Physics of quantum measurement and its applications I

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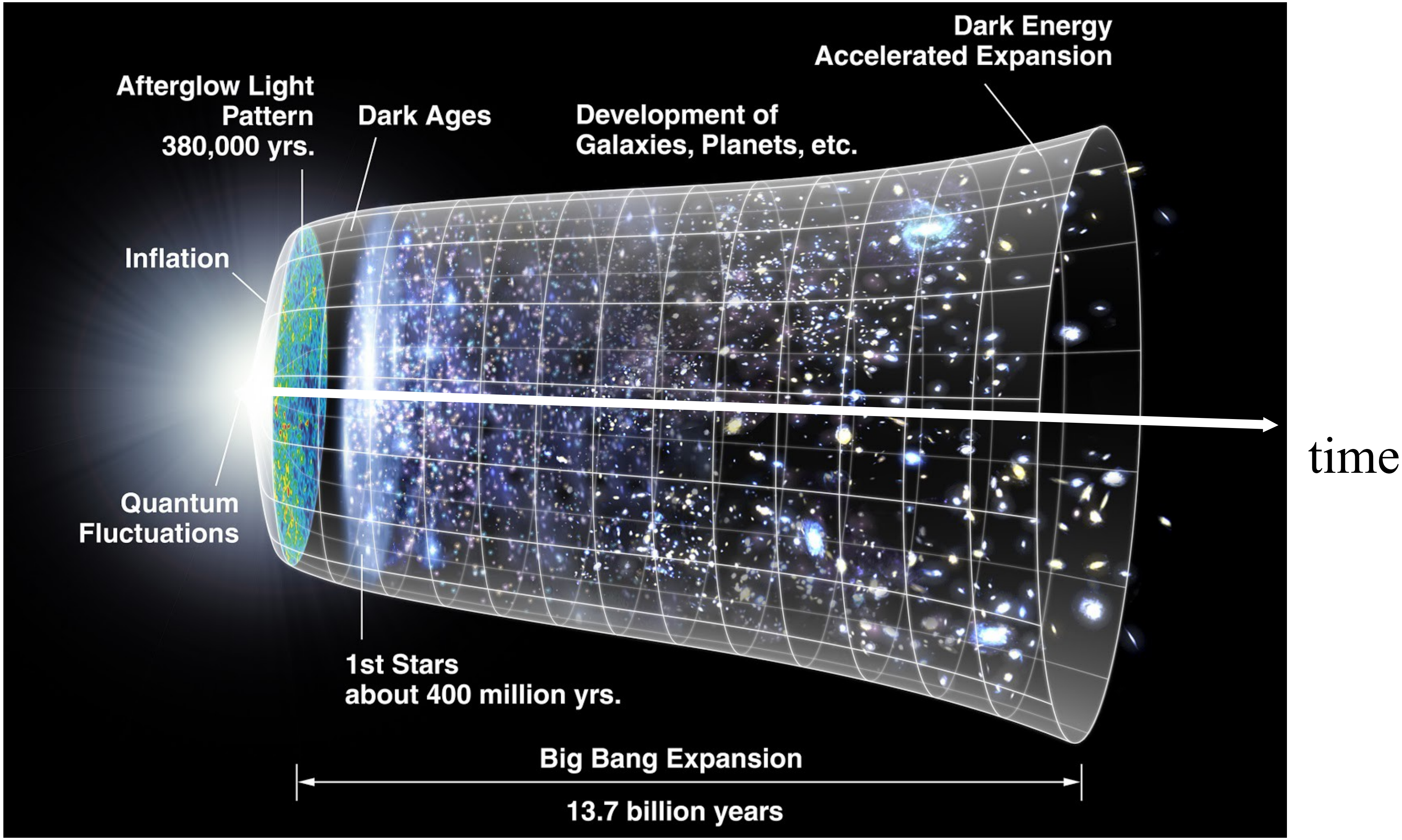
Quantum mechanics is an excellent theory to describe laboratory experiments. However its operational logic prevents us from applying it to the autonomous systems in which no explicit observer exists. The first-structure problem, i.e. generation of the primordial density fluctuations in the early Universe, belongs to such category.

In the first part of the presentation, we propose a physical description of the quantum measurement process based on the collective interaction of many degrees of freedom in the detector. This model describes a variety of quantum measurement processes including the quantum Zeno effect. In some cases this model yields the Born probability rule. By the way, the classical version of this model turns out to be a good model to describe the geomagnetic polarity flipping history over 160 Million years.

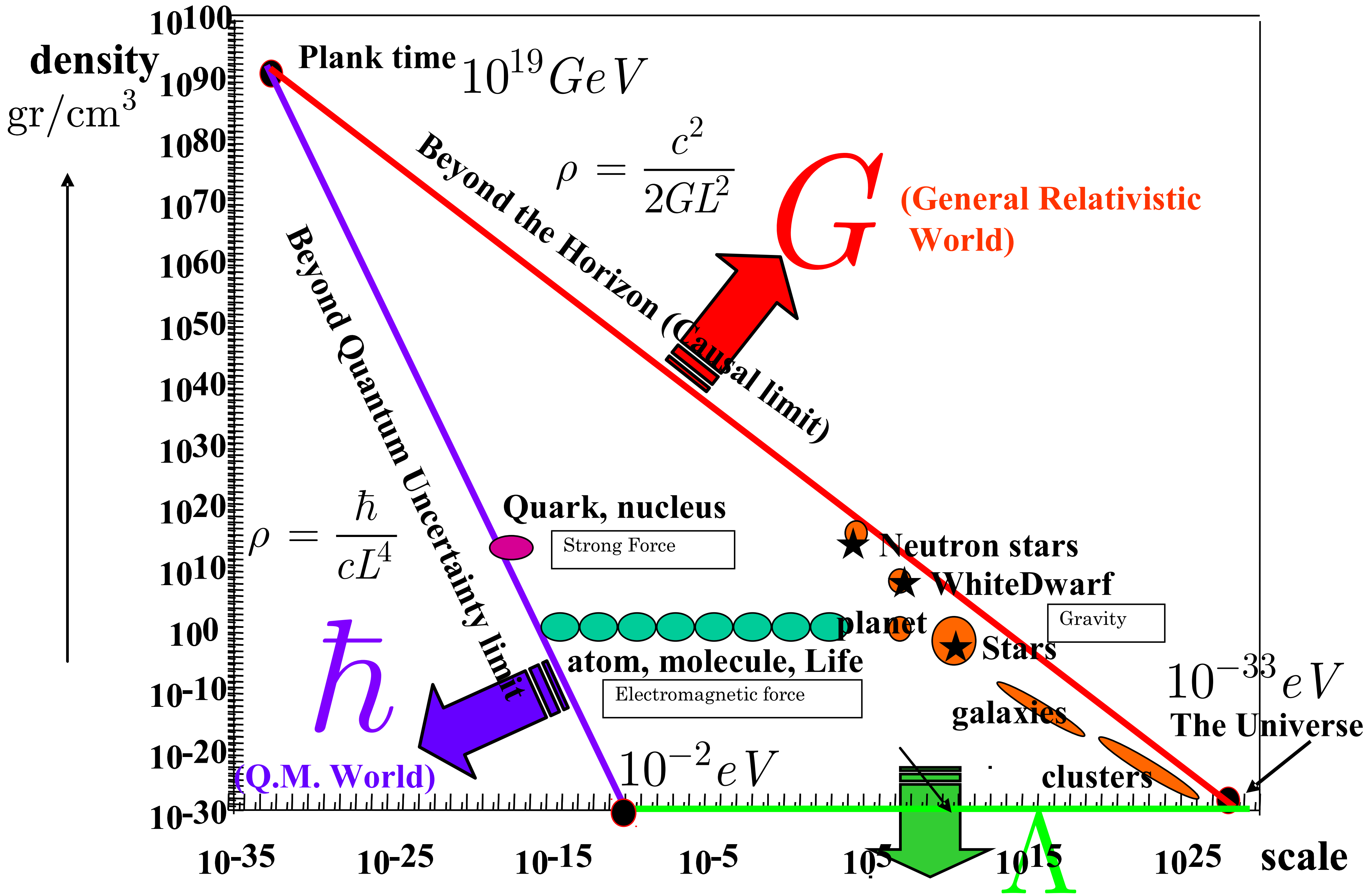
Then, in the next part, we generalize this model to quantum field theory based on the effective action method of Keldysh type, in which the imaginary part describes the rich variety of classical statistical properties. Applying this method, we describe (a) the EPR measurement as an autonomous process and (b) the primordial fluctuations from the highly squeezed state during inflation in the early Universe, in the same footing.

§1 Introduction - Expansion of the Universe

New born Universe produces all the structures while it cools:



Matter density vs. Length scale: de Vaucouleurs-Ikeuchi Diagram



Laboratory Physics is connected to the cosmic Physics:

1. **Quantum measurement physics – Geomagnetism**

2. **EPR measurement – Origin of the primordial fluctuations**

§2 Quantum mechanics in Universe

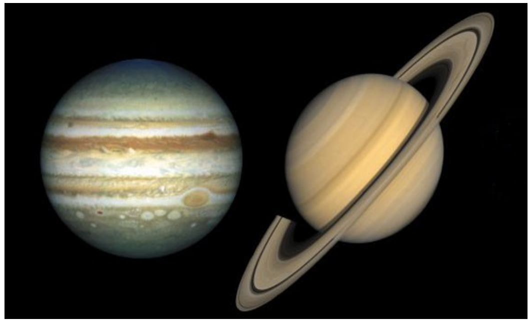
Quantum mechanics is an excellent theory to describe Universe.

■ $E_{atom} = \frac{p^2}{2m} - \frac{k_0 e^2}{r}$, $\Delta r \Delta p \approx \hbar$, $\frac{GM^2}{R} = -E_{atom}\left(\frac{M}{m_p}\right)$

yields

$$R_{\text{planet}} = \frac{\hbar^2}{e\sqrt{Gk_0} m_e m_p} \approx 1.0 \times 10^7 \,\text{m}$$

$$M_{\text{planet}} = \frac{e^3 k_0{}^{3/2}}{G^{3/2} m_p{}^2} \approx 8.1 \times 10^{26} \,\text{Kg}$$



→Planet (Jupiter)

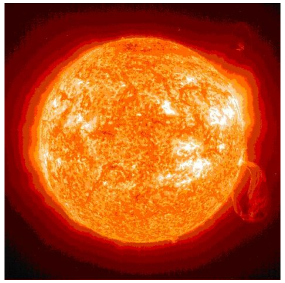
■ go beyond the p-p coulomb barrier by quantum uncertainty:
(coulomb energy)=(energy uncertain) i.e.

$E_{nf} = \frac{p^2}{2m_p} = \frac{\hbar^2}{2x^2 m_p}$ and gravitational energy $E_{gr} = \frac{4\pi G M^2}{x}$,

which yields $T = 5 \times 10^8$K. Furthermore,

$$R_{\rm star} = \frac{\hbar^2}{e\sqrt{Gk_0}{m_e}^{3/2}{m_p}^{1/2}} \approx 3.2 \times 10^8 \mathrm{m}$$

$$M_{\rm star} = \frac{e^3 {k_0}^{3/2}}{G^{3/2}{m_e}^{3/2}{m_p}^{1/2}} \approx 2.3 \times 10^{31} \mathrm{Kg}$$



→star(sun)

■ relativistic limit $v \to c, \quad m_e \to m_p$ yields the white dwarf and the neutron star, the structure of degenerate fermion

$$M = \frac{c^{3/2}\hbar^{3/2}}{2\sqrt{2}G^{3/2}{m_p}^2} \approx 1.3 \times 10^{30}\,\mathrm{Kg}$$

$$R = \frac{\hbar^{3/2}}{2\sqrt{2}\sqrt{c}G{m_p}^2} \approx 10^3\,\mathrm{m}$$

→neutron star

- Classical limit $\hbar \to 0$ is not realistic at all.
- Macroscopic structure is based on the microscopic quantum theory.
- The specific scale appears reflecting the non-extensivity of gravity.

- The structure by degenerate neutrino : $R = \frac{\hbar^{3/2}}{2\sqrt{2}\sqrt{cG}{m_\nu}^2} \approx 3 \times 10^{25}\,\mathrm{m}$

(almost cosmic radius)

- <u>Case of Boson:</u> if CDM is the boson of $m \leq 20\mathrm{eV}$, then the boson is degenerate

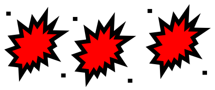
$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^2}\left(\rho_g + \rho_\phi + \rho_l\right)$$

$$\dot{\rho}_g = -3H\rho_g - \Gamma\rho_g$$

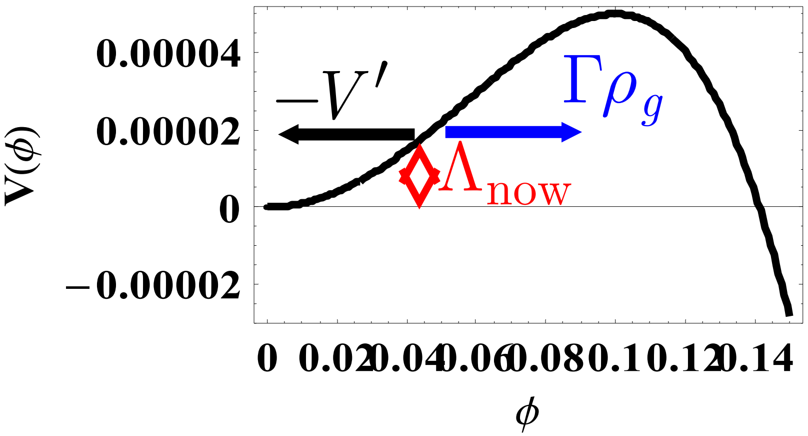
$$\dot{\rho}_\phi = -6H\left(\rho_\phi - V\right) + \Gamma\rho_g - \Gamma'\rho_\phi$$

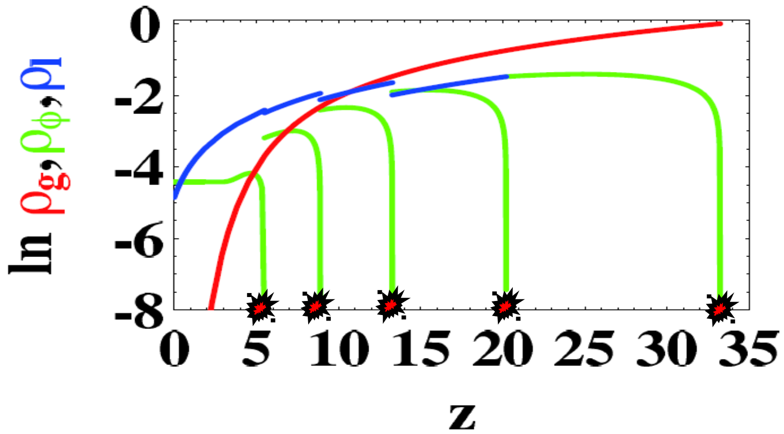
$$\dot{\rho}_l = -3H\rho_l + \Gamma'\rho_\phi$$

◆ This BEC field behaves as classical fields.

◆ If the potential V is unstable, the BEC suddenly collapses. 

◆ The force V' and the condensation force Γ balances w.e.o.⇒constant Λ_{now} (acc. Exp.) PTP115 (2006) 1047, Phys.Rev.D80:063520,2009





§ 3 Quantum mechanics in the universe – Deeper connection

- Quantum mechanics is an excellent theory to describe Universe.
- But in more fundamental level, autonomous irreversible Universe conflicts with the hybrid structure of QM, i.e.

Schrödinger equation + measurement process

- The ultimate description of the Universe is thought to be the wave function of the Universe $|\Psi(g_{\mu\nu}, \varphi, \psi, \ldots)\rangle$, which obeys

  $\hat{H}|\Psi(a, \varphi)\rangle = 0$ (mini-superspace app.)

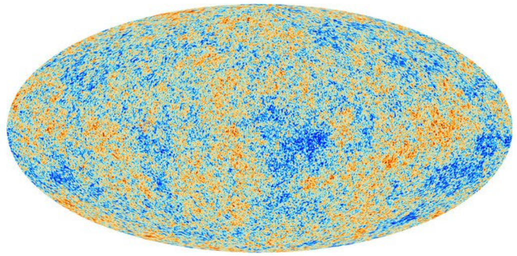
  (Wheeler–DeWitt eq. ←to operator← $H = 0$←canonical gravity)
- How to set initial (boundary) condition?
- No observer can reduce the wave function!

- The Universe never repeats. No one can reset the Universe for repeated measurements.
- A star begun to shine when someone once measured the star ??
- Primordial density fluctuations in the early Universe.

k-mode of inflaton obeys $v_k'' + \left(k^2 - 2H^2 e^{2Ht}\right) v_k = 0$, and the

squeezing amplifies the fluctuation: $v_k \approx k^{-1/2}\left(1 + ik^{-1} H e^{Ht}\right)$

$$\underbrace{\left\langle \hat{\phi}(x)\hat{\phi}(y)\right\rangle\Big|_k}_{\text{quantum mechanical}} = \underbrace{|\delta\varphi(k)|^2}_{\text{classical}} \rightarrow P(k) \rightarrow$$



A k-mode becomes classical when it goes beyond the horizon??

Does quantum fluctuations make spatial structures??

Are qu. fluctuations equivalent to the stat. fluctuations??

◆ Origin of the problem is the hybrid structure of QM:

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|A_0\rangle \Rightarrow \underbrace{\alpha|\uparrow\rangle|A_\uparrow\rangle + \beta|\downarrow\rangle|A_\downarrow\rangle}_{|\Psi_{\text{entangled}}\rangle}$$

$$\underset{measurement}{\Rightarrow} \begin{cases} \xrightarrow[|\alpha|^2]{} |\uparrow\rangle|A_\uparrow\rangle \\ \text{or} \\ \xrightarrow[|\beta|^2]{} |\downarrow\rangle|A_\downarrow\rangle \\ \text{either} \end{cases}$$

Schrödinger equation + measurement process

system & apparatus — observer POVM

This hybrid structure is an axiom of QM:

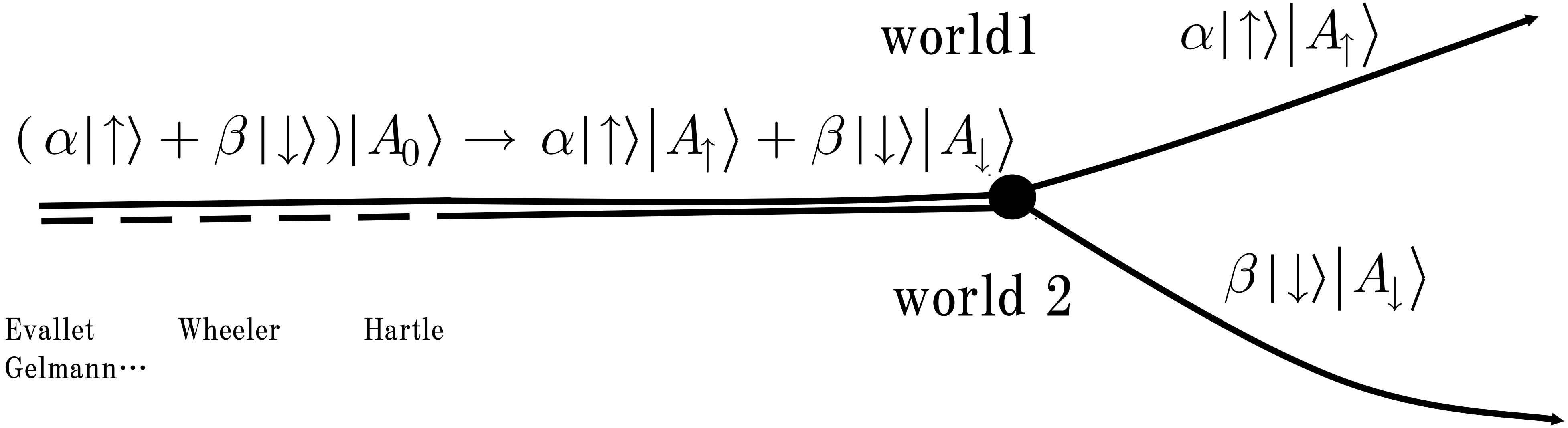
A wave function shrinks into an eigenstate when measured.

§ 4 Many world –a typical unified theory of QM
– cosmologists' favorite

◆ One-to-many map (Observer's indeterminism) can be brought into an one-to-one map if the domain is extended.

i.e. state reduction is regarded as the bifurcation to many correlated pairs (of a system and the observer)

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|A_0\rangle \rightarrow \alpha|\uparrow\rangle|A_\uparrow\rangle + \beta|\downarrow\rangle|A_\downarrow\rangle$$



◆ Problems of many world theory

1 bifurcation is ambiguous

$$\left|\Psi_{\text{entangled}}\right\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle) = \frac{1}{\sqrt{2}}(|\leftarrow\rangle|\rightarrow\rangle - |\rightarrow\rangle|\leftarrow\rangle)$$

A special base is needed(Zurek)

2 bifurcation dynamics is missing: When/How bifurcate?

3 How the decoherence between the worlds realize?

4 No intrinsic prediction of the theory; We cannot demonstrate the theory.

◆ A bit qualitative argument: Consistent historytheory (decoherence between the worlds)

$$D(\alpha,\alpha') = \int_{\alpha'} \delta q' \int_{\alpha} \delta q \delta(q'_f - q_f)$$
$$\times \exp(i\{S[q'(\tau)] - S[q(\tau)]\}/\hbar)\rho(q'_0, q_0)$$

coarse grain Q of the full set of variables $q = (x, Q)$ to yield the effective dynamics for x.

If $D(\alpha,\alpha') = 0,\ \alpha' \neq \alpha$, then two histories α, α' decohere and the consystent probability is assigned to each history.

M Gell-Mann, JB Hartle - Physical Review D, 1993 - APS

Actually, this is the Feynman-Vernon Influence Functional Theories or Caldeira-Legett formalism <u>in our world</u>.

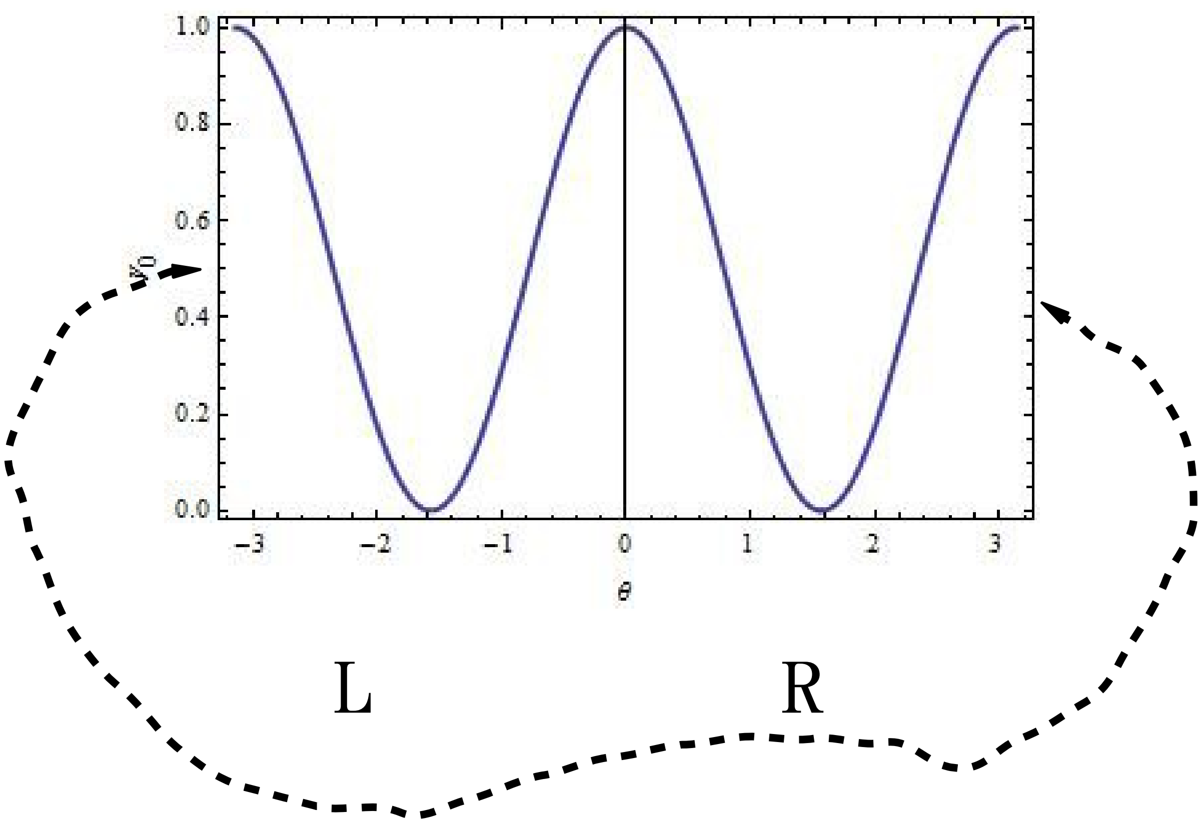
R. P. Feynman and F. L. Vernon, Ann. Phys. (N. Y.) 24, 118 (1963)   A. Caldeira and A. J. Leggett, Phys. Rev. Lett., vol. 46, p. 211, 1981

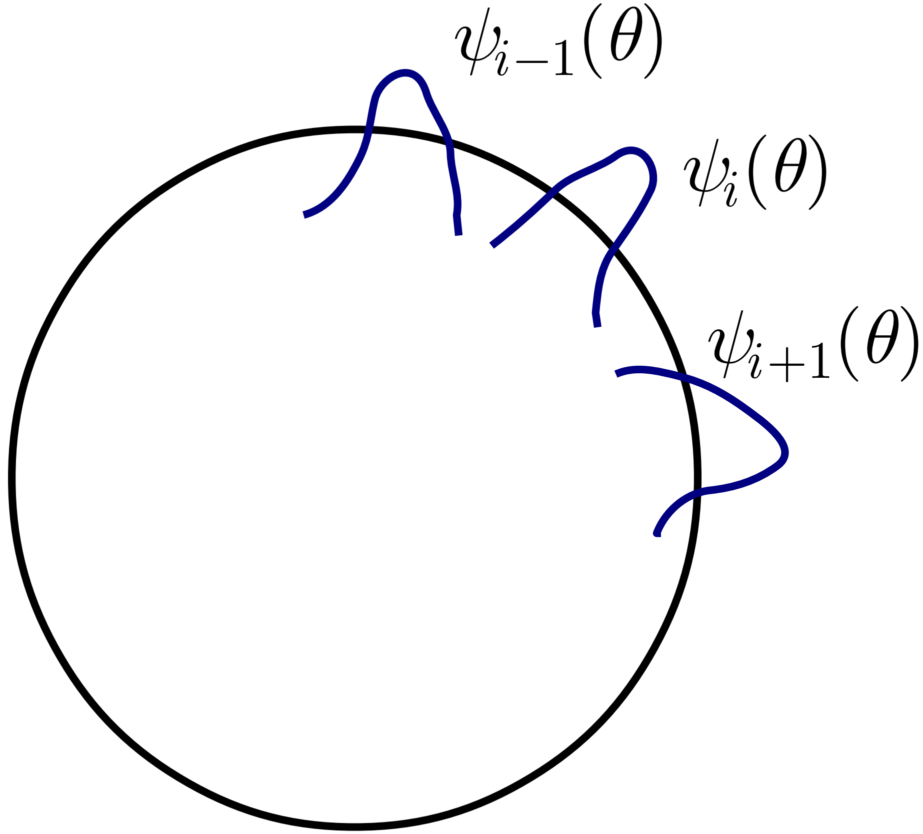
Therefore many worlds may be in our own world.

§ 5 Simple model of quantum measurement

- particles on a ring (1-dim.) measurement apparatus determines the location of a system particle: either right or left side of the ring.
- a system measured: $\psi_0(\theta)$ a prticle wave function

$V_0(\theta)$: ptential



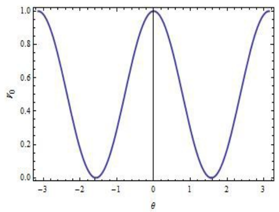


- measurement apparatus: many particles $\psi_i(\theta),\ i = 1, 2, \ldots, N$

- Equation of motion: $i = 0, 1, 2, \ldots, N$

$$i\hbar \frac{\partial \psi_i(t,\theta_i)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial \theta_i^2}\psi(t,\theta_i) + V_0(\theta_i)\,\psi(t,\theta_i) + V_{HF}(\theta_i)\,\psi_i(t,\theta_i)$$

where



$V_0(\theta_i) = cos(\theta_i)^2$: common potential

$V_{HF}(\theta_i) \equiv \lambda\left(\varphi(t,\theta)^2 - \frac{1}{N}\left|\psi_i(t,\theta)\right|^2\right)$: attractive force

$\varphi(t,\theta)^2 \equiv \frac{1}{N}\sum_{k=1}^{N}\left|\psi_k(t,\theta)\right|^2$: order variable i.e. meter readout

- initial condition for the apparatus $\psi_i(\theta), \quad i = 1, 2, \ldots., N$

$$\psi_i(t=0,\theta) = \frac{1}{\sqrt{2\pi s}} \exp\left(-\frac{(\theta-\xi_i)^2}{2s^2}\right) e^{i\xi_i'\theta}$$

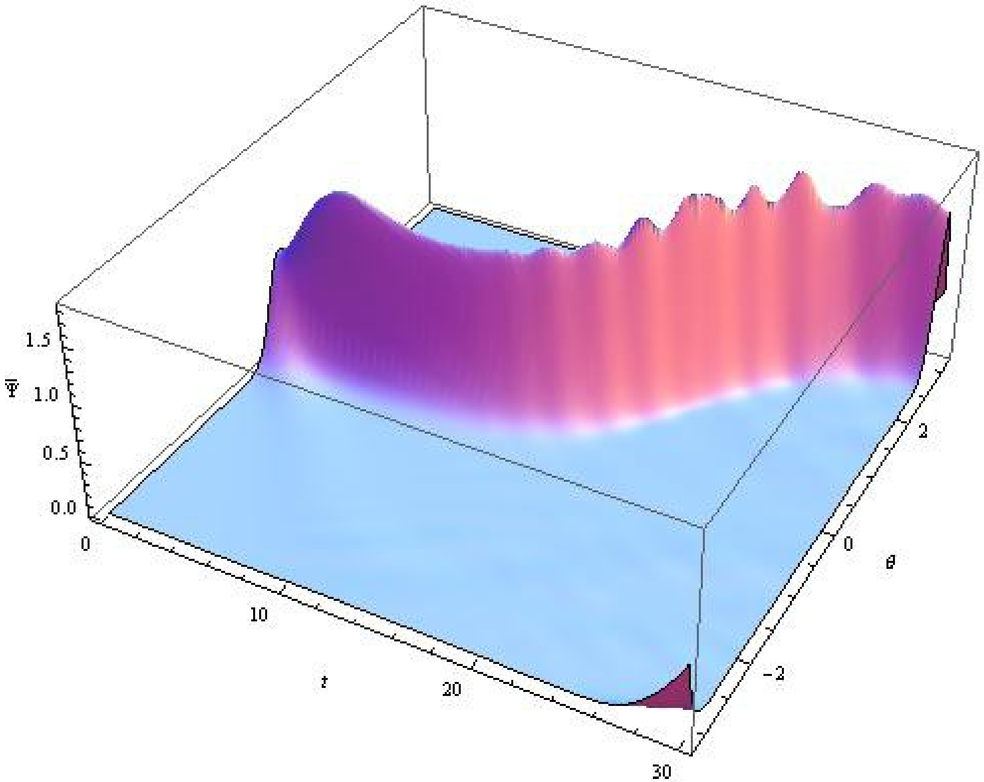
where ξ_i, ξ_i' are Gaussian random variables(dispersion σ)

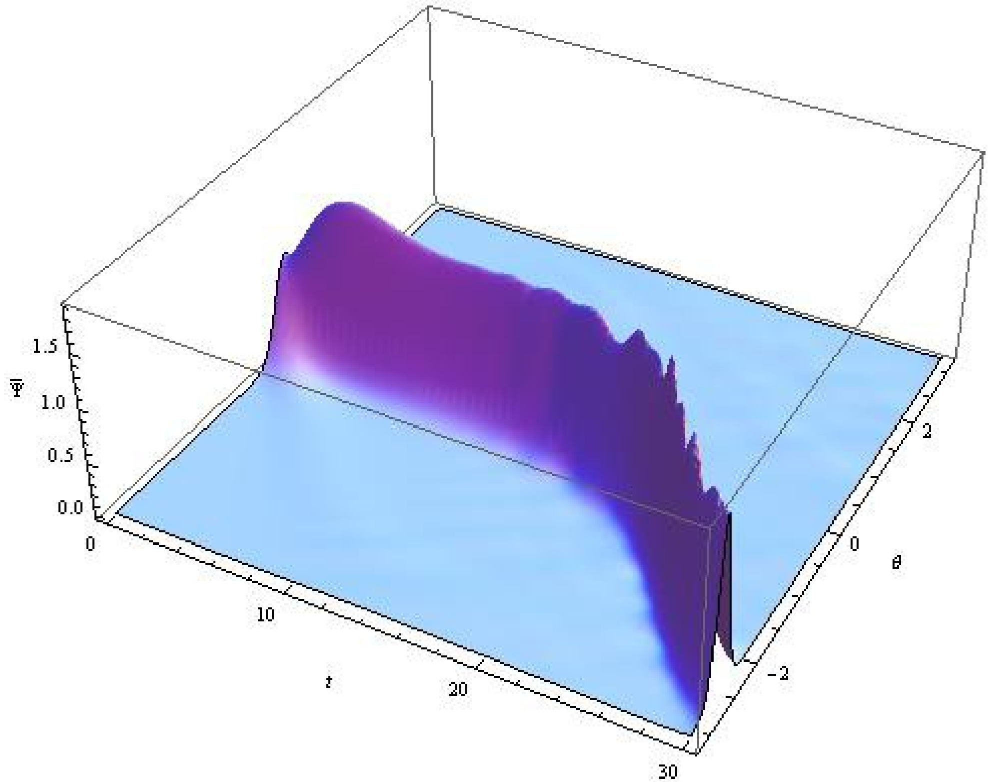
i.e. all the particles are set almost on the potential top $\theta \approx 0$.

- all the particles $\psi_i(\theta)$ obey <u>linear</u> equation of motion

- dynamics of apparatus $\psi_i(\theta), \quad i = 1, 2, ..., N$

$$m = 1, \hbar = 0.02, \sigma = 0.01, \lambda = -1$$

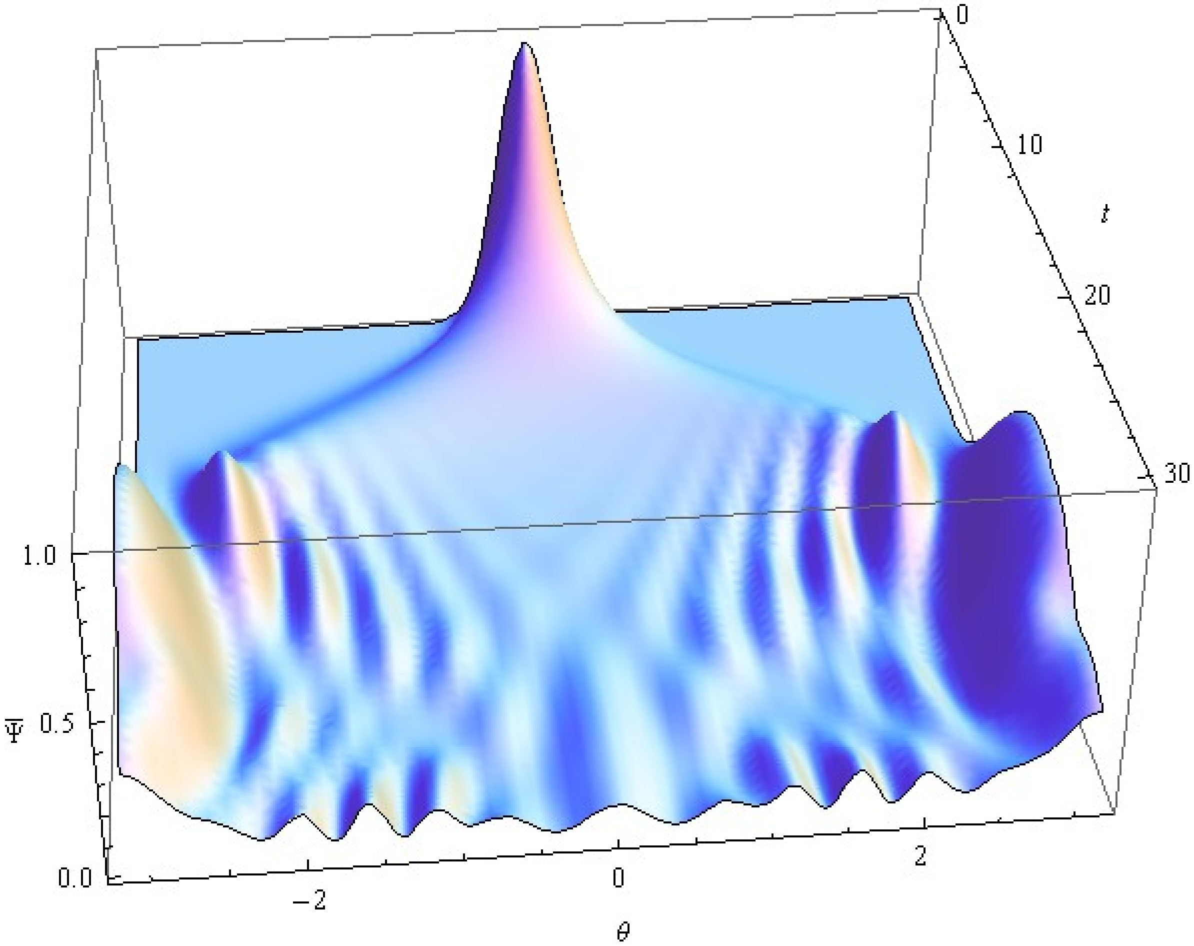




$\Rightarrow$ attractive force yields an almost single wave packet

- dynamics of apparatus $\psi_i(\theta), \quad i = 1, 2, \ldots., N$

$\lambda = 0 \;\Rightarrow$ the wave packet dissipates away if no attractive force



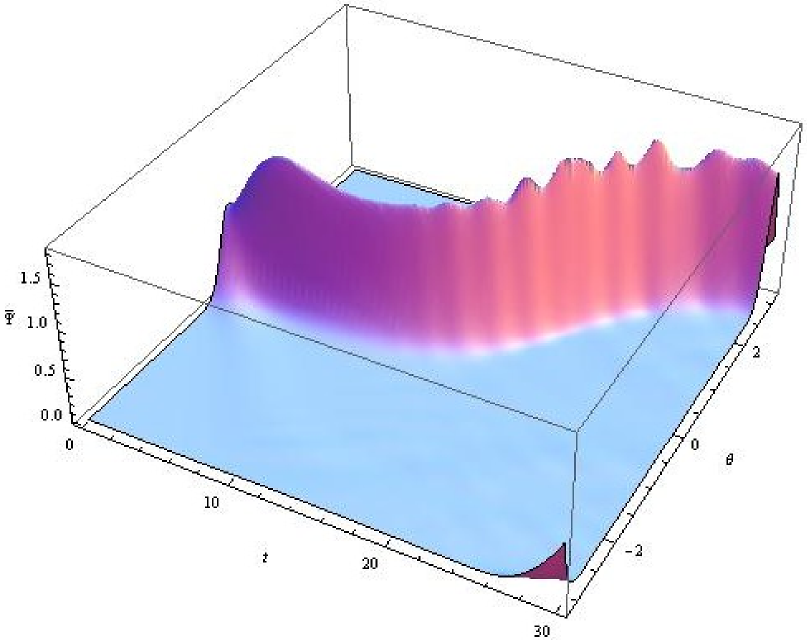
The condition for the wave packet to form a single peak:

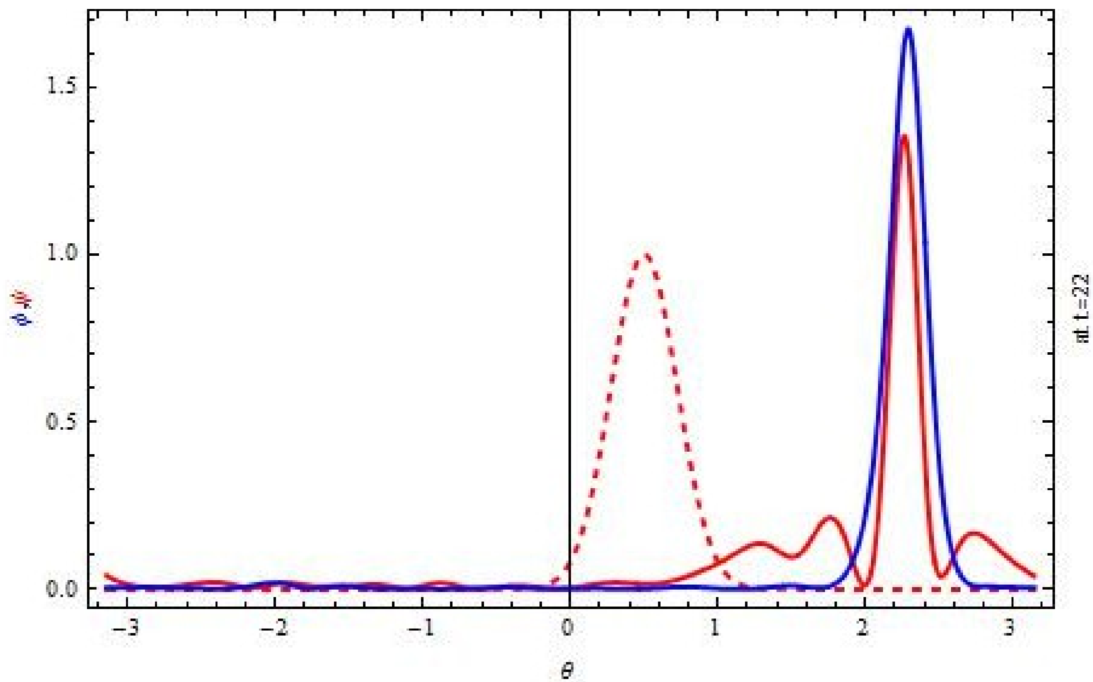
$$\frac{\sigma^2}{2m} + |\sigma V_0'(\sigma)| < \frac{|\lambda|}{2}$$

i.e. attractive force dominates kinetic energy/differential force

- case of system (located right)+ apparatus $(\alpha = 0)$

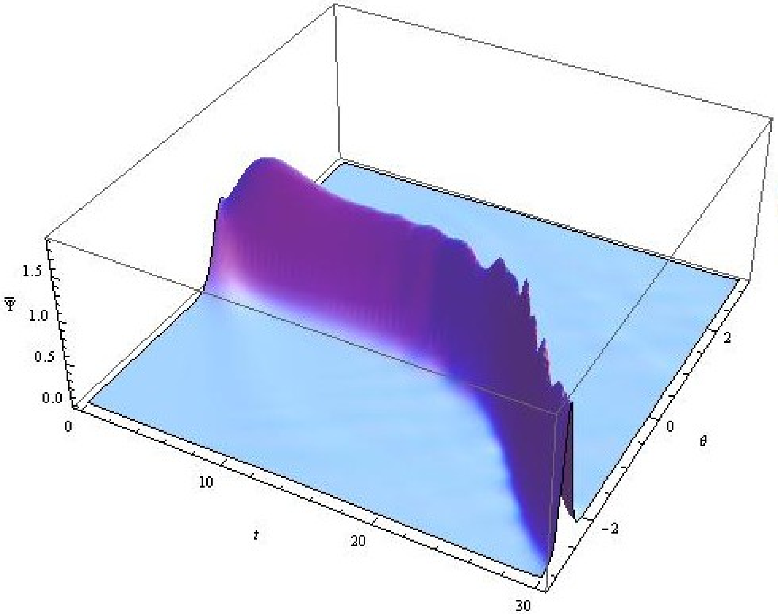
$$\psi_0(t, \theta_0) = \sin(\alpha) \exp\left(-\frac{(\theta + 0.5)^2}{2s^2}\right) + \cos(\alpha) \exp\left(-\frac{(\theta - 0.5)^2}{2s^2}\right)$$

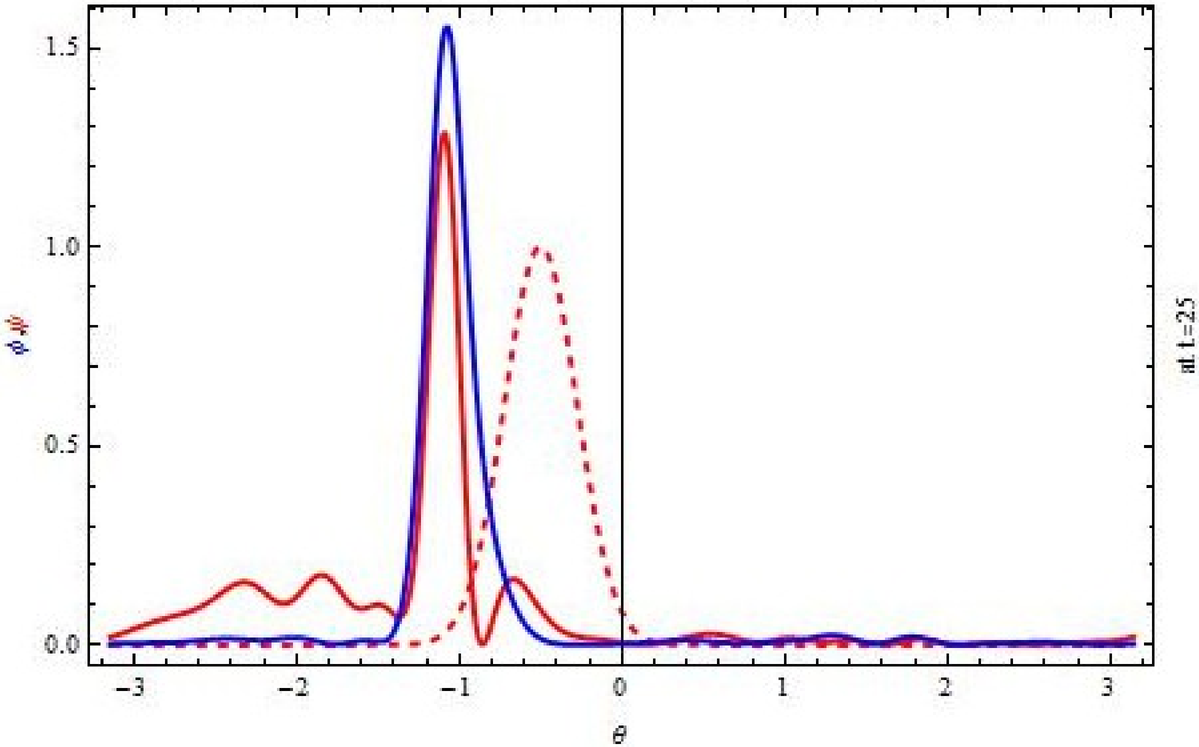




- case of system (located left)+ apparatus $(\alpha = \pi/2)$

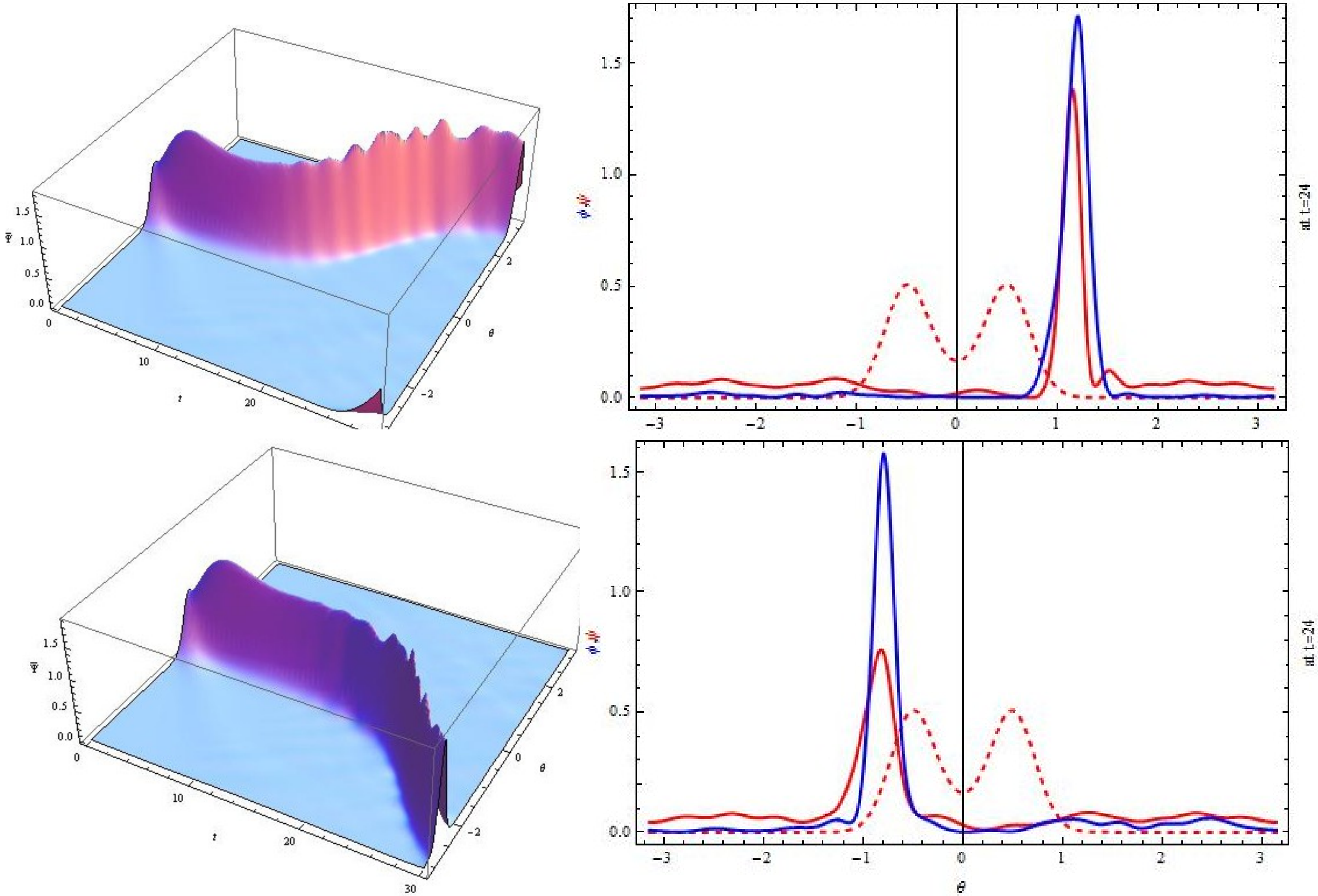
$$\psi_0(t,\theta_0) = \sin(\alpha)\exp\left(-\frac{(\theta+0.5)^2}{2s^2}\right) + \cos(\alpha)\exp\left(-\frac{(\theta-0.5)^2}{2s^2}\right)$$





- case of system (located both sides)+ apparatus $(\alpha = \pi/4)$

$$\psi_0(t,\theta_0) = \sin(\alpha)\exp\left(-\frac{(\theta+0.5)^2}{2s^2}\right) + \cos(\alpha)\exp\left(-\frac{(\theta-0.5)^2}{2s^2}\right)$$



case happens

The condition for the proper measurement :

$$\left[\exp\left(-\frac{\sqrt{Nm\triangle E/2a}}{\hbar}\right)\right]^{-1} \gg T_{measurement} > \left[\exp\left(-\frac{\sqrt{m\triangle E/2a}}{\hbar}\right)\right]^{-1}$$

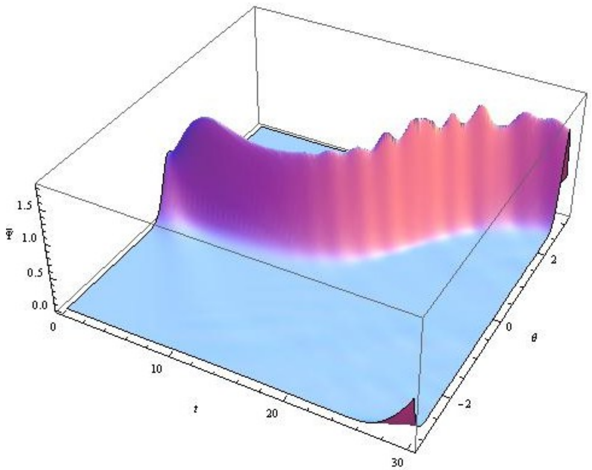
collective mode never tunnel & a single wave function tunnels

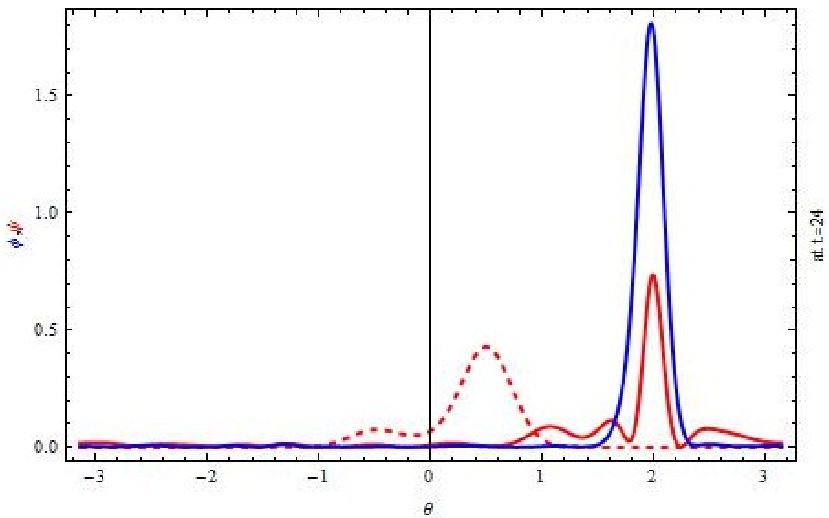
i.e. $\sqrt{N} \gg \hbar/\sqrt{m\langle\triangle E\rangle/(2a)} \approx \hbar/\sqrt{mV'(\sigma)/2}$

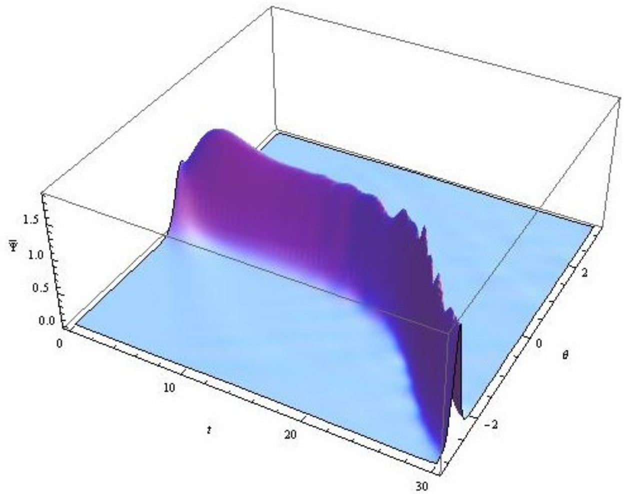
⇒ Coexistence of the extremely separated time scales will be necessary for proper measurement.

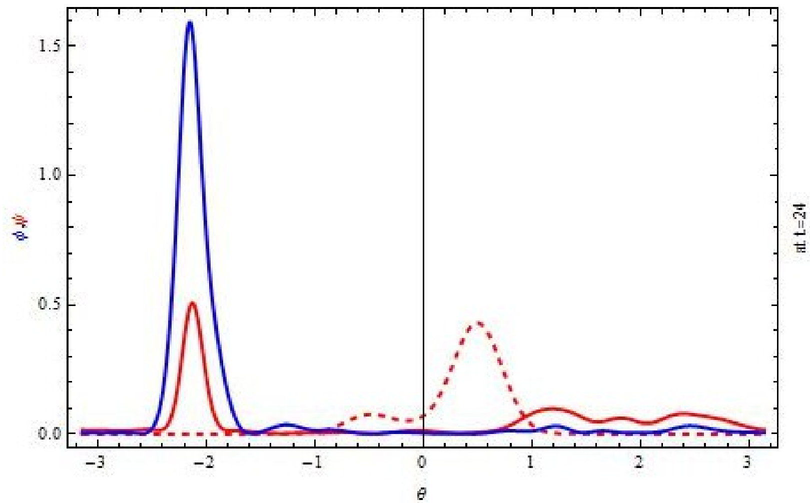
- case of system (general superposition)+apparatus $(0 \leq \alpha \leq \pi/2)$

$$\psi_0(t,\theta_0) = \sin(\alpha)\exp\left(-\frac{(\theta+0.5)^2}{2s^2}\right) + \cos(\alpha)\exp\left(-\frac{(\theta-0.5)^2}{2s^2}\right)$$





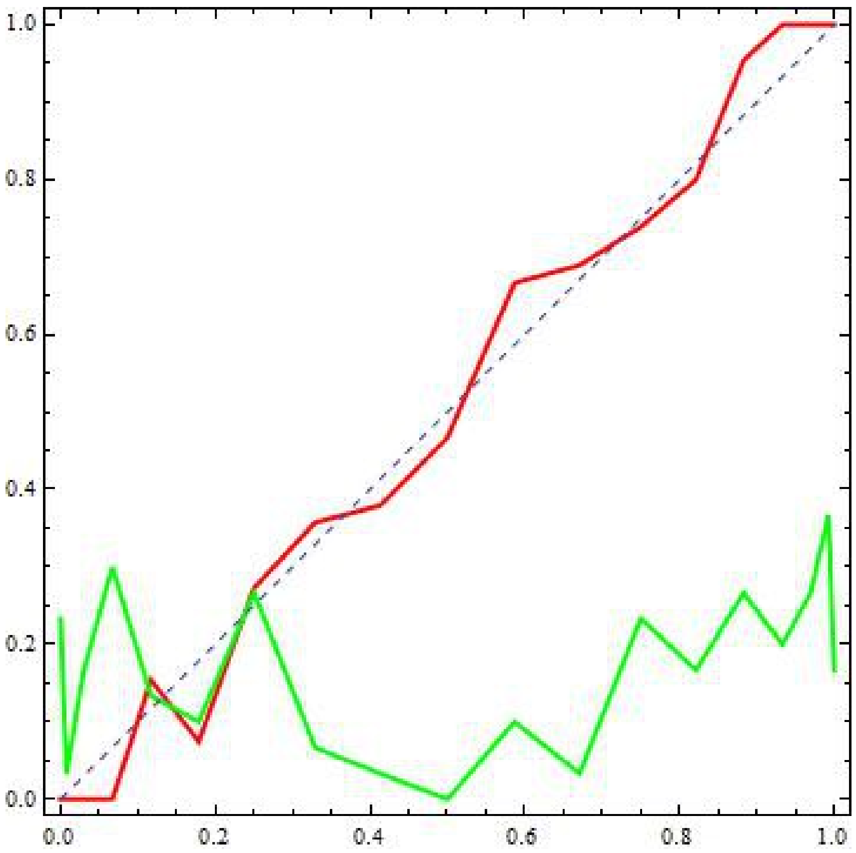




only a single case happens

- Probability distributions for $0 \leq \alpha \leq \pi/2$

frequency(after measurement)



--- frequency ratio of meter readout is `right`

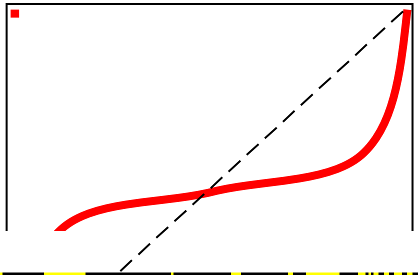
--- error ratio (inconsistent meter readout and the system state)

⋯ Born probability rule

$2\alpha / \pi$(system state bef. measurement)

- Born rule appears for this special model, but no guarantee that the relation becomes straight for N→∞ and error→0

★ A condition for faithful measurement



The balance between

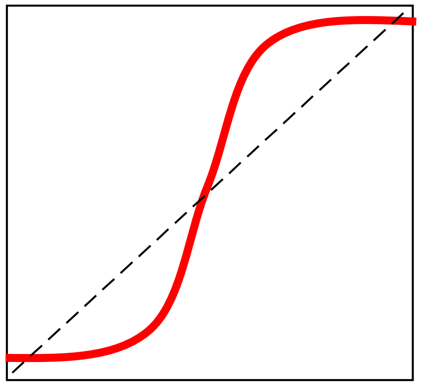
(1) $\varphi(t,\theta)^2$ can freely move on $V_0(\theta)$ (randomness)

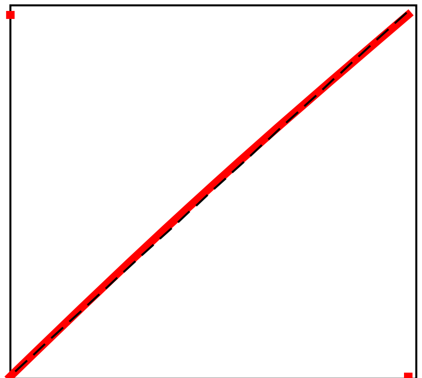
and

(2) system $\psi_0(t,\theta)$ can trigger $\varphi(t,\theta)^2$ through $V_{HF}(\theta)$ (trigger)

i.e.

$$\sigma V_0'(\sigma) N \approx \lambda \left(e^{-5(1/2)^2} \right)^2$$



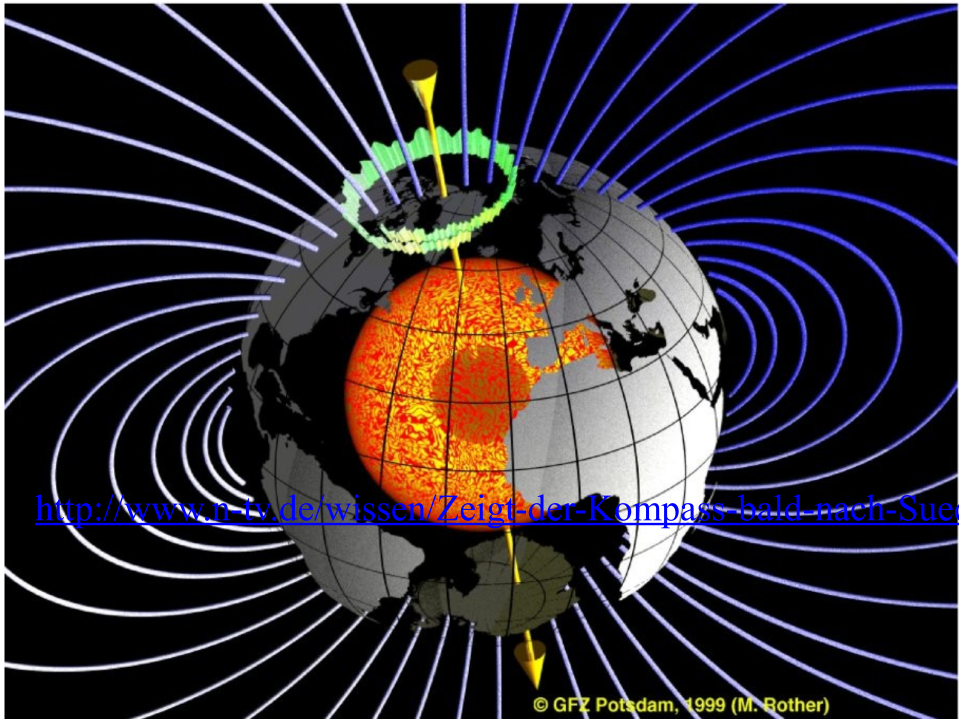


- Both classical and quantum nature of apparatus are simultaneously needed for the quantum measurement.

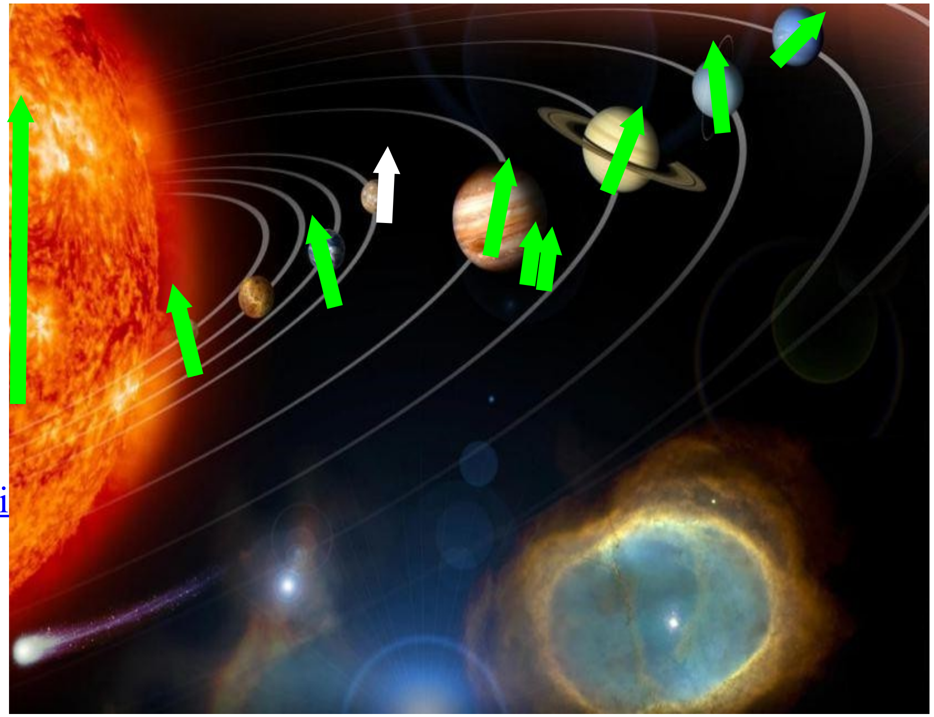
§ 6 Classical analog: geomagnetic flipping

- The same model describes the geomagnetic flipping history

↓ present S-N 160M years ago↓



http://www.n-tv.de/wissen/Zeigt-der-Kompass-bald-nach-Sueden-arti



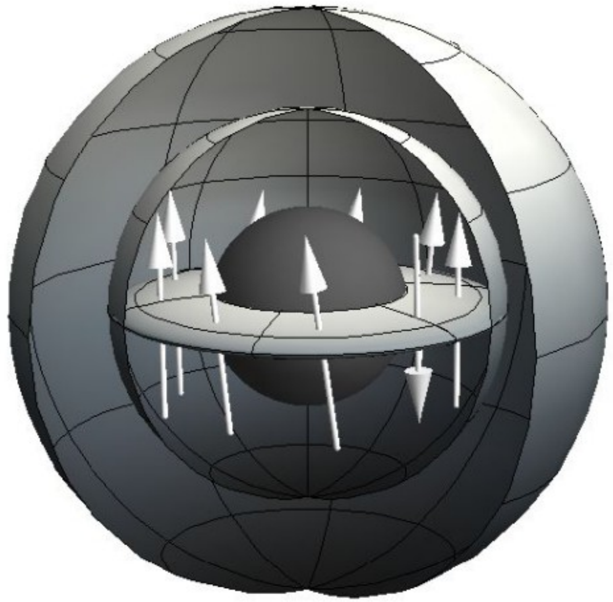
- Classical version of the Quantum measurement model describes the dynamics of geomagnetic variation including flipping

■ Long-range Coupled-Spin model (LCS)

$$K = \frac{1}{2}\sum_{i=1}^{N}\dot{\vec{s}}_i^{\,2} = \frac{1}{2}\sum_{i=1}^{N}\dot{\theta}_i^{2}, \quad V = \mu\sum_{i=1}^{N}\left(\vec{\Omega}\cdot\vec{s}_i\right)^2 + \lambda\sum_{j=1}^{N}\sum_{i=1}^{N}\vec{s}_i\cdot\vec{s}_j$$

$$L = K - V, \quad \frac{d\left(\partial L / \partial\dot{\theta}_i\right)}{dt} = \frac{\partial L}{\partial\theta_i} - \kappa\dot{\theta}_i + \xi$$

mean field: $M \equiv \frac{1}{N}\vec{\Omega}\cdot\left(\sum_{i=1}^{N}\vec{s}_i\right)$



This model describes random polarity flip despite being a deterministic model.

C

Computer simulations of the basic MHD equation yields vortex dynamo structures → LCS model

Kida, S. & Kitauchi, H., Prog. Theor. Phys. Suppl.130, 121

$$\nabla \vec{v} = 0$$

$$\rho_0\left(\frac{\partial \vec{v}}{\partial t} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla\left(p - \frac{1}{2}\rho_0\left|\vec{\Omega}\times\vec{r}\right|^2\right) + \rho_0\nu\Delta\vec{v}$$
$$+\rho\vec{g} - 2\rho_0\vec{\Omega}\times\vec{v} + \vec{j}\times\vec{B}$$

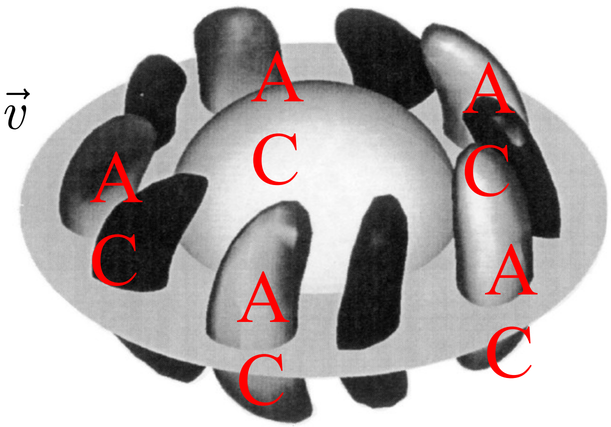


Fig. 3. Iso-surface of vorticity magnitude $|\boldsymbol{\omega}| = 52$ in region $1.15r_1 \le r \le 0.91r_2$ at $t = 25$. The black and gray surfaces represent the cyclones and the anti-cyclones, respectively.

$$\nabla\cdot\vec{B} = 0$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla\times\left(\vec{v}\times\vec{B}\right) + \eta\Delta\vec{B}$$

$$\frac{\partial T}{\partial t} + \vec{v}\cdot\nabla T = \kappa\Delta T + \varepsilon$$

$$\vec{J} = \frac{1}{\mu_0}\nabla\times\vec{B}$$

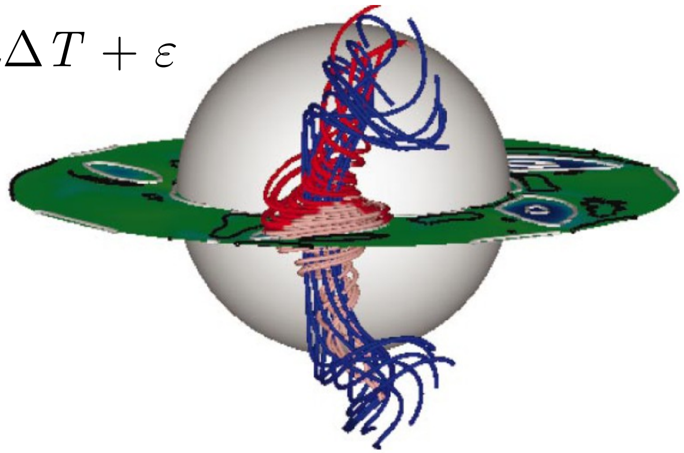


FIGURE 18. Intensification of magnetic field by stretching. Magnetic lines which pass through an anticyclone (7 in figure 13(a-ix) viewed from the right) are drawn with blue lines, the magnetic flux of which points to the south. Helical lines surrounding the magnetic lines represent streamlines. The red and pink indicate the northward and southward flows, respectively. Viewed from 10° north-latitude at $t = 61.4$.

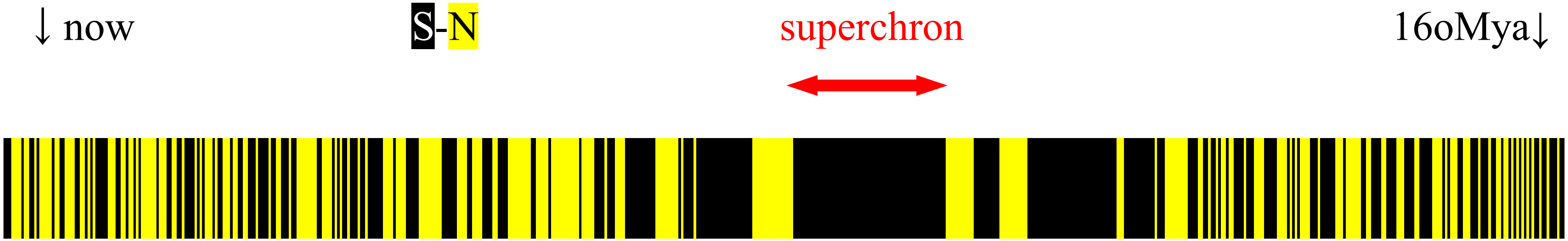
$$R_e = \left|\frac{\vec{v}\cdot\nabla\vec{v}}{\nu\Delta\vec{v}}\right| = O\left(10^8\right)$$

$$\sqrt{T_a} = \left|\frac{-2\vec{\Omega}\times\vec{v}}{\nu\Delta\vec{v}}\right| = O\left(10^{14}\right)$$

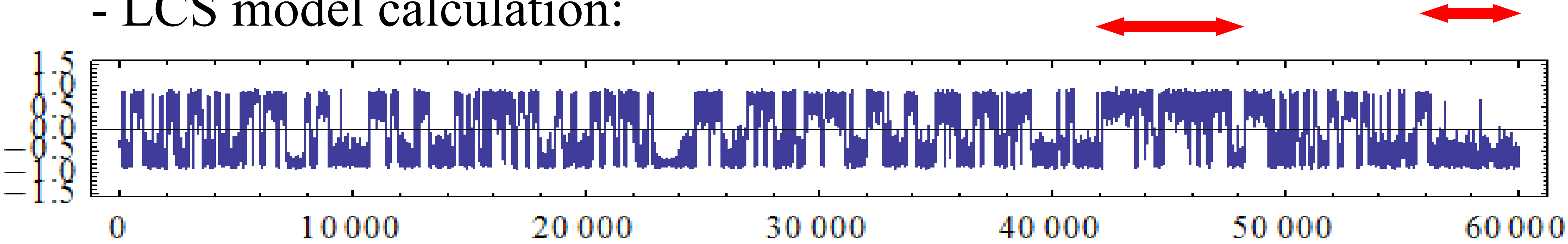
$$\sqrt{R_a} = \left|\frac{\vec{g}}{\sqrt{\nu\kappa_T}\Delta\vec{v}}\right| = O\left(10^8\right)$$

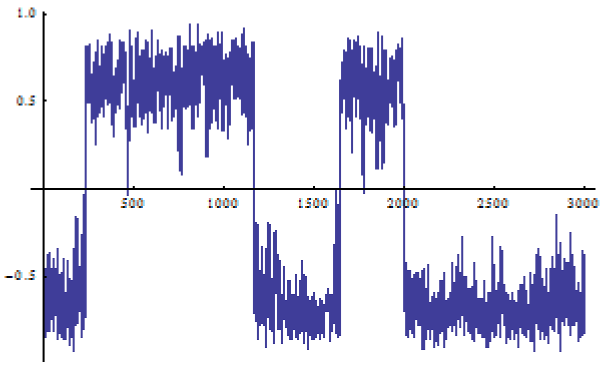
$$R_m = \left|\frac{\nabla\times\left(\vec{v}\times\vec{B}\right)}{\eta\Delta\vec{B}}\right| = O\left(10^2\right)$$

- Geomagnetic polarity flipping history over 160My:



- LCS model calculation:



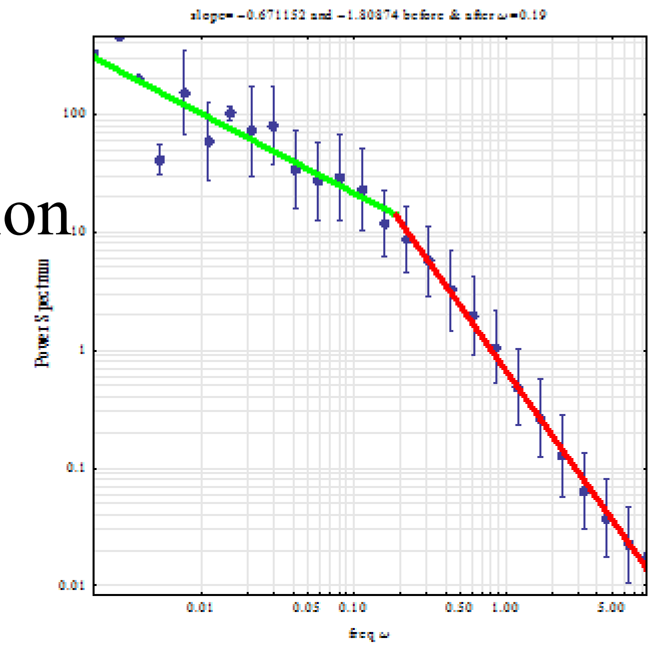


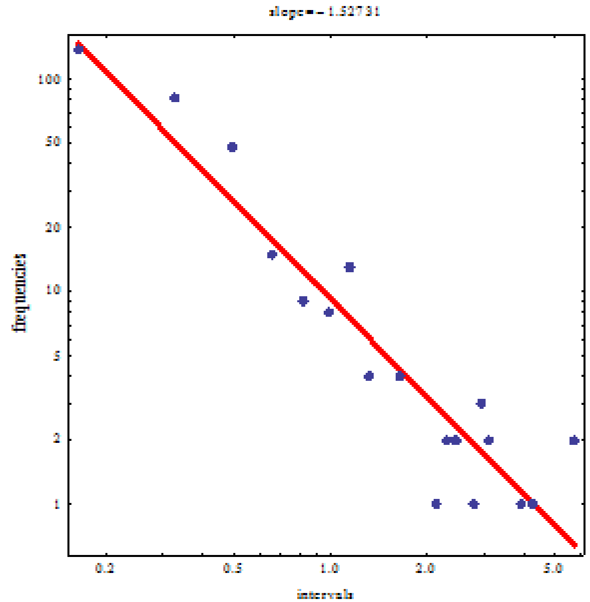
almost steady period($\sim 10^5$y) and rapid polarity flipping($\sim 10^3$y) coexists

Power spectrum

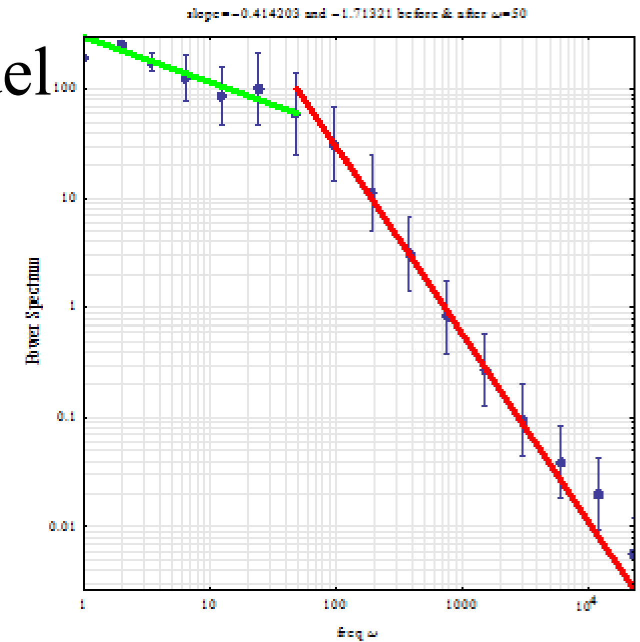
Interval distributions

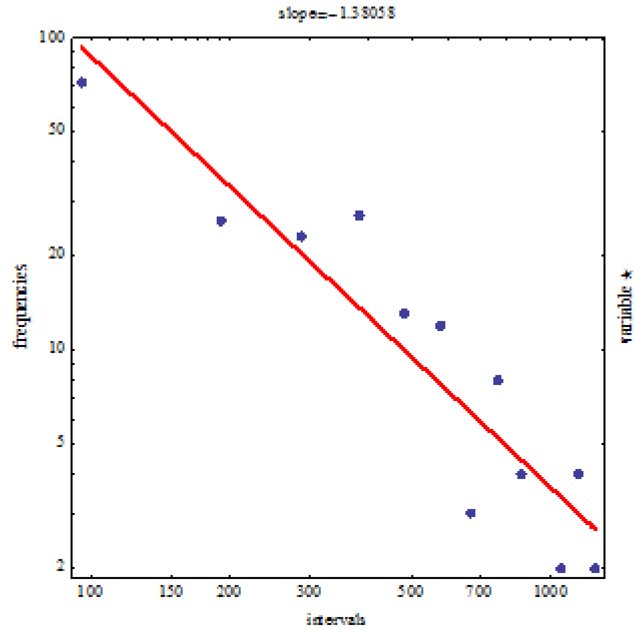
observation



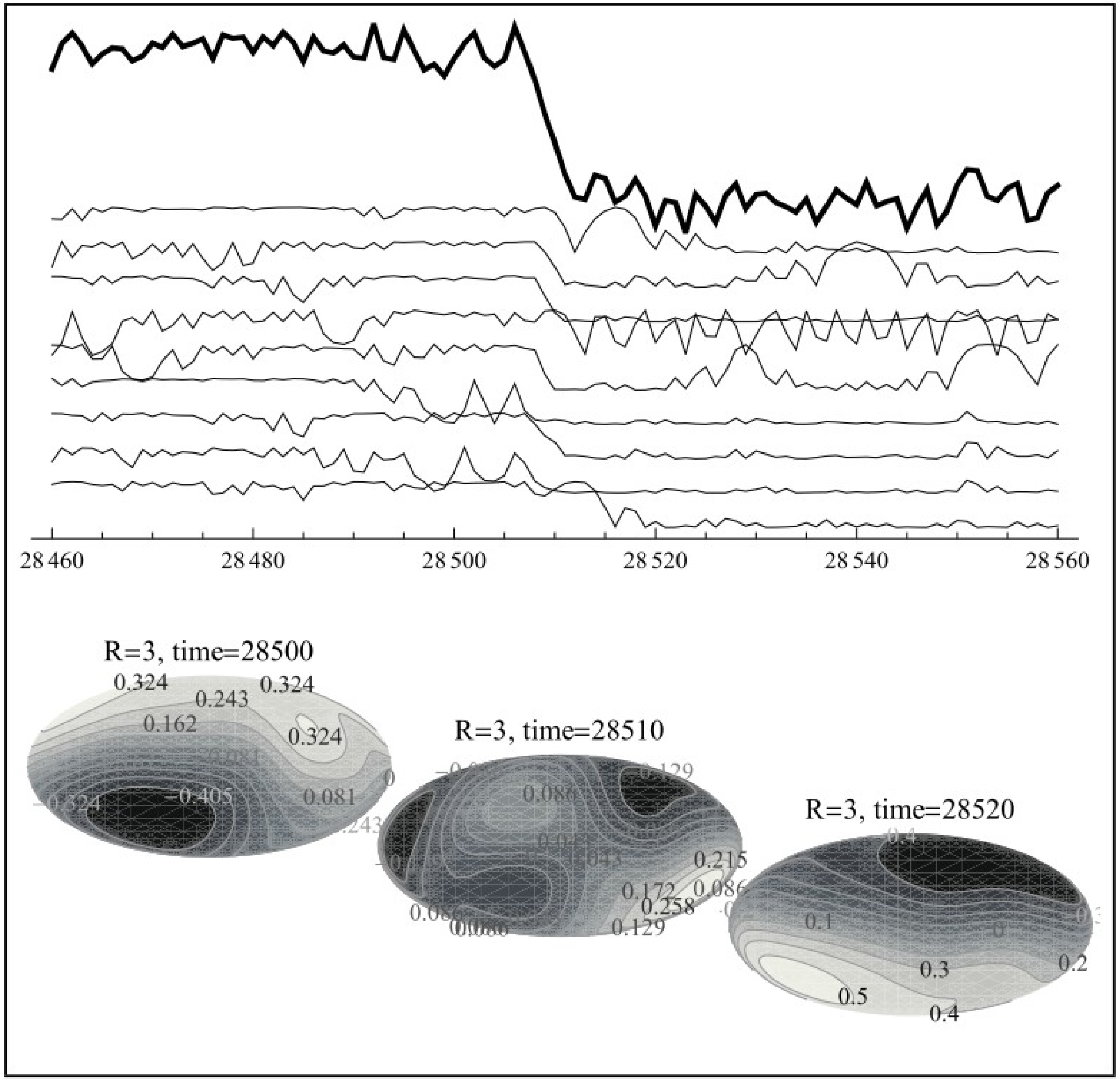


LCS model





A typical polarity flip dynamics:



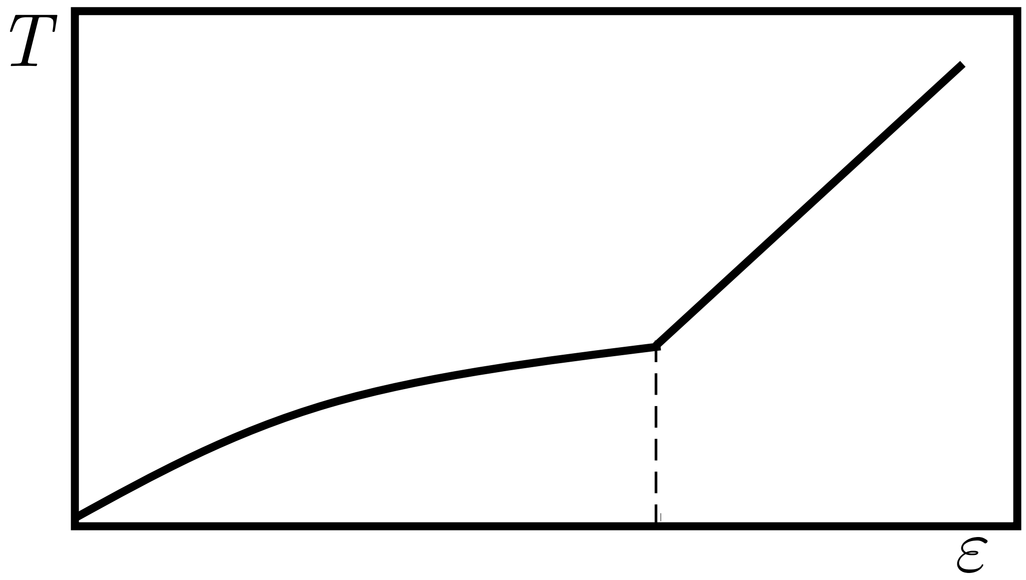
- Physical backgrounds:

<u>HMF model</u> (=Hamiltonian Mean Field) deterministic model which shows phase transition

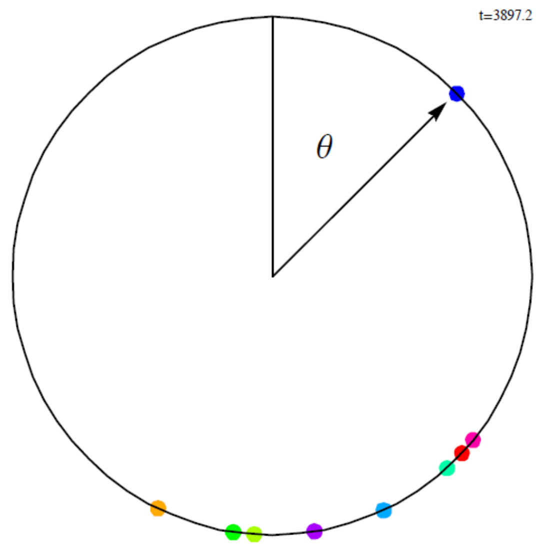
$$H = K + V = \sum_{i=1}^{N} \frac{p_i^2}{2} + \frac{1}{2N} \sum_{i,j=1}^{N} [1 - \cos(\theta_i - \theta_j)]$$

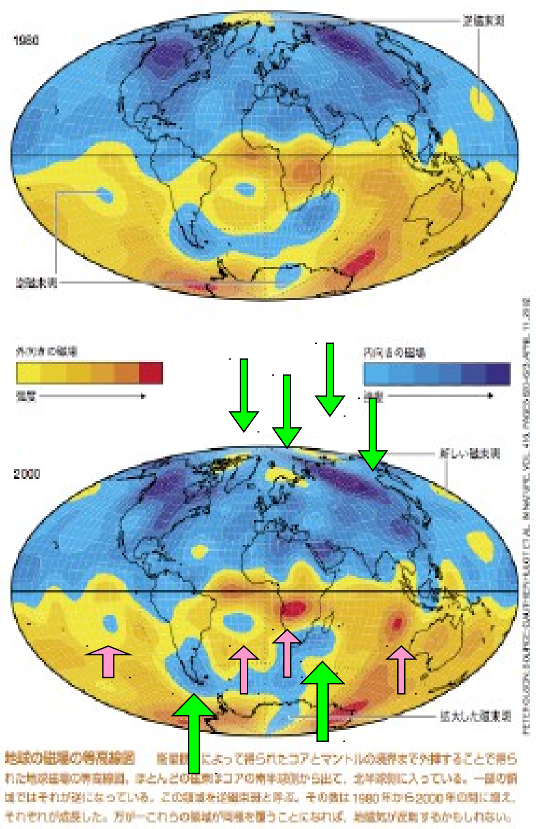
$$\mu \sum_{i=1}^{N} (\Omega \cdot \vec{s}_i)^2 \rightarrow 0$$

- yields phase transition



- Core-halo structure is formed
    - Core: almost stable dipole ↔meter $\varphi(t,\theta)^2$
    - Halo: rapid variation ↔system $\psi_0(t,\theta)$



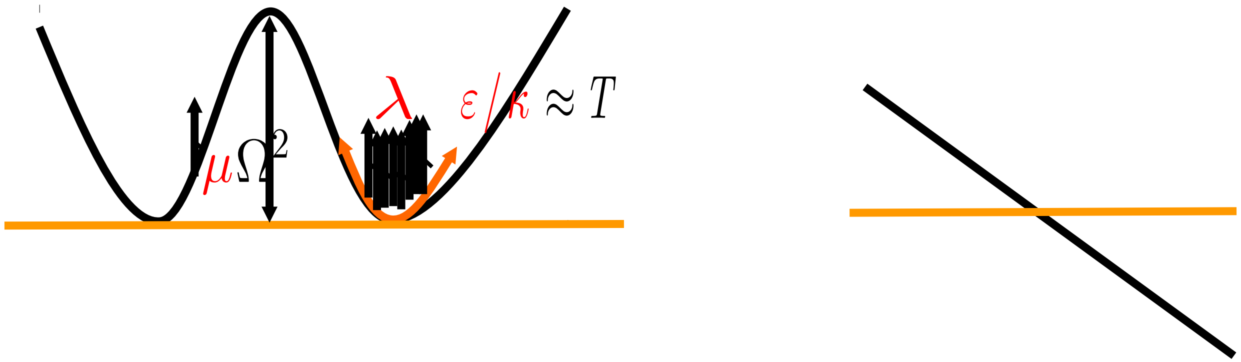


spots of reverse polarity moves rapidly

magnetic field distribution on the core-mantle boundary (upper: 1980, lower: 2000)

- <u>Kuramoto model</u> analogy (describe synchronization)

$$\mu \sum_{i=1}^{N} \left(\vec{\Omega} \cdot \vec{s}_i\right)^2 \rightarrow -\sum_{i=1}^{N} \omega_i \theta_i$$



Kuramoto model $\frac{\partial \dot{\theta}_i}{\partial t} = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin(\theta_j - \theta_i), \quad 1 \le i \le N$

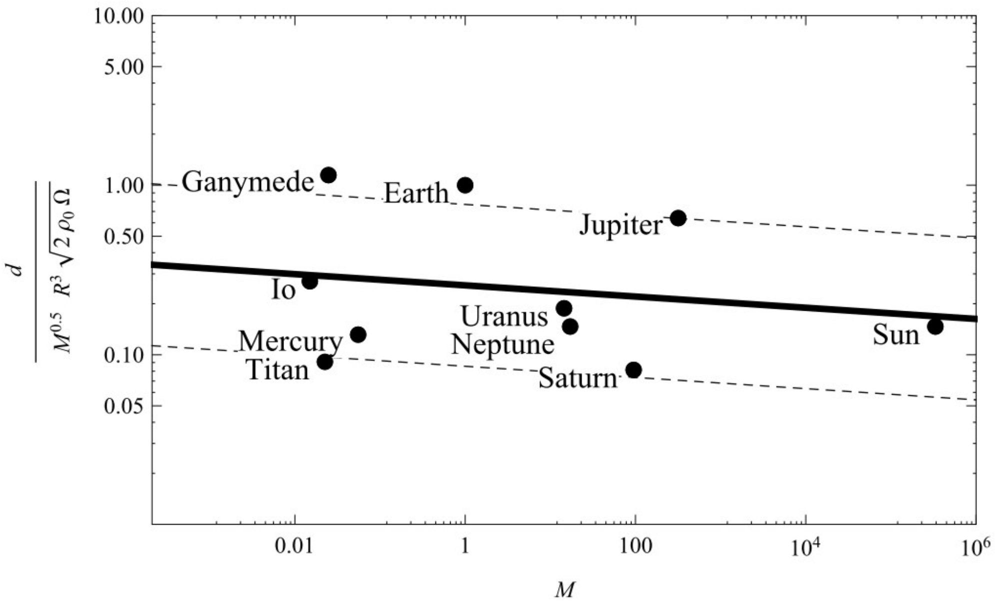
synchronization: An interaction between many oscillators yields a single stable rhythm globally.

§ 7 Other planets, satellites, and stars

- balance of generation/diffusion of magnetic fields, balance of collioris force and magnetic pressure, yield dipole moment:

$$d \equiv B_{out} R^3 = N\gamma^2 R_c^2 R \left(\frac{2\rho_0 \Omega}{\sigma} \right)^{1/2}$$

- plot of d for various planets, satellites, and the sun



yields the scaling:

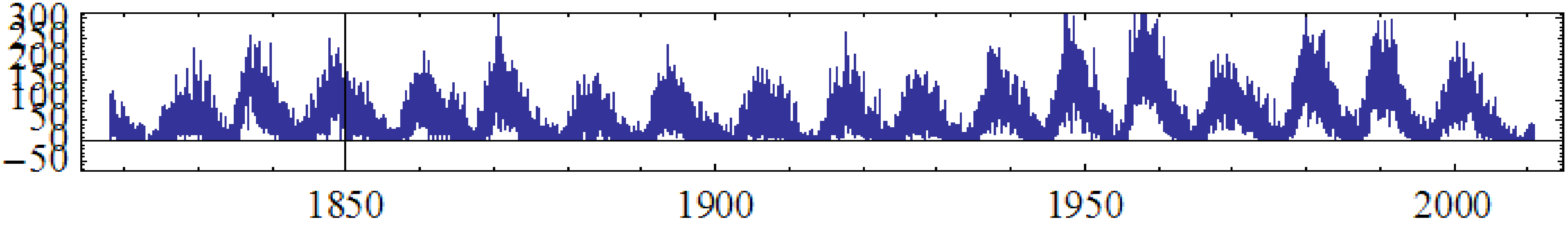
$$N\gamma^2 \propto M^{1/2}$$

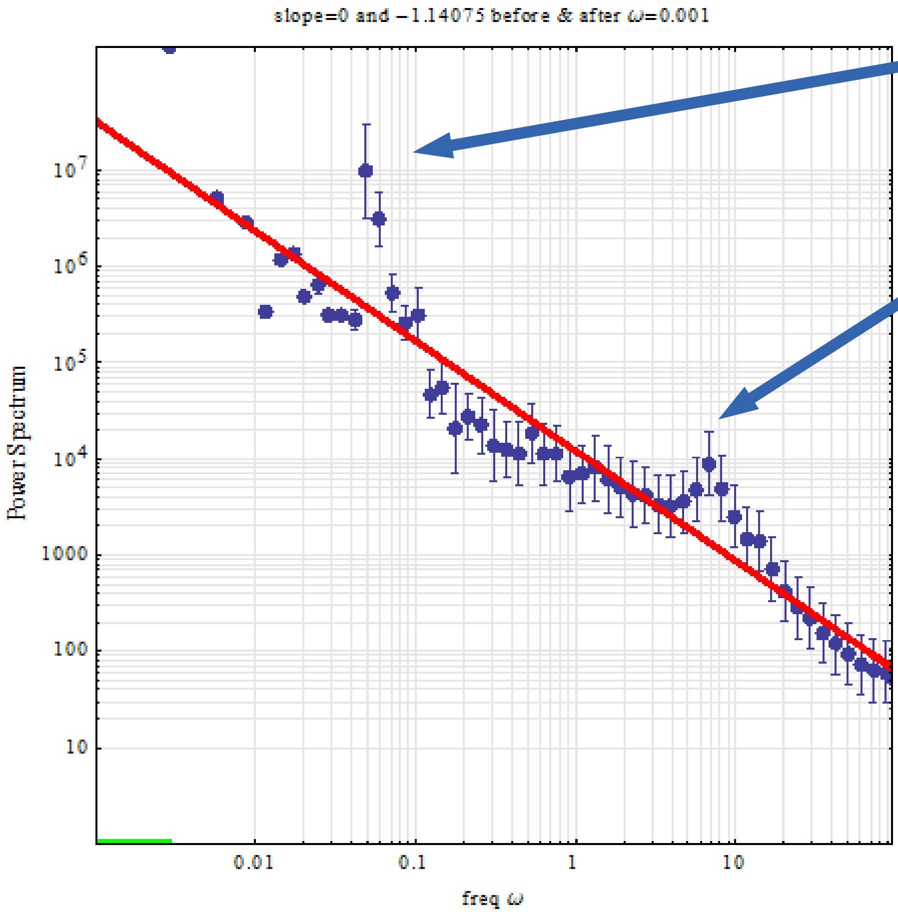
This scaling implies for the sun,

#Taylor cell: $N = 9 \times (M_\odot / M_{\rm earth})^{1/2} \approx 5 \times 10^3$

§ 8 The sun

solar activity ~ magnetism ~ #sun spots,   polarity flips every 11 years



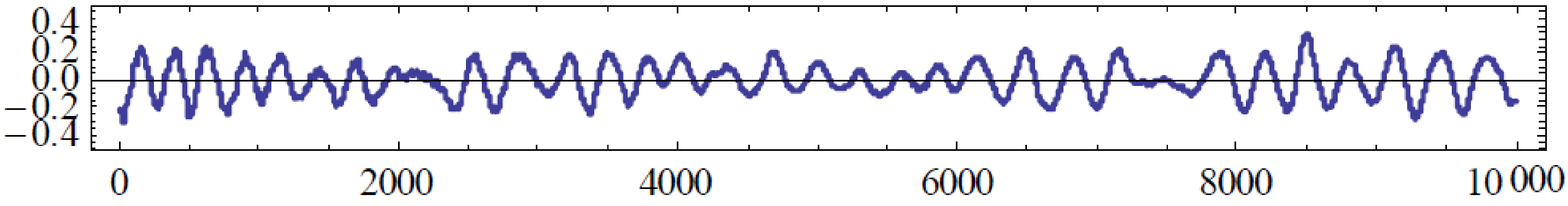


almost 11 years

(fake: month spin period)

1/f fluctuation + 11y period

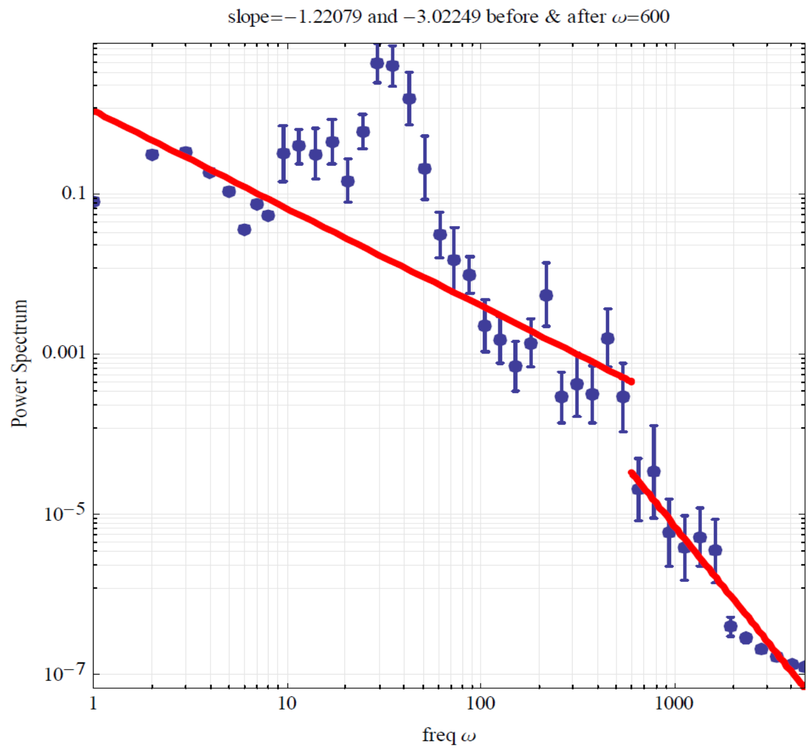
- LCS model with N=101



clear period and
$f^{-1.2}$ fluctuation

synchronization of
element spins?

http://xxx.yukawa.kyoto-u.ac.jp/abs/1104.5093
A. Nakamichi, H. Mouri, D. Schmitt,
A. Ferriz-Mas, J. Wicht, M. Morikawa



- The same model describes the dynamics of celestial magnetic fields including polarity flip.

§ 9 summary

1. - No observer exists in the Universe and QM should be described without external operation.

   - We proposed a model of quantum observation, as a physical process.

     - coexistence of extremely separated time scales.

       mean field and the system(attractive force/many particles)

     - localized mean field describes the classical meter

     - process of measurement: positive field back

       1. the system triggers the mean field
       2. the mean field synchronizes with the system

     - Born rule is derived in restricted cases

     - Ozawa quartet $\left\{K, \tilde{X}, \alpha, U\right\}$ is $\psi_i \in K$, $\tilde{X} = \varphi^2$, α:initial random distribution, U:Unitary evolution(HF app.)

2. The classical version of the same model describes geomagnetism

- coexistence of extremely separated time scales.

  core and halo (←spin-spin interaction)

- almost stable dipole component = localized mean field
- rapidly moving component = individual mode
- process of polarity flip: positive field back
    1. halo triggers the core
    2. the core synchronizes with the halo
- planets, satellites, stars may have common physics of their activity and magnetism

■ Core-halo structure is common, in QM, celestial magnetism, gravity...

Physics of quantum measurement and its applications II

Masahiro Morikawa(Ocha Univ.)

Collaboration: A. Nakamichi (Koyama Observatory Kyoto)

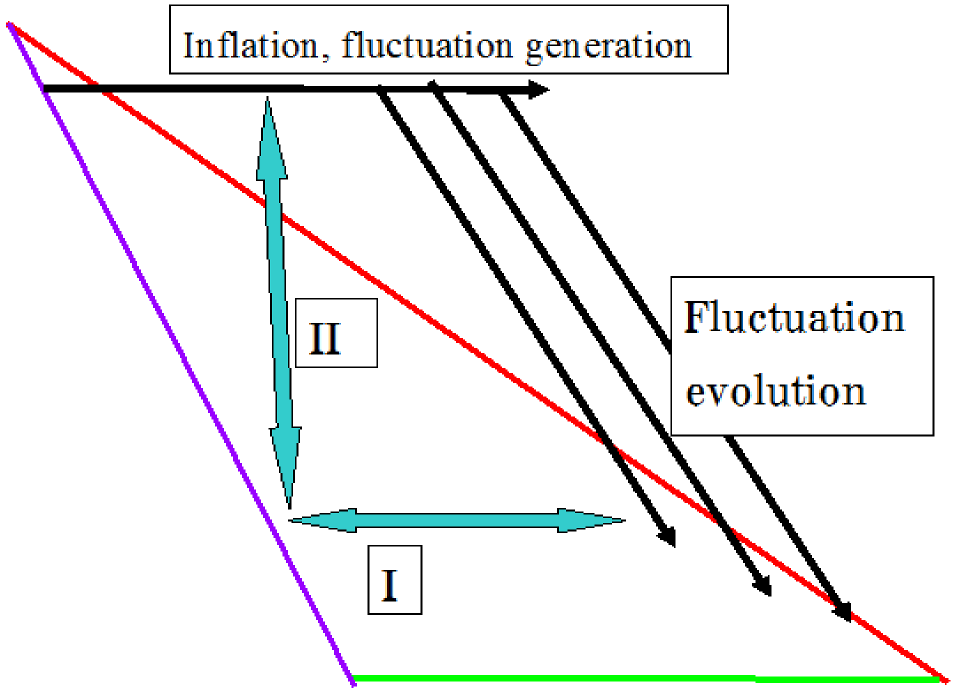
Quantum mechanics is an excellent theory to describe laboratory experiments. However its operational logic prevents us from applying it to the autonomous systems in which no explicit observer exists. The first-structure problem, i.e. generation of the primordial density fluctuations in the early Universe, belongs to such category.

In the first part of the presentation, we propose a physical description of the quantum measurement process based on the collective interaction of many degrees of freedom in the detector. This model describes a variety of quantum measurement processes including the quantum Zeno effect. In some cases this model yields the Born probability rule. By the way, the classical version of this model turns out to be a good model to describe the geomagnetic polarity flipping history over 160 Million years.

Then, in the next part, we generalize this model to quantum field theory based on the effective action method of Keldysh type, in which the imaginary part describes the rich variety of classical statistical properties. Applying this method, we describe (a) the EPR measurement as an autonomous process and (b) the primordial fluctuations from the highly squeezed state during inflation in the early Universe, in the same footing.

§1 Introduction – Universe and QM, Stat Mech.

- cosmological structures are not separated with each other but micro-macro scales are connected.



§2 Statistical and Quantum mechanics

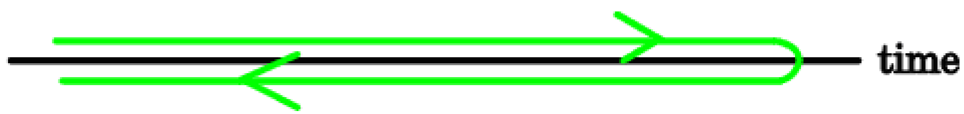
- Entropy and action ...analogy

$$\begin{cases} S_{entropy} = -k_B \mathrm{Tr}\,(\rho \ln \rho) & \to \text{maximized} \\ S_{action} = -i\hbar \ln \Psi & \to \text{minimized} \end{cases}$$

- Then the entropy for a dynamical system is implied as

$$S_{entropy} \approx \frac{k_B}{\hbar} \mathrm{Im} \left(\underbrace{\sum \left(S_{action} - S_{action}{}^{*(time\ \ reversal)} \right)}_{CPT-effective\ \ action\ \ \tilde{\Gamma}} \right)$$

where: $S_{entropy} \approx \dfrac{k_B}{\hbar} \mathrm{Im}\tilde{\Gamma}$



$$\exp\left(i\frac{\tilde{\Gamma}}{\hbar}\right) = \exp\left(\frac{i}{\hbar}\tilde{\Gamma}^{\mathrm{Re}} - \underbrace{\frac{1}{\hbar}\tilde{\Gamma}^{\mathrm{Im}}}_{S_{entropy}\,/\,k_B}\right)$$

- $\tilde{\Gamma}^{\mathrm{Im}}$: will imply statistical information (probabilistic)
  $\rightarrow$ a system actually evolves toward its maximum
- $\tilde{\Gamma}^{\mathrm{Re}}$: will imply mechanical information (deterministic)
  $\rightarrow$ only the extremum is relevant for evolution

- $\tilde{\Gamma}^{\mathrm{Im}} \neq 0 \leftarrow$ finite temperature/density, particle production, unstable systems, ...
- for the case of (meta-)stable vacuum, $\tilde{\Gamma}^{\mathrm{Im}} = 0$ if time scale is not considered

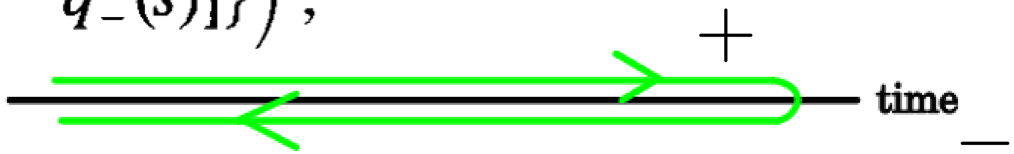
■ relation with the Caldeira-Leggett model (1980)

Full evolution $\rho(t) = \hat{S}(t)\,\rho(0)$ interaction: $H_{\mathrm{I}} = qQ$

Coarse grain Q and yields effective action for the reduced density matrix for q:

$$\hat{S}(t) = \hat{T}\exp\Bigl(-\frac{1}{\hbar}\int_0^t \mathrm{d}\tau \int_0^\tau \mathrm{d}s\ \{\mathrm{i}[q_+(\tau) - q_-(\tau)]\alpha_{\mathrm{I}}(\tau - s)\,[q_+(s) + q_-(s)]$$

$$+ [q_+(\tau) - q_-(\tau)]\alpha_{\mathrm{R}}(\tau - s)\,[q_+(s) - q_-(s)]\}\Bigr),$$



$$\left\{\begin{aligned} \alpha_{\mathrm{R}}(\tau) &= \frac{\eta}{\pi}\int_0^{\Omega} \omega \coth(\hbar\omega/2kT)\cos(\omega\tau)\,\mathrm{d}\omega\,, & &\leftrightarrow \tilde{\Gamma}^{\mathrm{Im}}\text{: fluctuation} \\ \alpha_{\mathrm{I}}(\tau) &= -\frac{\eta}{\pi}\int_0^{\Omega} \omega \sin(\omega\tau)\,\mathrm{d}\omega\,, & &\leftrightarrow \tilde{\Gamma}^{\mathrm{Re}}\text{: dissipation} \end{aligned}\right.$$

■ generalized closed contour path integral method(CPT)

Keldysh1965

$$\tilde{Z}[\tilde{J}] \equiv \mathrm{Tr}[\tilde{T}(\mathrm{Exp}[i\int \tilde{J}\tilde{\phi}]\rho)] \equiv \mathrm{Exp}[i\tilde{W}[\tilde{J}]]$$

$$\tilde{Z}[\tilde{J}] = \mathrm{Tr}[\tilde{T}(\mathrm{Exp}[i\int \tilde{J}\tilde{\phi} - i\int V[\phi]]\rho)]$$

$$=\mathrm{Exp}[-i\int \mathrm{V}[\frac{\delta}{1\delta\tilde{J}}]]\mathrm{Exp}[-\frac{i}{2}\iint \tilde{J}(x)\tilde{G}_0(x,y)\tilde{J}(y)]\mathrm{Tr}[:\mathrm{Exp}[i\int \tilde{J}\tilde{\phi}]:\rho].$$

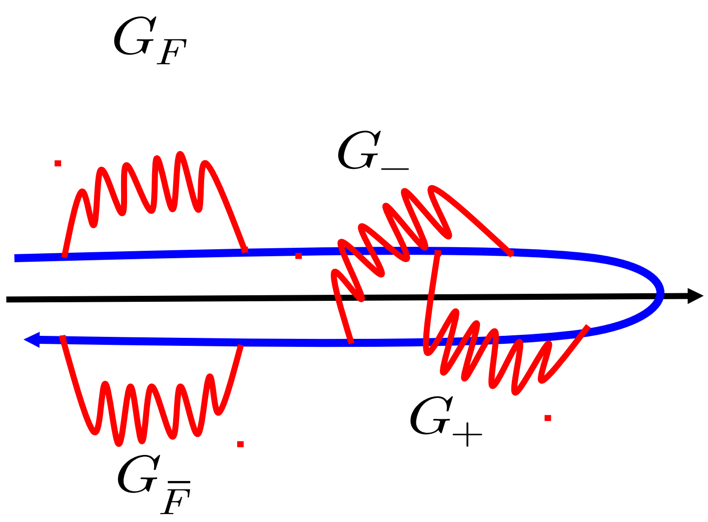
$$L[\phi] = L_0[\phi] - V[\phi],\ \tilde{J}(x) = J_+(x) - J_-(x), \qquad \tilde{\phi}(x) = \phi_+(x) - \phi_-(x)$$

$$\int \tilde{J}\tilde{\phi} = \int dx J_+(x)\phi_+(x) - \int dx J_-(x)\phi_-(x)$$

$$\tilde{G}_0(x,y) = \begin{pmatrix} G_F(x,y) & G_+(x,y) \\ G_-(x,y) & G_{\overline{F}}(x,y) \end{pmatrix}$$

$$\hat{G}_F(k) = \frac{D(k) - iB(k)}{D(k)^2 + A(k)^2}, \quad \hat{G}_{\overline{F}}(k) = \frac{-D(k) - iB(k)}{D(k)^2 + A(k)^2},$$

$$\hat{G}_\pm(k) = -i\frac{D(k) \mp iB(k)}{D(k)^2 + A(k)^2}.$$



D(k): renormalozation for the wave function and mass

A(k): odd for time reversal, new in CTP $\Sigma_F^R(x-x') \Leftrightarrow \mathrm{Im}\,\alpha$ (friction, irreversible)

B(k): even for time reversal, new in CTP $\Sigma_F^I(x-x') \Leftrightarrow \mathrm{Re}\,\alpha$ (classical fluctuations)

Defining the order variable $\tilde{\varphi}(x) \equiv \dfrac{\delta \tilde{W}}{\delta \tilde{J}} = \langle \Psi_i, t_i \left| T\hat{\phi}(y) \exp\left[i \int d^4x J(x) \hat{\phi}(x) \right] \right| \Psi_i, t_i \rangle$

and effective action $\tilde{\Gamma}[\tilde{\varphi}] \equiv W[\tilde{J}] - \int \tilde{J}\tilde{\varphi}$, we have

$$e^{i\tilde{\Gamma}[\varphi]} = \int [d\xi] P[\xi] \mathrm{Exp}[i(\tilde{\Gamma}^{\mathrm{Re}} + \xi\varphi_\Delta)] \quad \text{where } P[\xi] = \mathrm{Exp}[-\frac{1}{2}\iint \xi B^{-1}\xi]$$

Further $\dfrac{\delta \tilde{\Gamma}^{\mathrm{Re}}}{\delta \tilde{\varphi}_\Delta(x)} = -\tilde{J}_c(x)$ follows and becomes the (classical) Langevin equation:

$$(\partial_x \partial^x + m^2)\varphi(x) = -V' + \int_{-\infty}^{t} dt' \int dx' A(x-x')\varphi(x') + \xi(x)$$

- In general. $P[\xi]$ includes non-Gaussian terms
- \`We do not extremize $\tilde{\Gamma}^{\mathrm{Im}}$ (as it is statistical factor)\` is a big assumption.
- $\varphi(x)$ and $\xi(x)$ are classical fields

§3 Physics of quantum measurement

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|A_0\rangle \Rightarrow \underbrace{\alpha|\uparrow\rangle|A_\uparrow\rangle + \beta|\downarrow\rangle|A_\downarrow\rangle}_{|\Psi_{\text{entangled}}\rangle} \underset{measure}{\Rightarrow} \begin{cases} \xrightarrow[|\alpha|^2]{} |\uparrow\rangle|A_\uparrow\rangle \\ \quad or \\ \xrightarrow[|\beta|^2]{} |\downarrow\rangle|A_\downarrow\rangle \\ \quad either \end{cases}$$

3-1. one-to-many (non-deterministic time evolution)

3-2. mixed state ⇔pure state intergradation

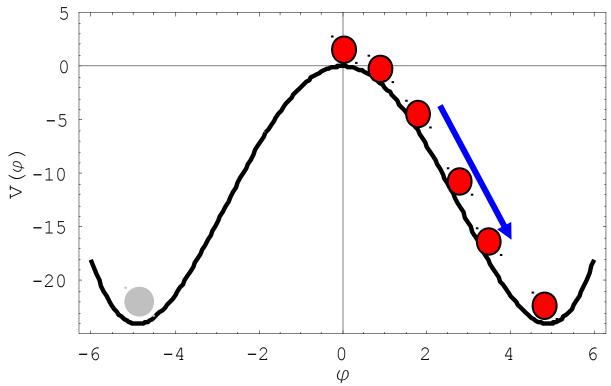
3-3. quantum and classical variables coexistance

3-4. from quantum to classical evolution

3-1. one-to-many

- extension of the domain (=many world) makes the map one-to-one although non-deterministic bifurcation should be described

- A typical process would be the dynamics of spontaneous symmetry breaking or phase transition



- This is essentially infinite degrees of freedom

- Thus, we need QFT beyond QM → vacuum changes in time

3-2. Mixed state ↔ pure state intergradation

- a spin (½) in the environment yields pure ↔ mixed evolution

$$\frac{d\rho}{dt} = -i\omega[S_3, \rho(t)] + a[S_+\rho(t), S_-] + b[S_-\rho(t), S_+]$$
$$+ c[S_3\rho(t), S_3] + h.c.$$

- If (spin temperature) < (environment temperature)

  ρ evolves from pure ⇒ mixed

  well studied relaxation/decoherence process

- If (spin temperature) > (environment temperature)

  ρ evolves from mixed ⇒ pure

- Actually $a = n, b = 1 + n$ and $a = \exp[-\lambda \hbar E_{\text{int}} / (kT)] b$

$$\rho \underset{t\to\infty}{\longrightarrow} \begin{pmatrix} \frac{b}{a+b} & 0 \\ 0 & \frac{a}{a+b} \end{pmatrix}$$

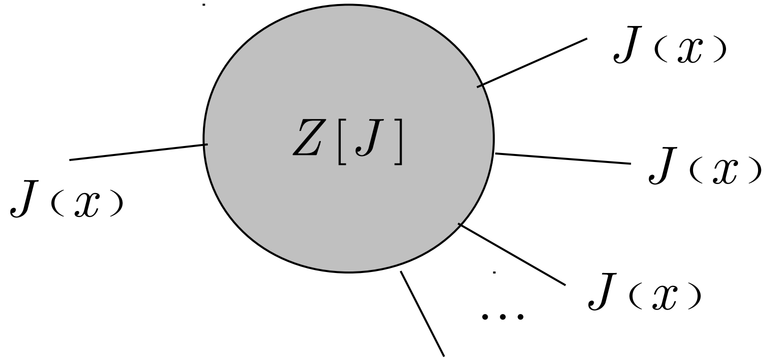
Thus $\begin{cases} \lambda > 0, T \to 0 & \text{yields } |u\rangle \\ \lambda < 0, T \to 0 & \text{yields } |d\rangle \end{cases}$

This is pro-coherence.

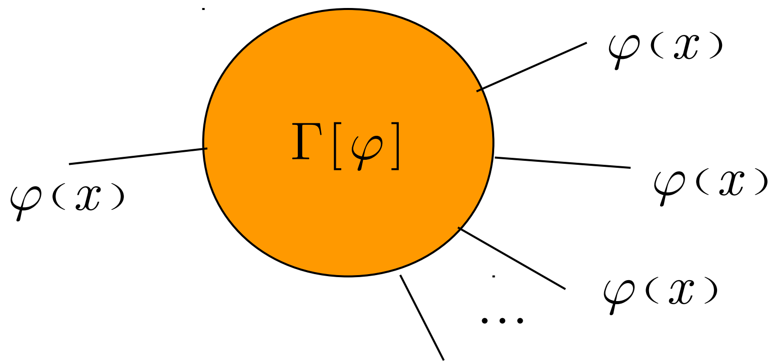
3-3. Quantum & classical variables coexistence

Partition function of QFT connects classical and quantum:

$$Z[J] = \langle \Psi_f, t_f | T \exp\left[i \int J \hat{\phi} \right] | \Psi_i, t_i \rangle$$



Also the effective action $\Gamma[\varphi]$



… These are the whole collection of fluctuations:

$$\int \underbrace{\varphi(x)}_{classical} \underbrace{\langle \Psi_f, t_f | \hat{\phi}(x) \ldots \hat{A}(y) \hat{\phi}(z) | \Psi_i, t_i \rangle}_{quantum} \underbrace{\varphi(z)}_{classical} \, dxdydz$$

3-4. Emergence of classical variable in QFT

- Keldysh effective action yields stochastic eq. for $\varphi(x)$:

  - $\mathrm{Im}\,\tilde{\Gamma} = (\text{even in } \varphi_\Delta(x)) = \frac{1}{2}\iint \varphi_\Delta(x) B(x-y) \varphi_\Delta(y) + \dots$

  This kernel $B(x-y)$ is positive. Therefore within Gaussian approximation, the effective action can be written as

  $$e^{i\Gamma[\varphi]} = \int [d\xi] P[\xi] \mathrm{Exp}[i\,\mathrm{Re}\,\Gamma + i\xi\varphi_\Delta], P[\xi] \equiv \mathrm{Exp}[-\frac{1}{2}\iint \xi B^{-1}\xi]$$

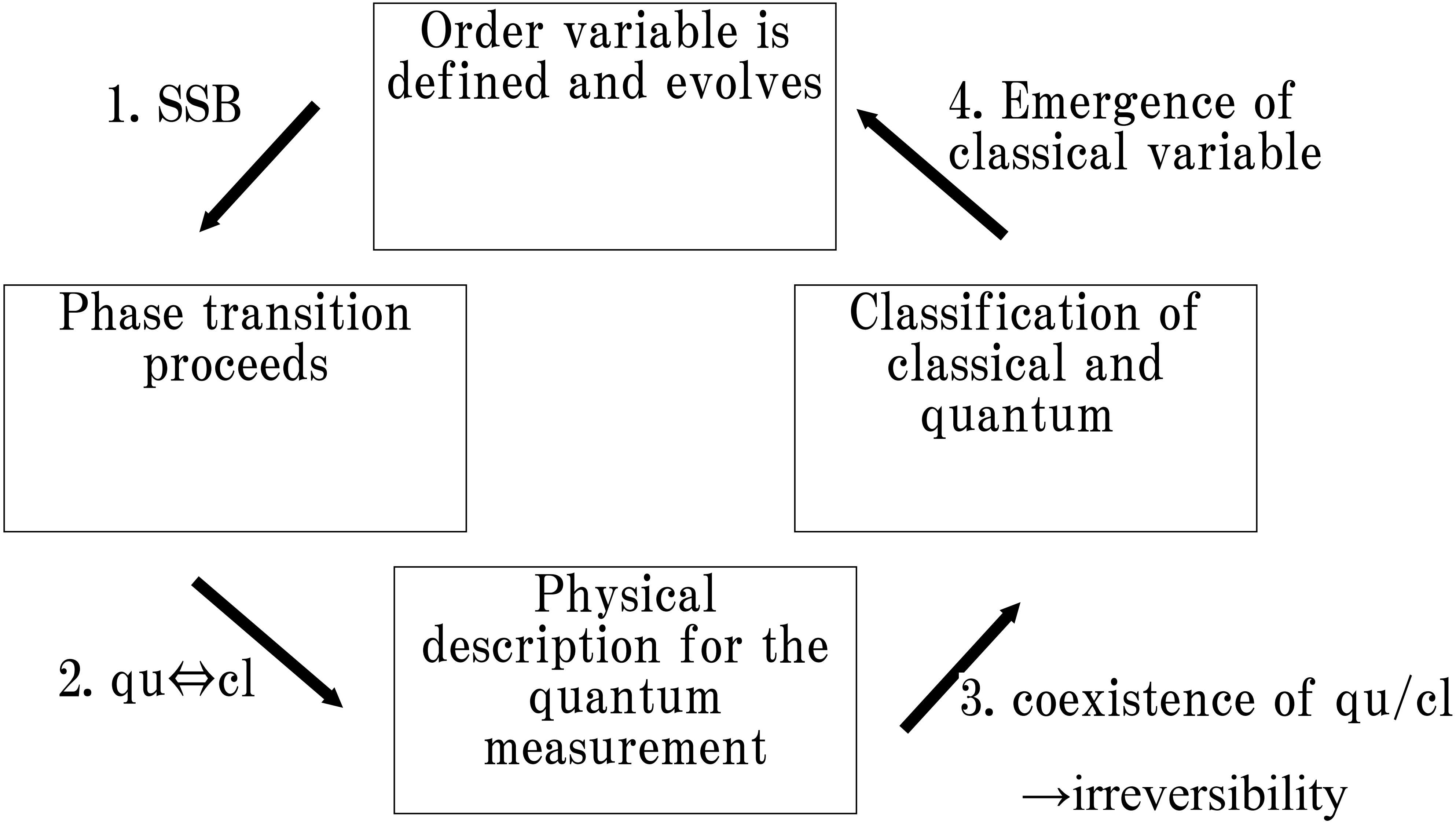
  is the Gaussian weight function for random field $\xi(x)$.

  On the other hand, $\mathrm{Re}\,\Gamma + \xi\varphi_\Delta \equiv S_{\mathrm{eff}}$ is real, and the variational principle for $\varphi_\Delta(x)$ is applied to yield

$\delta S_{\mathrm{eff}} \,/\, \delta\varphi_\Delta(x) = -\tilde{J}$ which is the Langevin eq. for classical fields:

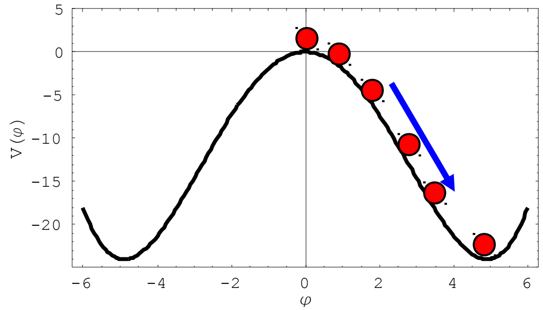
$$(\Box + m^2)\varphi_c = -V' + \int_{-\infty}^{t} dt' \int dx' A(x-x')\varphi_c(x') + \xi$$

3-5. Common origin for all the above problems?



§4 Quantum measurement model in QFT

- starting points
  - measurement apparatus is the fields
  - all dynamical process, including measurement process, is caused by the field interactions.
  - information obtained by a measurement is a pattern on the field
- A simple model of measurement



Field ϕ measures a spin $\vec{S}$:

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + \mu\phi\vec{S}\cdot\vec{B} + (\text{bath}) \quad \text{where } \mu > 0.$$

Progress of Theoretical Physics Vol. 116 No. 4 (2006) pp. 679-698 :

Quantum Measurement Driven by Spontaneous Symmetry Breaking ・・・ Masahiro Morikawa and Akika Nakamichi

- The minimu model which keeps the essence of the original mdel.

- Generalized effective action method yields for $\hat{\phi} = \varphi + \delta\hat{\phi}$,

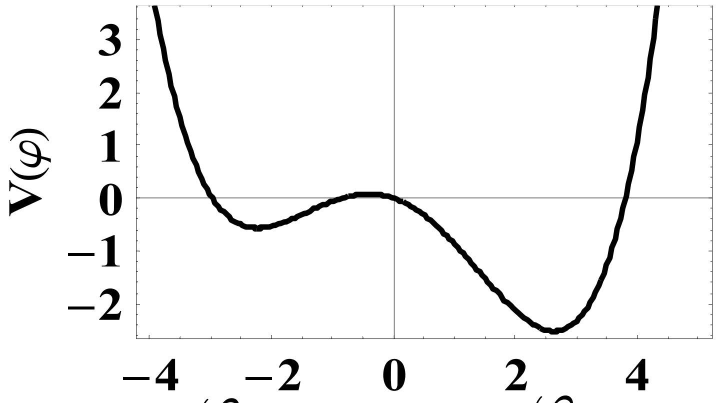
$$\dot{\varphi} = \gamma\varphi - \frac{\lambda}{3!}\varphi^3 + \mu\left\langle \vec{S} \right\rangle \cdot \vec{B} + \xi, \ \left\langle \xi(t)\xi(t')\right\rangle = \varepsilon\delta(t - t')$$

- spin density matrix in the environment,

N. Hashitsume, F. Shibata and M. Shingu, J. Stat. Phys. 17 (1977), 155.

$$\frac{d\rho}{dt} = -i\omega[S_3, \rho(t)] + a[S_+\rho(t), S_-] + b[S_-\rho(t), S_+] + c[S_3\rho(t), S_3] + h.c.$$

■ ξ triggers SSB initially $\langle \vec{S} \rangle \cdot \vec{B} > 0 \Rightarrow$ linear bias $\Rightarrow$ force φ toward $\varphi_+ > 0$



φ_- φ_+

■ mixed→pure

in the factor $a = \exp[-\hbar\mu\varphi(t)B/(kT)]b$, the effective temperature $T/\varphi(t) \to 0 \Rightarrow$ spin is brought into a pure state↑ $|\uparrow\rangle$

■ positive feedback linear bias is enhanced furthr $\Rightarrow$ spin $\vec{S}$ tends to be more parallel to $\vec{B}$.

- This positive feedback fixes the spin ↑, order variable $\varphi \to \varphi_+$.

■ On the other hand if initially $\left\langle \vec{S} \right\rangle \cdot \vec{B} < 0$, then the above process fixes the spin↓, order variable φ_{-}.

■ results: we solved the coupled equations for ρ and φ.

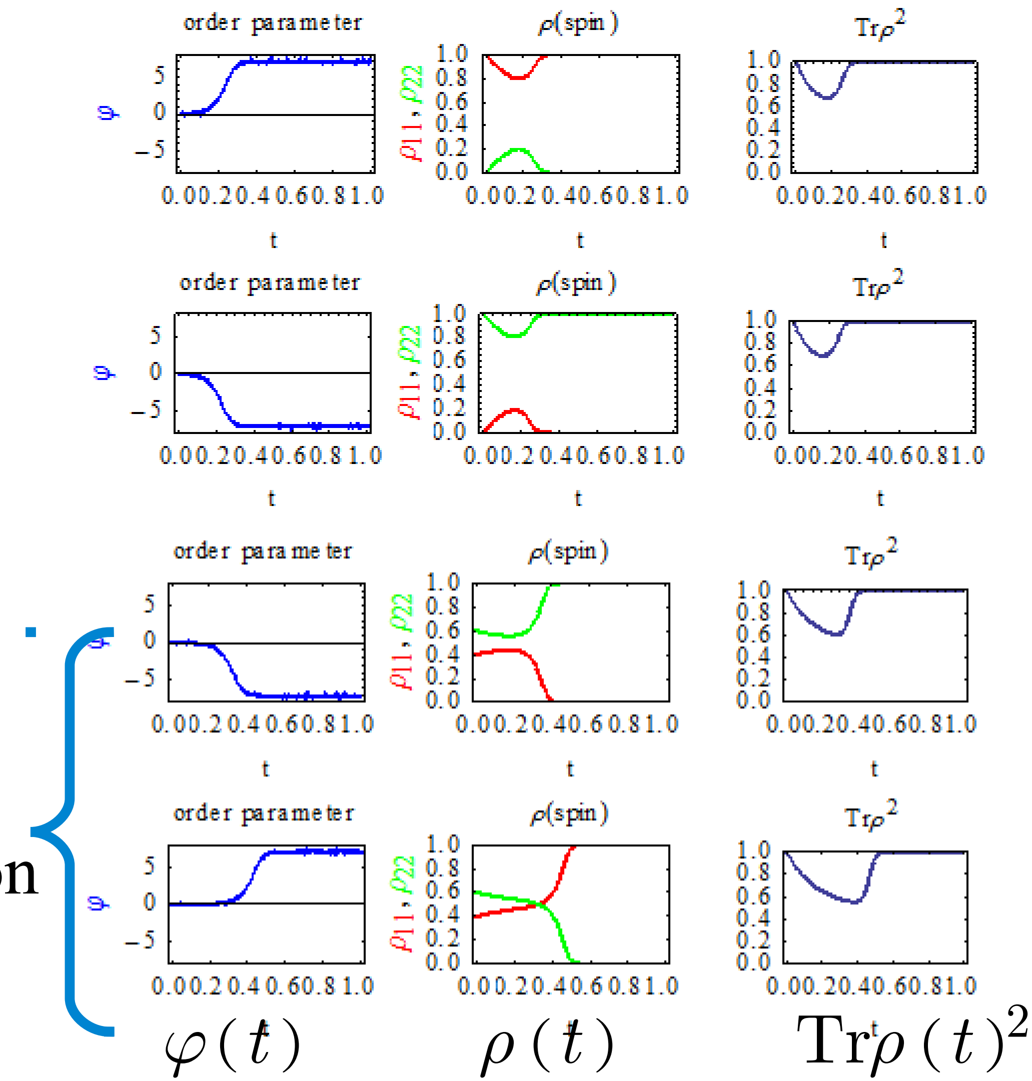
If $\rho(0) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, then

If $\rho(0) = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$, then

Further,

If $\rho(0) = \begin{pmatrix} 0.4 & 0.49 \\ 0.49 & 0.6 \end{pmatrix}$ etc.

then, the frequency distribution yields QM results



- comments

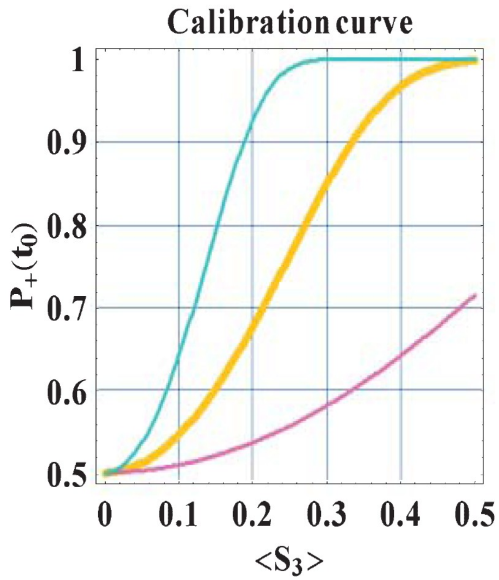
1. `world bifurcation`(many world)→`SSB`(our world)
2. For the general initial condition $\alpha|\uparrow\rangle + \beta|\downarrow\rangle$, bias $\delta = (\mu/\gamma)\langle\vec{S}\rangle\cdot\vec{B}$ affects $P(\varphi, t)$ non-linearly.

   a faithful measurement needs a calibration.

3. a relevant time scale:

$$t_0 = \frac{1}{2\gamma}\ln\left[\frac{g}{\gamma}\left(\frac{\varepsilon}{\gamma}+\delta^2\right)\right]^{-1}$$

   i.e. the completion time needed for SSB.

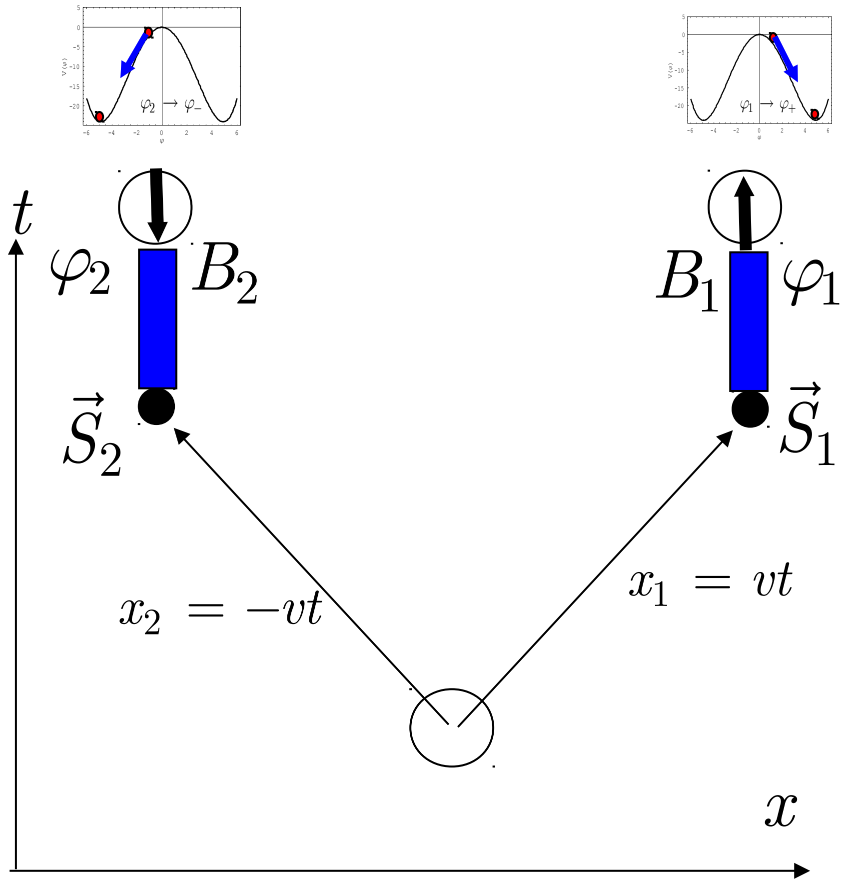


4. Application to various measurement processes are possible.

§5 EPR measurement (two spins $\vec{S}_1, \vec{S}_2$)

$$L = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

$$+ \mu\phi\vec{S}\cdot\vec{B} + (\text{bath})$$



- spatially separated spins
- localized apparatus ϕ_1, ϕ_2 measure each spin under individual magnetic fields B_1, B_2. are applied
- only causal interaction
- initial state of the system measured:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\right)$$

- Correlation terms in generalized effective action

$$\int \mu\phi(x)\vec{S}(x)\cdot\vec{B}(x)\,dx \rightarrow \int \mu\varphi \mathrm{Tr}\left(\rho\vec{S}\cdot\vec{B}(x)\right)dx$$

$$\iint \left(\mu\widehat{\phi}\widehat{\vec{S}}\cdot\vec{B}\right)_{x_1}\left(\mu\widehat{\phi}\widehat{\vec{S}}\cdot\vec{B}\right)_{x2} \rightarrow i\iint \mu^2\varphi(x_1)B_i(x_1)\mathrm{Tr}\left(\rho\widehat{S}_i(x_1)\widehat{S}_j(x_2)\right)B_j(x_2)\varphi(x_2)$$

...yields stochastic diff. Eequations:

$$\dot{\varphi}_1 = \gamma\varphi_1 - \frac{\lambda}{3!}\varphi_1^3 + \mu\mathrm{Tr}\left(\rho\vec{S}_1\cdot\vec{B}_1\right) + \vec{\xi}_1\cdot\vec{B}_1$$

$$\dot{\varphi}_2 = \gamma\varphi_2 - \frac{\lambda}{3!}\varphi_2^3 + \mu\mathrm{Tr}\left(\rho\vec{S}_2\cdot\vec{B}_2\right) + \vec{\xi}_2\cdot\vec{B}_2$$

$$\left\langle \xi_{1i}\xi_{2j}\right\rangle \leftarrow Tr\left(\rho\hat{S}_i^{(1)}\hat{S}_j^{(2)}\right)$$ ($-\delta_{ij}$ intially, quantum corelation)

For the state $\rho \equiv \rho_1 \otimes \rho_2$

$$\frac{d\rho}{dt} = -i\omega[S_3,\rho(t)] + a[S_+\rho(t),S_-] + b[S_-\rho(t),S_+] + c[S_3\rho(t),S_3] + h.c.$$

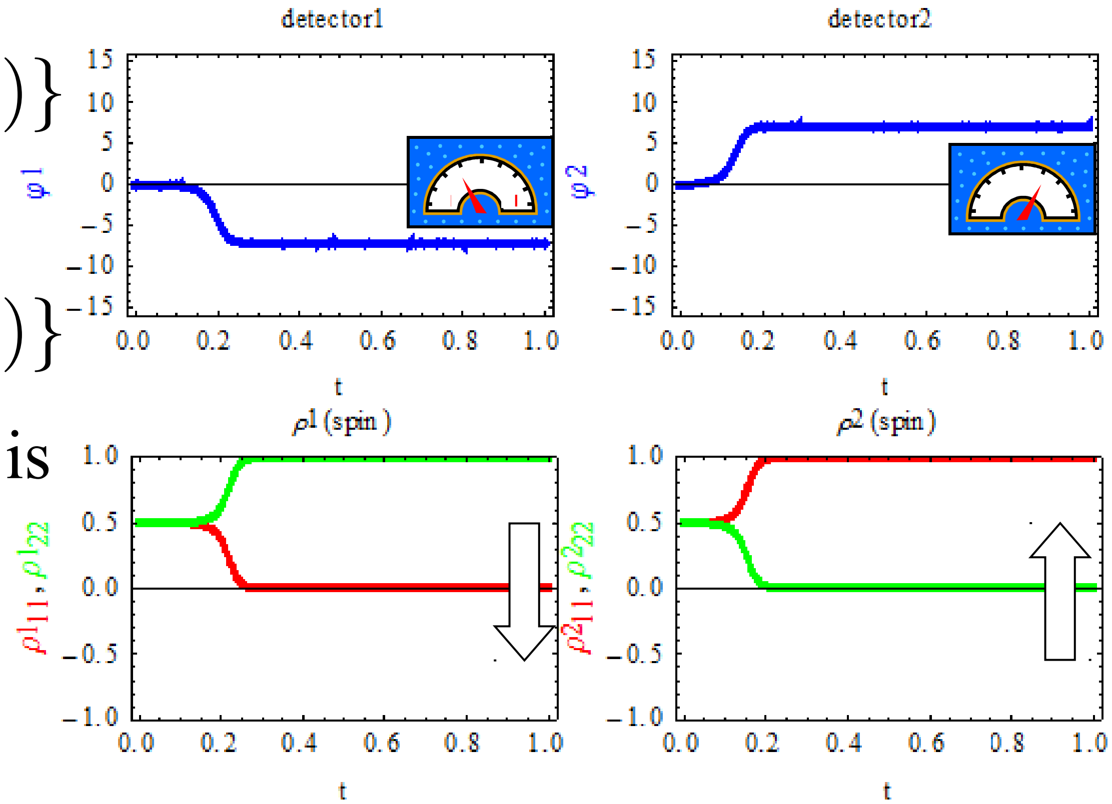
- From the initial state $|\psi\rangle = \dfrac{1}{\sqrt{2}}\left(|\uparrow\rangle_1 \otimes |\downarrow\rangle_2 - |\downarrow\rangle_1 \otimes |\uparrow\rangle_2\right)$,

$\{\vec{B}_1 = (0,0,1),\ \vec{B}_2 = (0,0,1)\}$

or

$\{\vec{B}_1 = (1,0,0),\ \vec{B}_2 = (1,0,0)\}$

etc. complete anti-correlation is obtained.



Spin state ρ and meter read φ are consistent with each other.

■ Bell in-equallity is violated?

- in probability $P(B_1, B_2) = \int \varphi_{1final}(\lambda, B_1)\varphi_{2final}(\lambda, B_2)\rho d\lambda$ the weight ρ depends on $\varphi_1, B_1, \varphi_2, B_2$. Therefore this is not the `hidden variable theory`.

- $\langle \xi_{1i}\xi_{2j} \rangle \leftarrow Tr\left(\rho \hat{S}_i^{(1)} \hat{S}_j^{(2)}\right)$ all the correlations of quantum theory is included.

- Evolutions of the classical field φ and the density matrix ρ couple with each other. Positive feedback is the essence.

- Actually

$$\delta = \vec{\xi}_1 \cdot \vec{B}_1 / \gamma, \quad \sqrt{2\varepsilon(t)} \approx \gamma^{-1/2}$$ (γ: friction M. Suzuki, Adv. Chem. Phys. **46** (1981), 195.) and

$\left\langle P^{(1)}{}_{+}\left(\vec{B}_1\right) P^{(2)}{}_{-}\left(\vec{B}_2\right)\right\rangle = P_{+-}$ yield

$$C\left(\vec{B}_1, \vec{B}_2\right) \equiv P_{++} + P_{--} - \left(P_{+-} + P_{-+}\right)$$

$$= \left\langle \mathrm{erf}\left(\gamma^{-1/2}\vec{\xi}_1 \cdot \vec{B}_1\right) \mathrm{erf}\left(\gamma^{-1/2}\vec{\xi}_2 \cdot \vec{B}_2\right)\right\rangle.$$

$$\approx -\gamma^{-1}\vec{B}_1 \cdot \vec{B}_2$$

Setting the magnetic field $\left|\vec{B}_1\right|\left|\vec{B}_2\right| = \gamma$ makes $C\left(\vec{B}_1, \vec{B}_2\right) \approx -\cos\theta_{12}$.

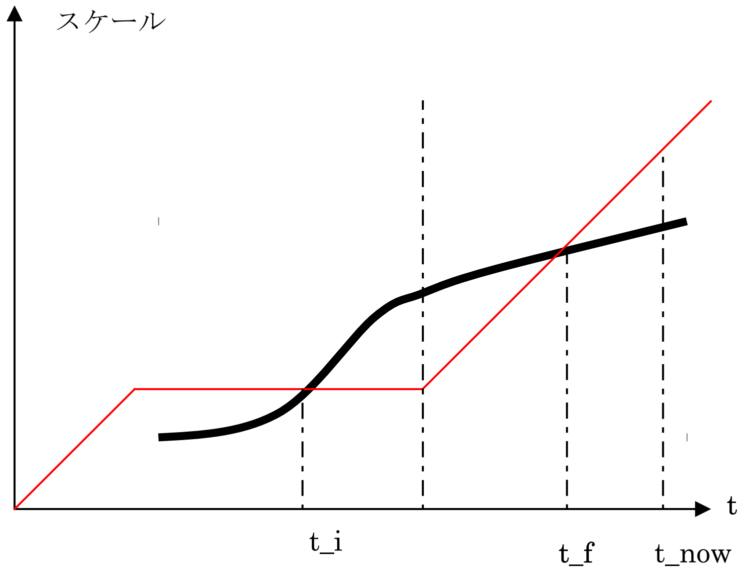
Thus if the \`accuracy\` ($\mathrm{erf}(\ ), \gamma, \ldots$) are sufficient, we have

$$\left|C\left(\vec{B}_1, \vec{B}_2\right) + C\left(\vec{B}_1', \vec{B}_2\right) + C\left(\vec{B}_1, \vec{B}_2'\right) - C\left(\vec{B}_1', \vec{B}_2'\right)\right| > 2$$

■ There is a possibility that the Bell inequality is violated.

§6 Origin of the primordial density fluctuations

■ The inflationary model in the early Universe



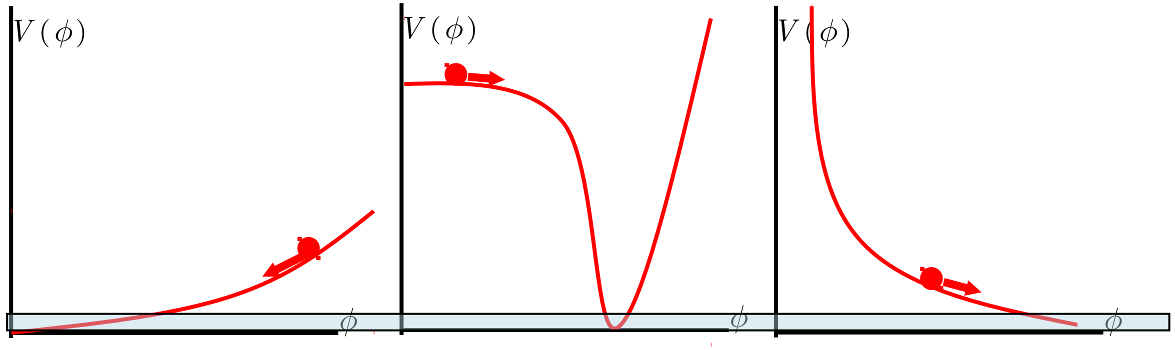
- The scenario:
  from FRW expansion $\left(a(t) \propto \sqrt{t}\right)$
  then, de Sitter expansion $\left(a(t) \propto e^{Ht}\right)$
  connected to FRW $\left(a(t) \propto \sqrt{t}\right)$
- Inflaton field (scalar field) promotes the inflation

- k-mode obeys $$v_k'' + \left(k^2 - 2H^2 e^{2Ht}\right) v_k = 0$$

and highly squeezes, yielding IR-fluctuations.

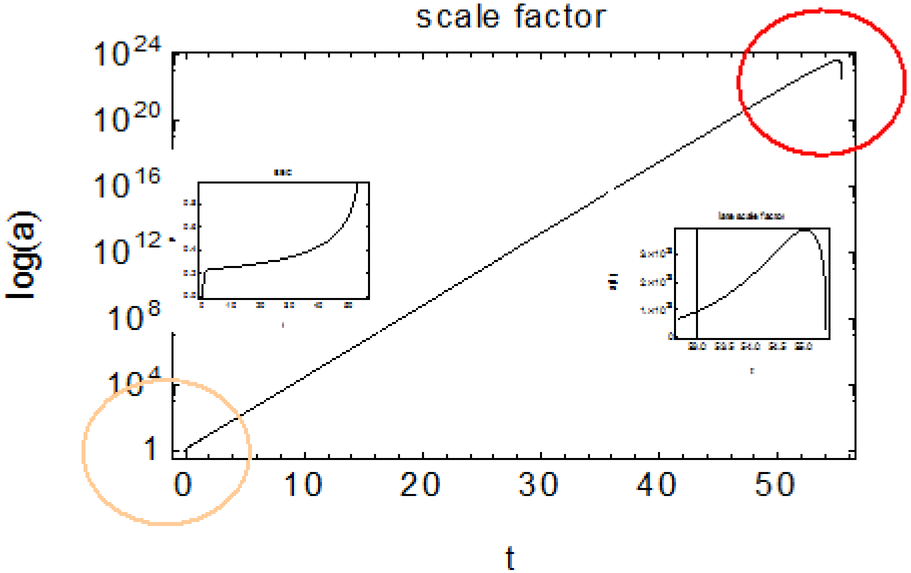
$$v_k \approx k^{-1/2}\left(1 + ik^{-1} H e^{Ht}\right) \qquad (v_k \equiv a\varphi_k)$$

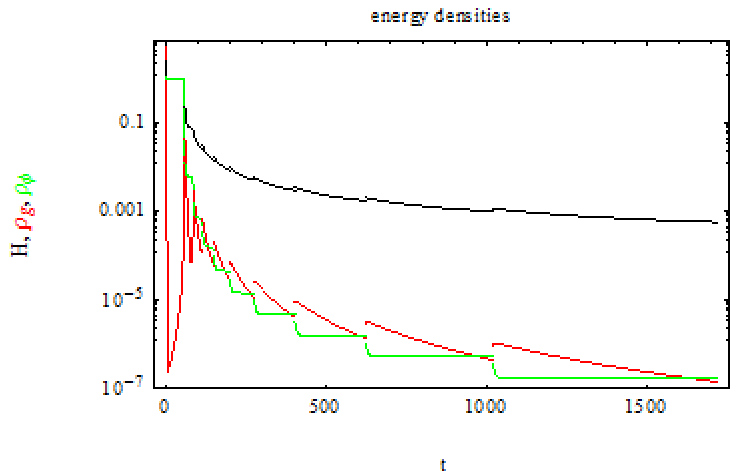
■ Universal inflation model?



A standard model but allows negative region of the potential. Then, this model always enters into the period of stagflation ($\rho_\phi = \dot{\rho}_\phi = H = 0$ but $\varphi \to \infty$), where the expansion stops. Then the uniform mode $\varphi_0$becomes unstable and decays → FRuniverse

⇒Λ-self regulation Phys.Rev.D80:063520,2009

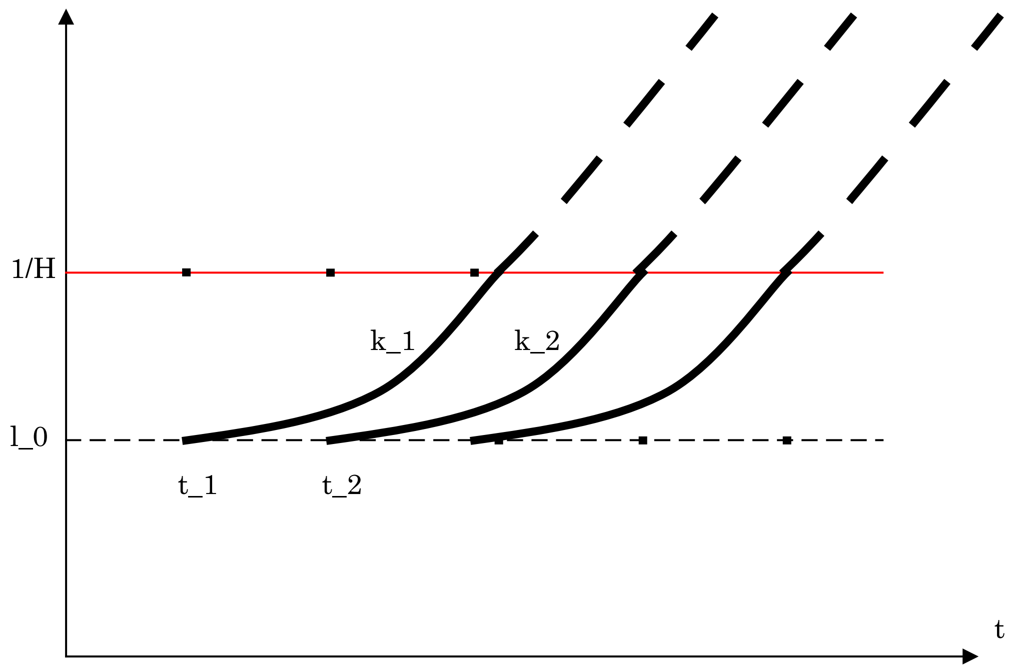




- standard theory assumes $\underbrace{\left\langle \hat{\phi}(x)\hat{\phi}(y)\right\rangle\Big|_k}_{quantum\ \ theory} = \underbrace{|\delta\varphi(k)|^2}_{\substack{clasical \\ stat.\ \ mech.}}, \quad \varphi_k \equiv \frac{v_k}{a}$

i.e. a mode becomes classical when goes beyond the horizon.

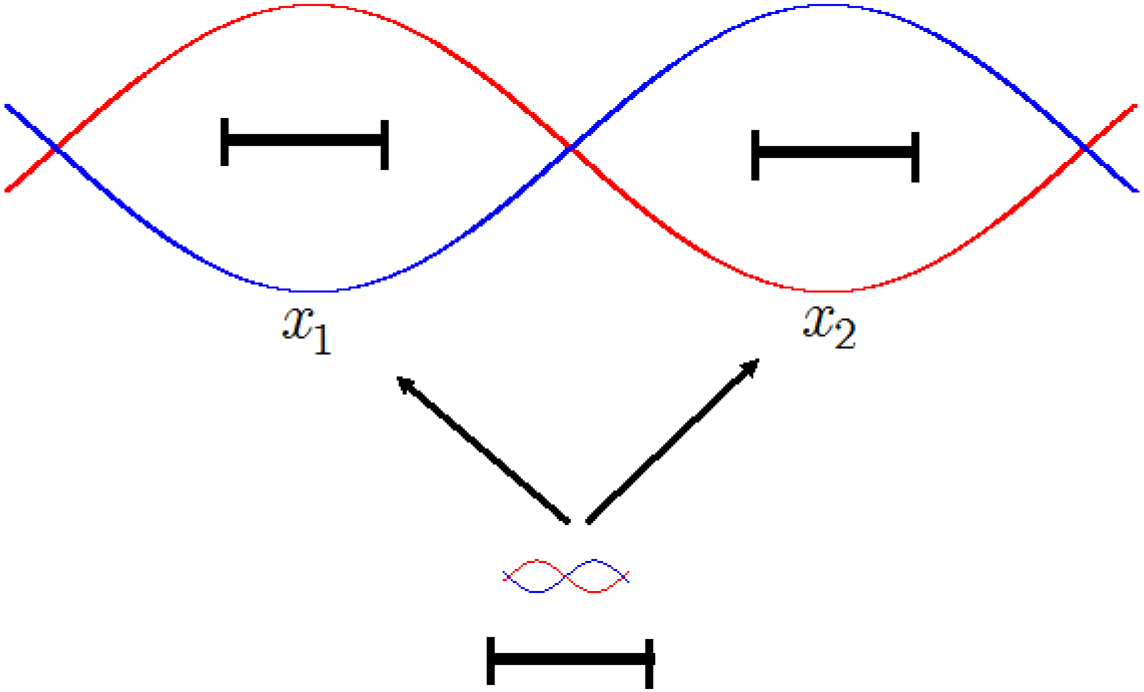
Therefore $P_{\delta\varphi} = \frac{4\pi k^3}{(2\pi)^3}\left|\frac{v_k}{a}\right|^2 \xrightarrow[k/(aH)\to 0]{} \left(\frac{H}{2\pi}\right)^2$ (scale invariant)



each mode evolves analogously

→ scale invariant fluctuations

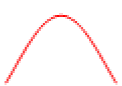
■ quantum → classical fluctuation: analogous to EPR measurement

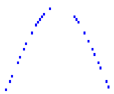


de Sitterexpansion

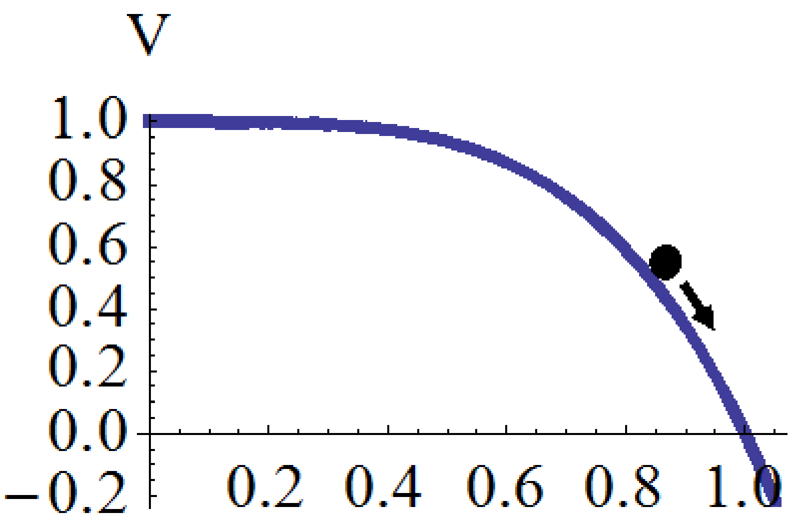
horizon size (causal limit)

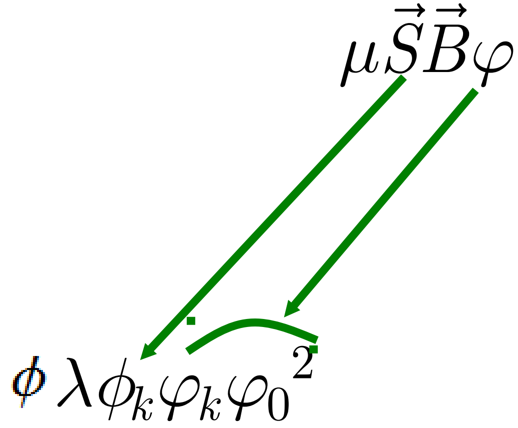
squeezed state (after complete measurement)

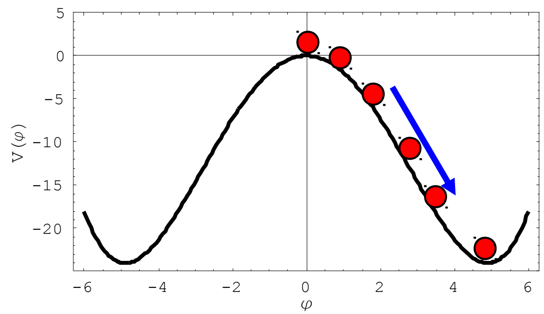
If  at x_1, then  at x_2 or,

If  at x_1, then  at x_2

■ Interaction in the Universe at reheating stage.







case of spin

interaction $\lambda\phi^4$, $\phi = \varphi_0 + \delta\varphi$

the term ${\varphi_0}^2\delta\varphi(k)$ ——— ${\varphi_0}^2\delta\varphi(k)$ yields

$$\tilde{\Gamma} = \tilde{\Gamma}_0 + \int \lambda{\tilde{\varphi}_0}^2\delta\tilde{\varphi}(k)\left\langle \tilde{\phi}(x)\,\tilde{\phi}(y) \right\rangle \lambda{\tilde{\varphi}_0}^2\tilde{\varphi}(-k)\,dk$$

$$\mathrm{Re}\langle\phi(\vec{z})\phi(0)\rangle = \int_0^\infty dk\,\frac{\sin[kz]}{4\pi^2 z}\left[\left(\frac{\eta'^3-\eta^3}{3\eta'\eta}\right)+O\left(k^3\right)\right]$$

...IR($k \to 0$) converges $\qquad \to G_{\mathrm{ret}}$: mechanics

$$\mathrm{Im}\langle\phi(\vec{z})\phi(0)\rangle = \int_0^\infty dk\,\frac{\sin[kz]}{4\pi^2 z}\left[\frac{-1}{\eta'\eta k^2}-\frac{1}{2}\left(\frac{\eta}{\eta'}+\frac{\eta'}{\eta}\right) + O\left(k^2\right)\right]$$

… IR-divergent $\qquad \to G_{\mathrm{c}}$: statistical mech.

yields scale free fluctuations

■ Applying the least action method on $\mathrm{Re}\,\tilde{\Gamma}$, (not $\mathrm{Im}\,\tilde{\Gamma}$, stat. mech.)

$$\Box\varphi(x) = -V'(\varphi(x)) + \int_{-\infty}^{t_x} dy G_{\mathrm{ret}}(x-y)\varphi(y) + \xi(x)$$

where $\langle \xi(x)\xi(y)\rangle = G_{\mathrm{c}}(x-y)\lambda^2\varphi_0(x)^2\varphi_0(y)^2$

IR-divergent statistical fluctuations

Leaving the most relevant part $\ddot{\varphi}_k \approx \xi_k$, we have

$$P_{\delta\varphi} = \frac{4\pi k^3}{(2\pi)^3}|\varphi_k|^2 \xrightarrow[k/(aH)=1]{} \lambda^2\left(\Delta t\,\varphi_0\right)^4\left(\frac{H}{2\pi}\right)^2$$

i.e. scale invariance is universal but the amplitude is not

present case, setting $\Delta t \approx \varphi_0/\dot{\varphi}_0$, $V(\varphi_0)\approx 0$ yield $P_{\delta\varphi} = O(1)\left(\frac{H}{2\pi}\right)^2$

chaotic $\lambda\phi^4$ model ($\varphi_0 \approx 0$) yields no fluctuations

In general, the amplitude depends on the interaction at reheating

■ quantum theory in the inflationary era

- correlation in the quantum system $\hat{\phi}(x)$ is traced in the macroscopic pattern in apparatus $\varphi(t,\vec{x})$. This evolves into the present large scale structure.

i.e. the quantum system $\hat{\phi}(x)$ disappears and only apparatus remains in the universe.

- IR-divergence in $\Gamma^{\mathrm{Re}}, \Gamma^{\mathrm{Im}}$: does it factorize? ...not solved

$$Z[J] = \exp\left[i \int L\left(\frac{\delta}{i\delta J} \right) \right] \exp\left[\frac{1}{2} \int J \left(G^{\mathrm{Re}} + G^{\mathrm{Im}} \right) J \right]$$

- variety of reheating mechanism → selection of the inflation models

§7 summary

1. We constructed a physical quantum measurement model.
   - phase transition (SSB), de/pro coherence, positive feedback
   - measurement process is a phase transition (SSB)
   it is deterministic ($-V'$) as well as non-determinsitic($\xi(t, \vec{x})$)
   - EPR measurement process, Bell inequality
2. Origin of the primordial density fluctuations
   - classical apparatus variable determines the large scale structure
   - statistical fluctuations are autonomously generated as in the case of EPR measurement.
   - density fluctuations depends on the reheating interaction and the potential

- Cosmological structures are not separated with each other but micro-macro hierarchy are connected:
  measurement process,
  EPR,
  celestial magnetism,
  primordial density fluctuations
  ...

  - A system can be non-equilibrium and dissipative at any scale.