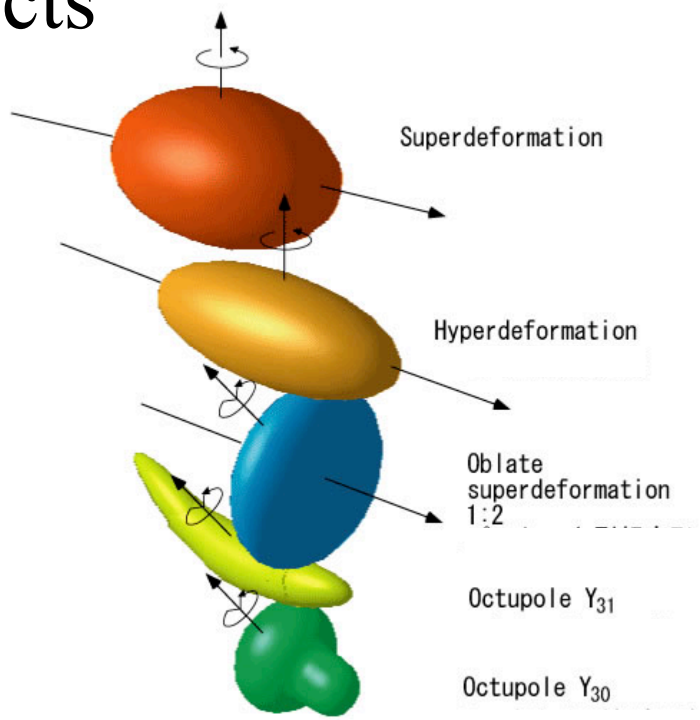
$$\approx |\Psi_{pair}(\vec{r}\uparrow,\vec{r}'\downarrow)|^2$$



M. Matsuo (Niigata Univ.)

Summary

- Nucleus is a wonderful laboratory of finite many-body systems of quantum liquid
- Nucleus shows different aspects in different time scales
- A variety of spontaneous symmetry breakings are observed in nuclei
- A new type of shape phase transition is predicted in superdeformed nuclei



CNO Cycle

- Slowest reaction determines the energy-production efficiency
  - p+p → d (pp chain)
  - Proton capture (CNO)