

**Theoretical Science Colloquium  
Okochi Memorial Hall, Riken,  
May 31, 2013**



SCIENCE

**Quantum Leap: Scientists  
Teleport Bits of Light**

By [Clara Moskowitz](#)  
Published April 14, 2011



**Quantum information processing  
for coherent communication**

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# Collaborators

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Physical process of

# Information processing

encoding in  
physical systems



state transformation  
of physical systems

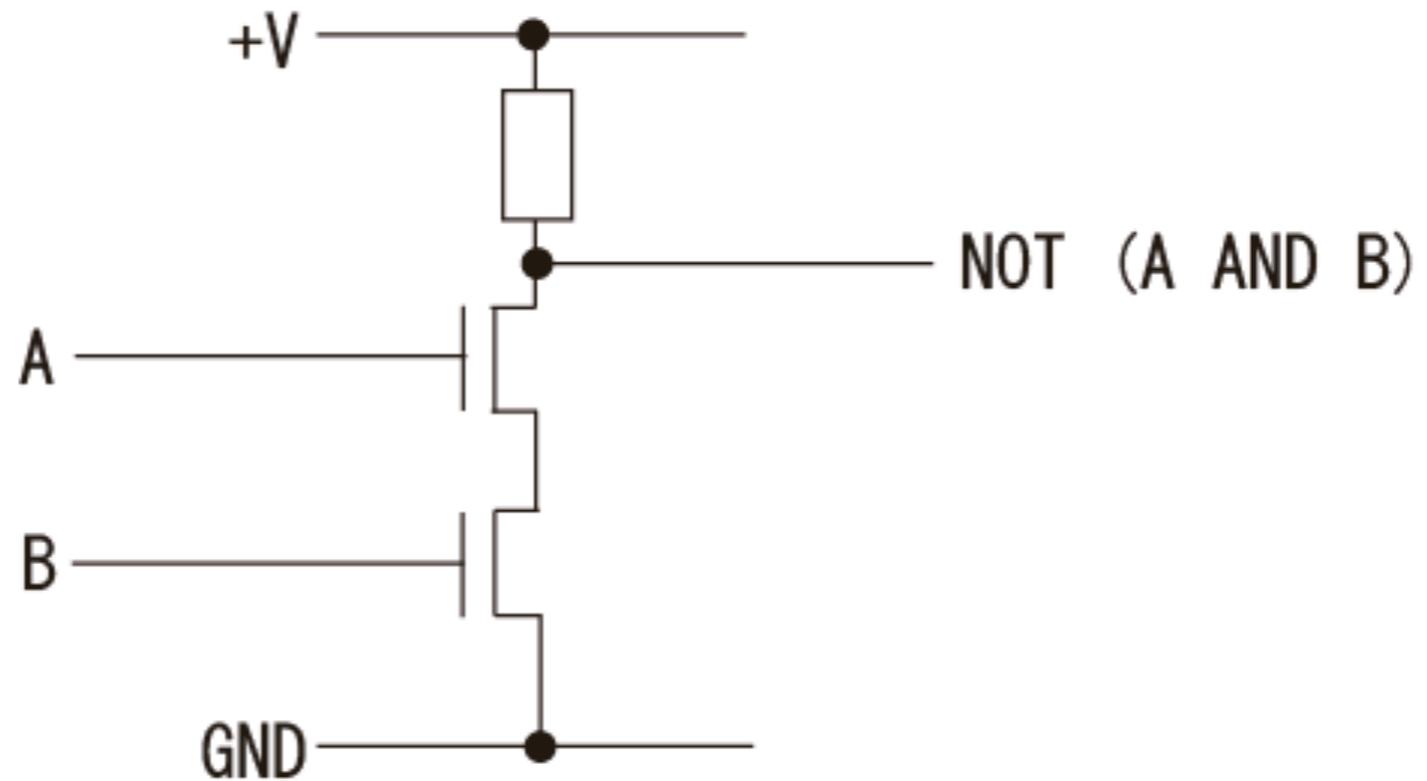


decoding from  
physical systems

Capacitor voltage

0 :  $V < V_S$

1 :  $V > V_S$



NAND gate

Physical process of

# Quantum information processing

encoding in  
physical systems

**quantum**



state transformation  
of physical systems

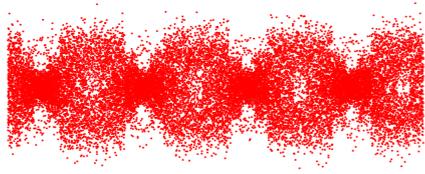
**quantum**



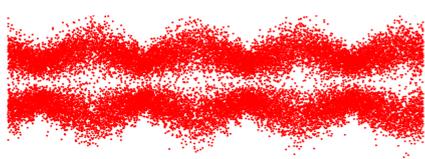
decoding from  
physical systems

**quantum**

$|0_L\rangle$



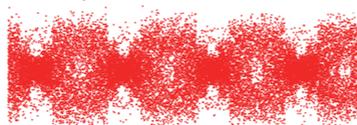
$|1_L\rangle$



Inputs



$|1_L\rangle$



$|0_L\rangle$

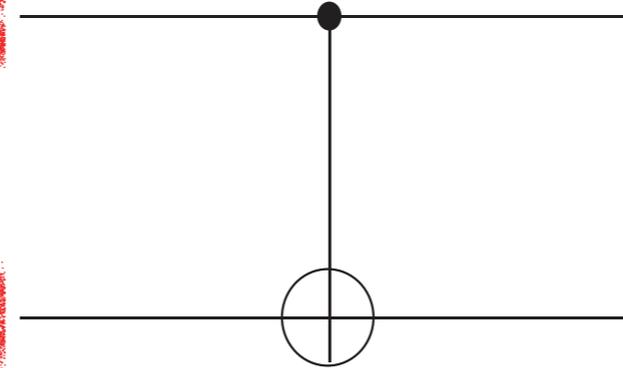
Outputs



$|1_L\rangle$



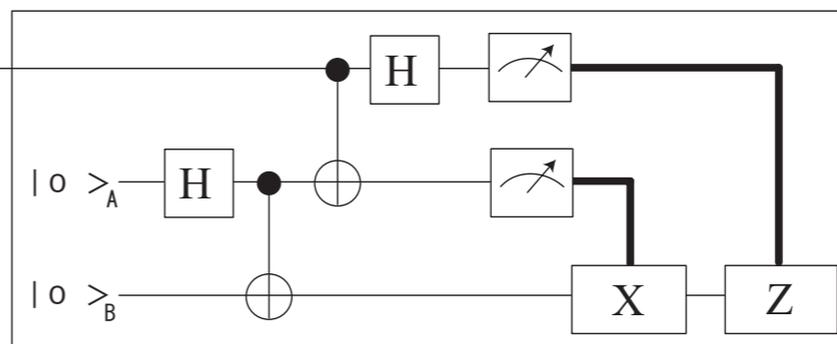
$|1_L\rangle$



Input



**teleporter**



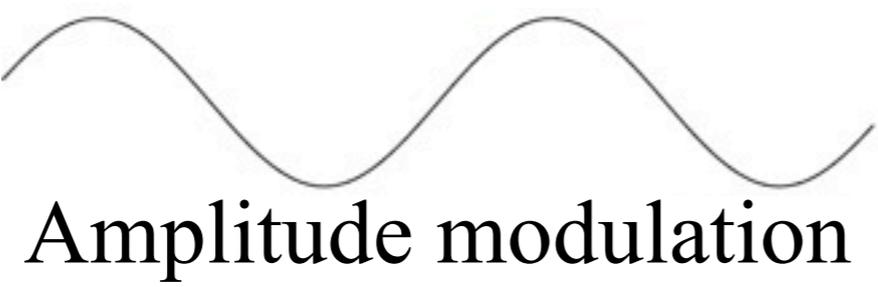
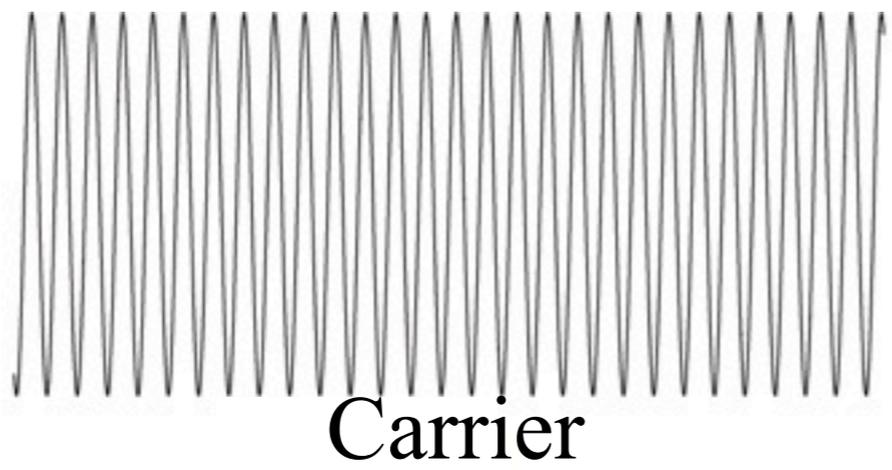
Output



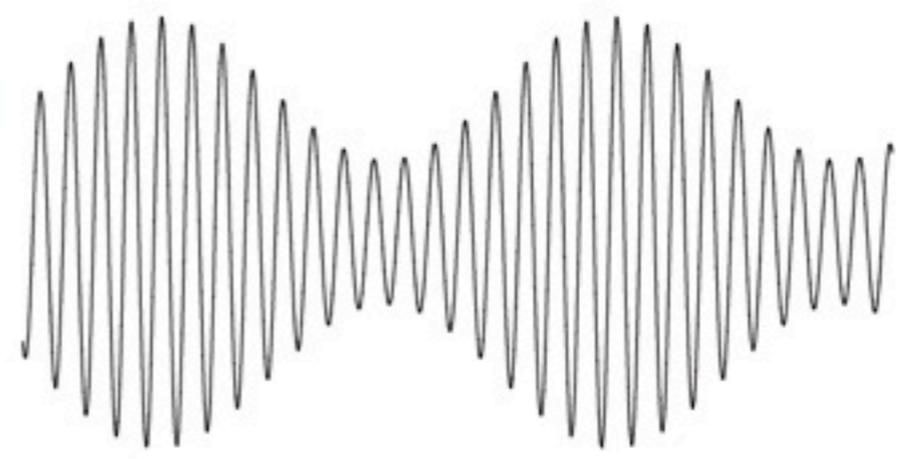
Coherent communication  
and  
Quantum information processing

# AM and FM signals

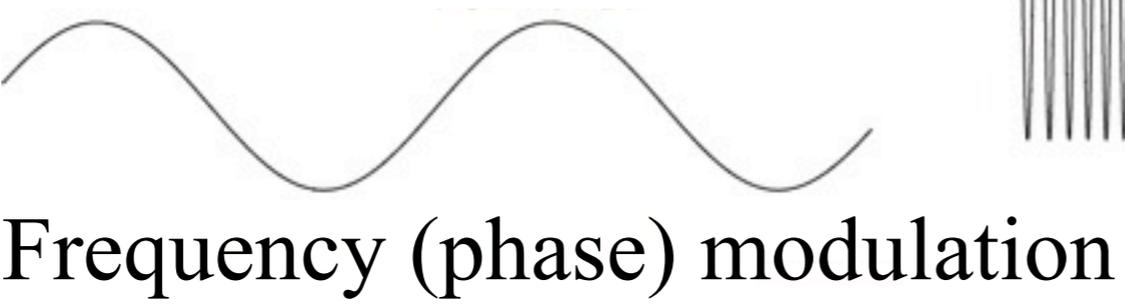
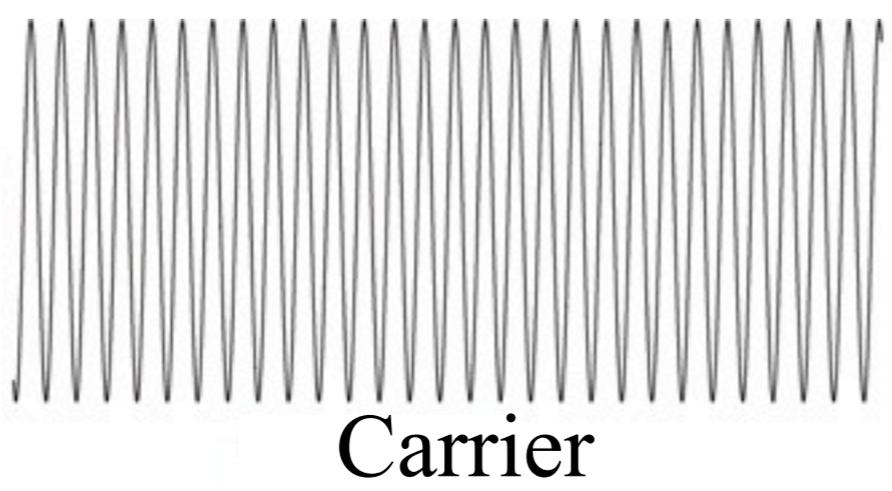
NHK  
(AM)  
594kHz



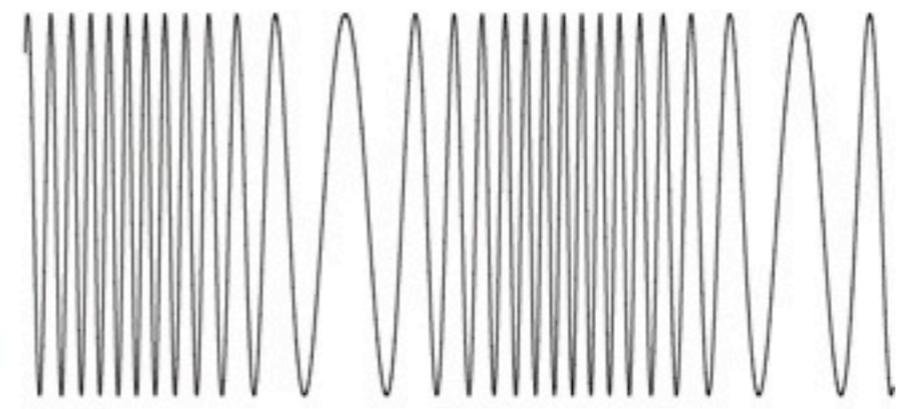
AM signal



J-WAVE  
(FM)  
81.3MHz

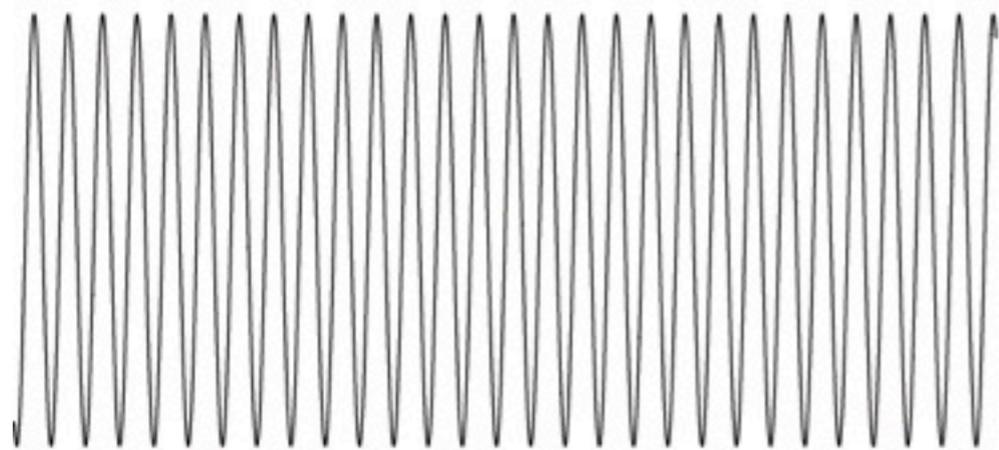


FM signal



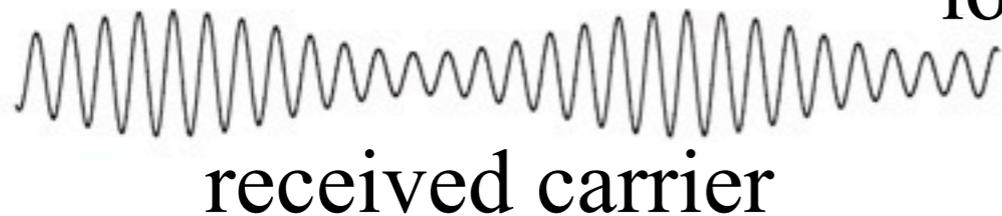
# Homodyne detection

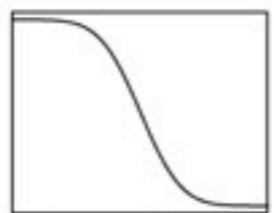
 **Local oscillator (LO)**  
same frequency as carrier

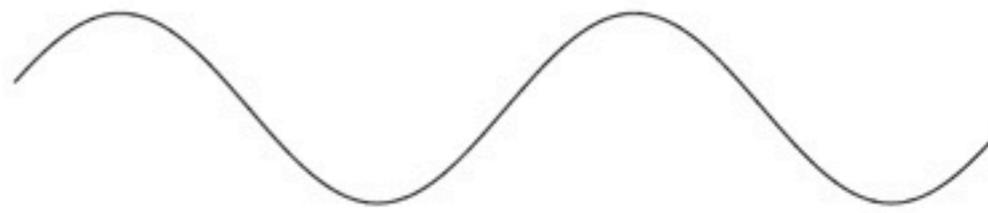


**We can select AM or FM signal by changing the LO phase.**

 **Mixer**  
multiply

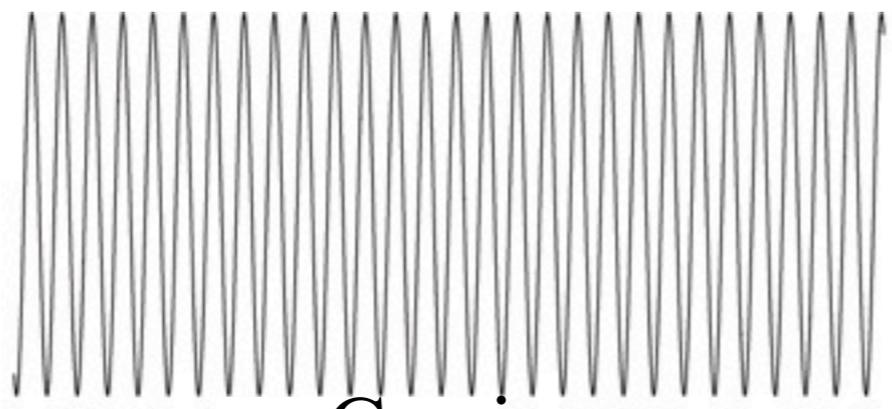


  
**low-pass filter**

  
**demodulated signal**

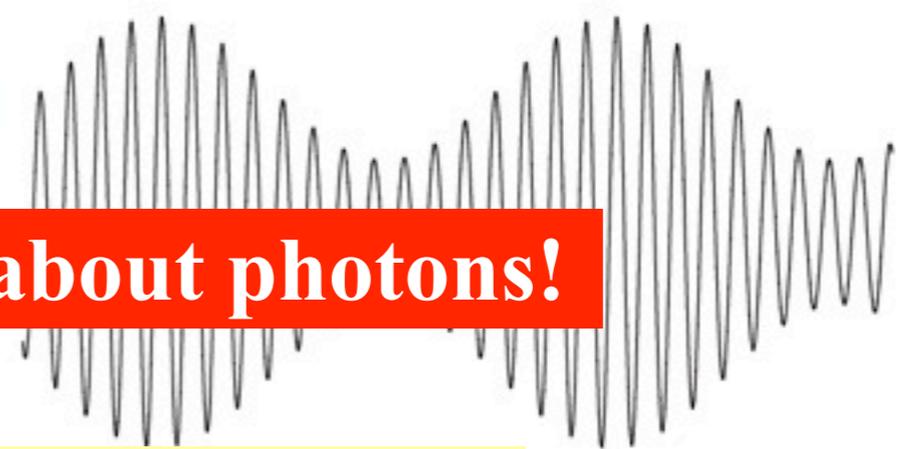
# AM and FM signals

NHK  
(AM)  
594kHz



Carrier

AM signal



We don't have to think about photons!

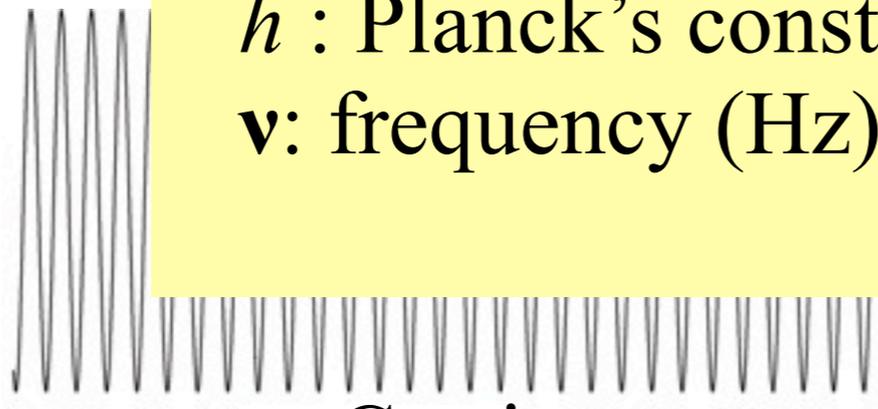


Am

$$\text{photon energy} = h\nu$$

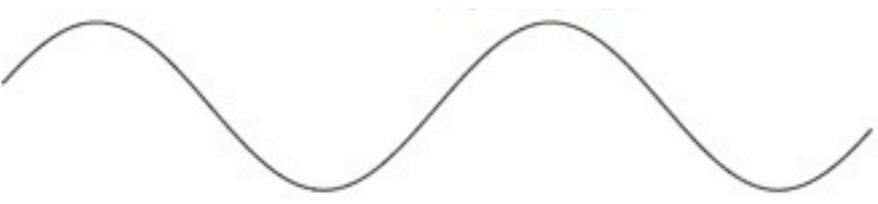
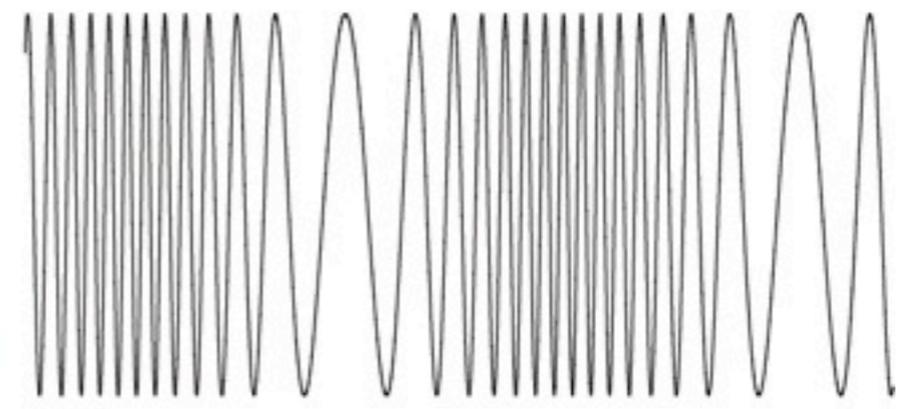
$h$  : Planck's const ( $6.6 \times 10^{-34}$  Js)  
 $\nu$ : frequency (Hz)

J-WAVE  
(FM)  
81.3MHz



Carrier

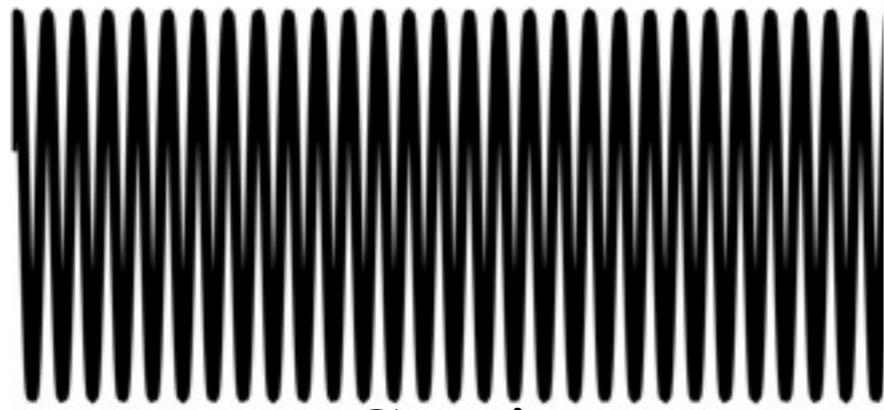
FM signal



Frequency (phase) modulation

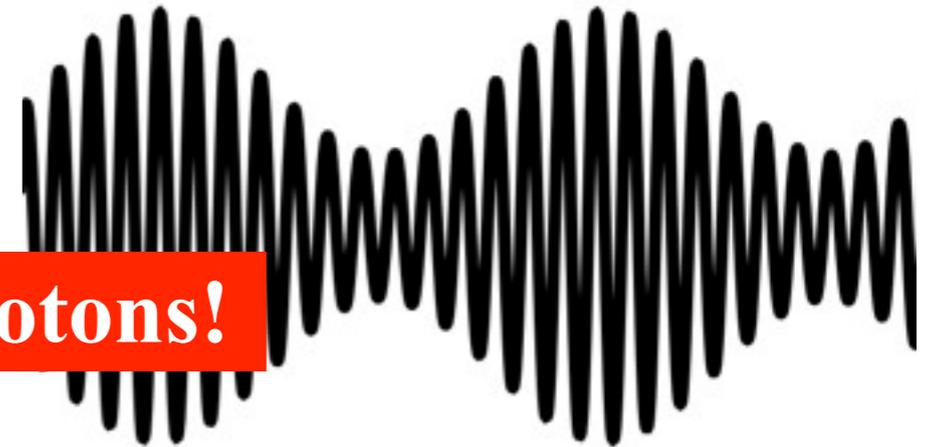
# AM and FM signals

Freq of vis light  
100THz



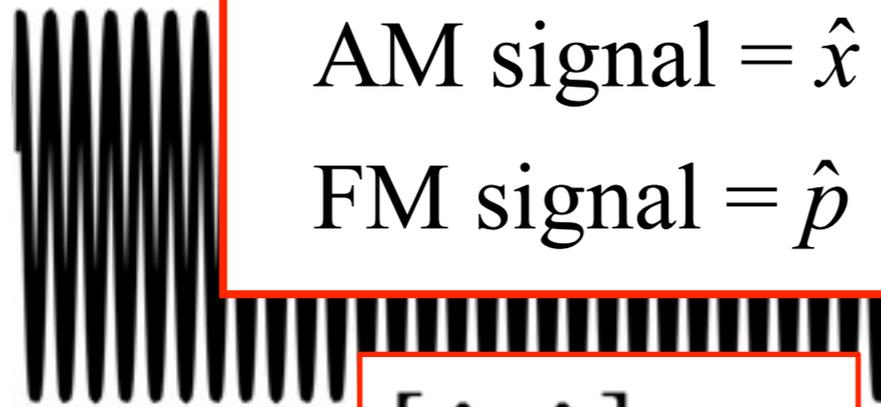
Carrier

AM signal



We have to think about photons!

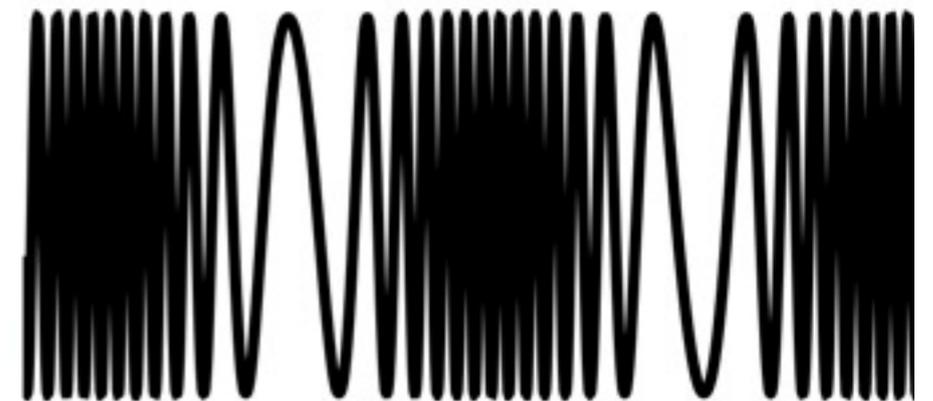
AM and FM signals become conjugate variables.



$$\begin{aligned} \text{AM signal} &= \hat{x} \\ \text{FM signal} &= \hat{p} \end{aligned}$$

$$[\hat{x}, \hat{p}] = i\hbar$$

FM signal



Frequency (phase) modulation

# Quantum optics

annihilation operator  $\hat{a}$

$$[\hat{a}, \hat{a}^\dagger] = 1 \quad \left( \hbar = \frac{1}{2} \right)$$

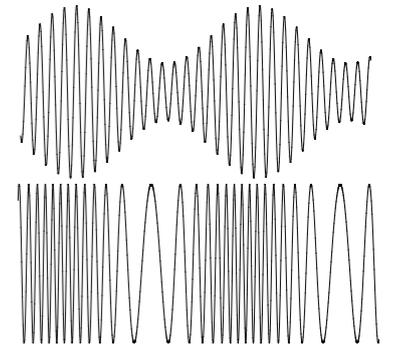
quantum complex amplitude

$$\hat{a} = \hat{x} + i\hat{p}$$

Photon-number units

$\hat{x}$  : AM signal

$\hat{p}$  : FM signal



$$[\hat{x}, \hat{p}] = i\hbar$$

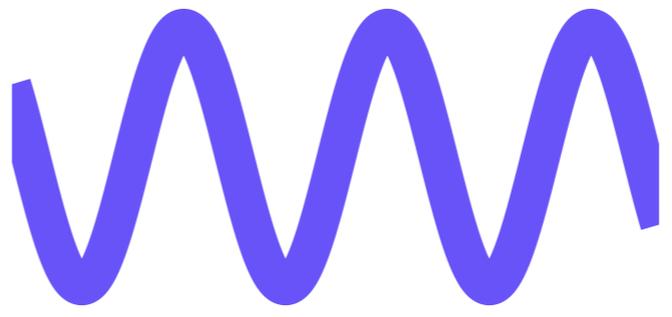
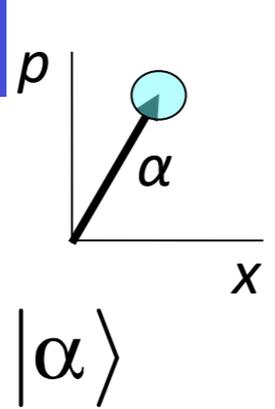
$\hat{x}$ : position

$\hat{p}$ : momentum

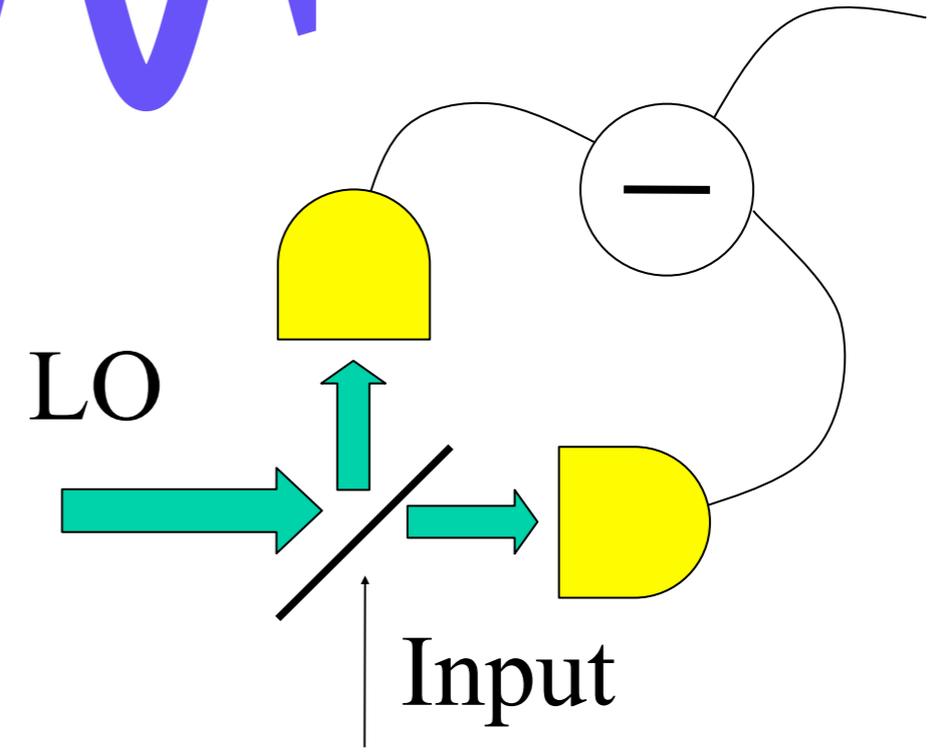
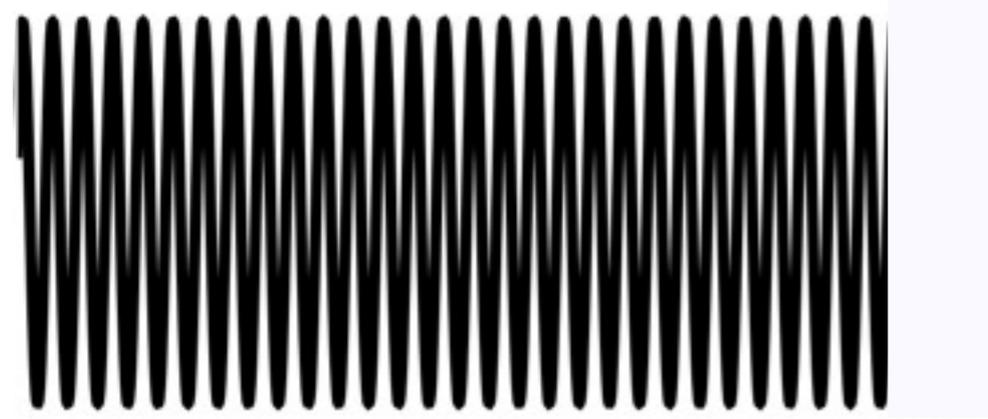
$$[\hat{x}, \hat{p}] = \frac{i}{2}$$

Canonically conjugate variables

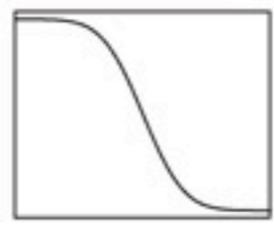
# Homodyne detection



**Laser** oscillator (LO)  
same frequency as carrier



**Beam splitter**



low-pass filter



demodulated signal

**With shot noise!**



received carrier

**Coherent communication**

**Shannon limit**

# An example of quantum version of coherent communication

**Channel capacity beyond the Shannon limit**

**Beyond the shot-noise limit!**

Sending station

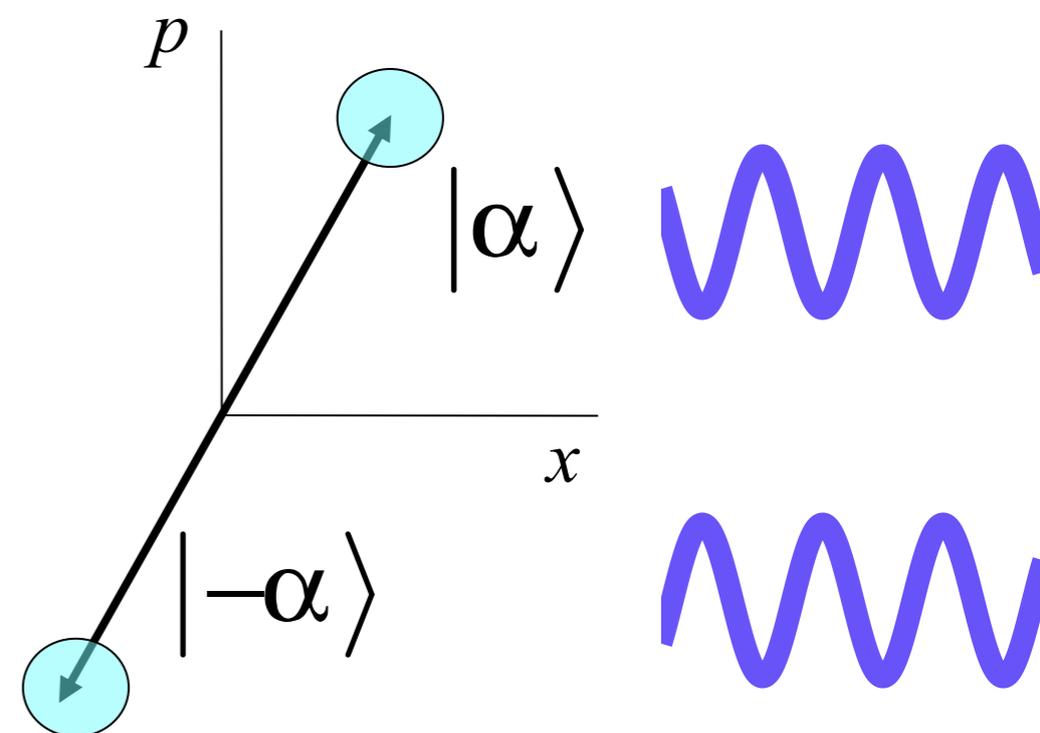
**Encode**

fiber

Receiving station

**Decode**

$$\begin{aligned} |00\rangle &= |S_0\rangle = |\alpha\rangle|\alpha\rangle|\alpha\rangle \\ |01\rangle &= |S_1\rangle = |\alpha\rangle|-\alpha\rangle|-\alpha\rangle \\ |10\rangle &= |S_2\rangle = |-\alpha\rangle|-\alpha\rangle|\alpha\rangle \\ |11\rangle &= |S_3\rangle = |-\alpha\rangle|\alpha\rangle|-\alpha\rangle \end{aligned}$$



**Collective measurement**

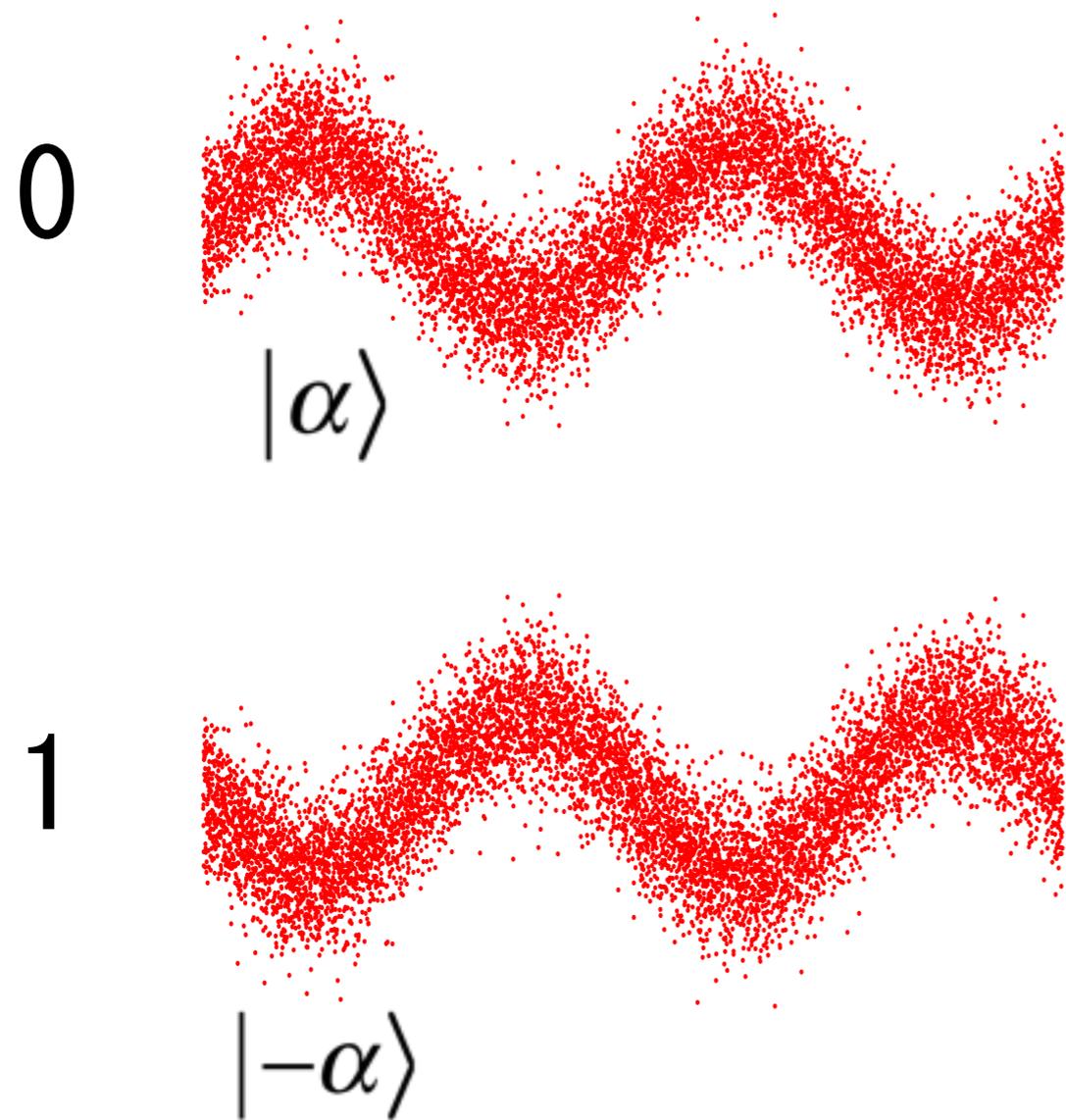
**Projection onto**

$$|\mu_i\rangle = \left( \sum_{k=0}^3 |S_k\rangle\langle S_k| \right)^{-1/2} |S_i\rangle \quad (i = 0, 1, 2, 3)$$

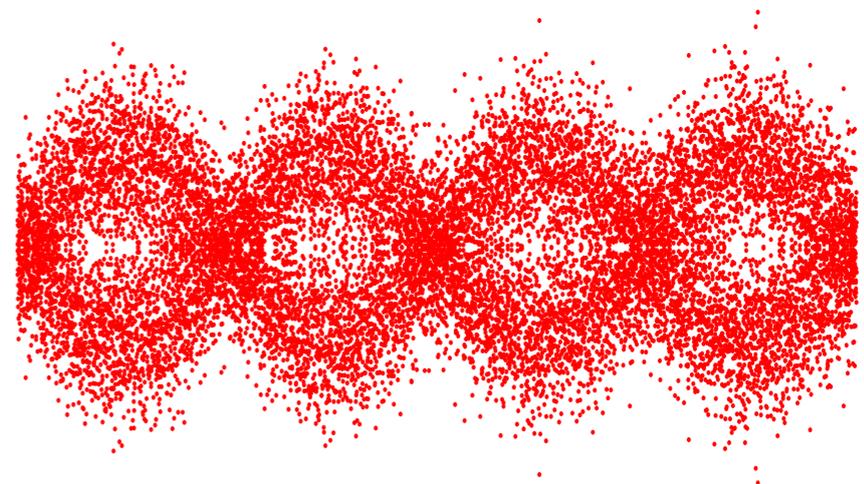
**Orthogonal bases**

M. Sasaki et al., Phys. Lett. A **236**, 1 (1997)

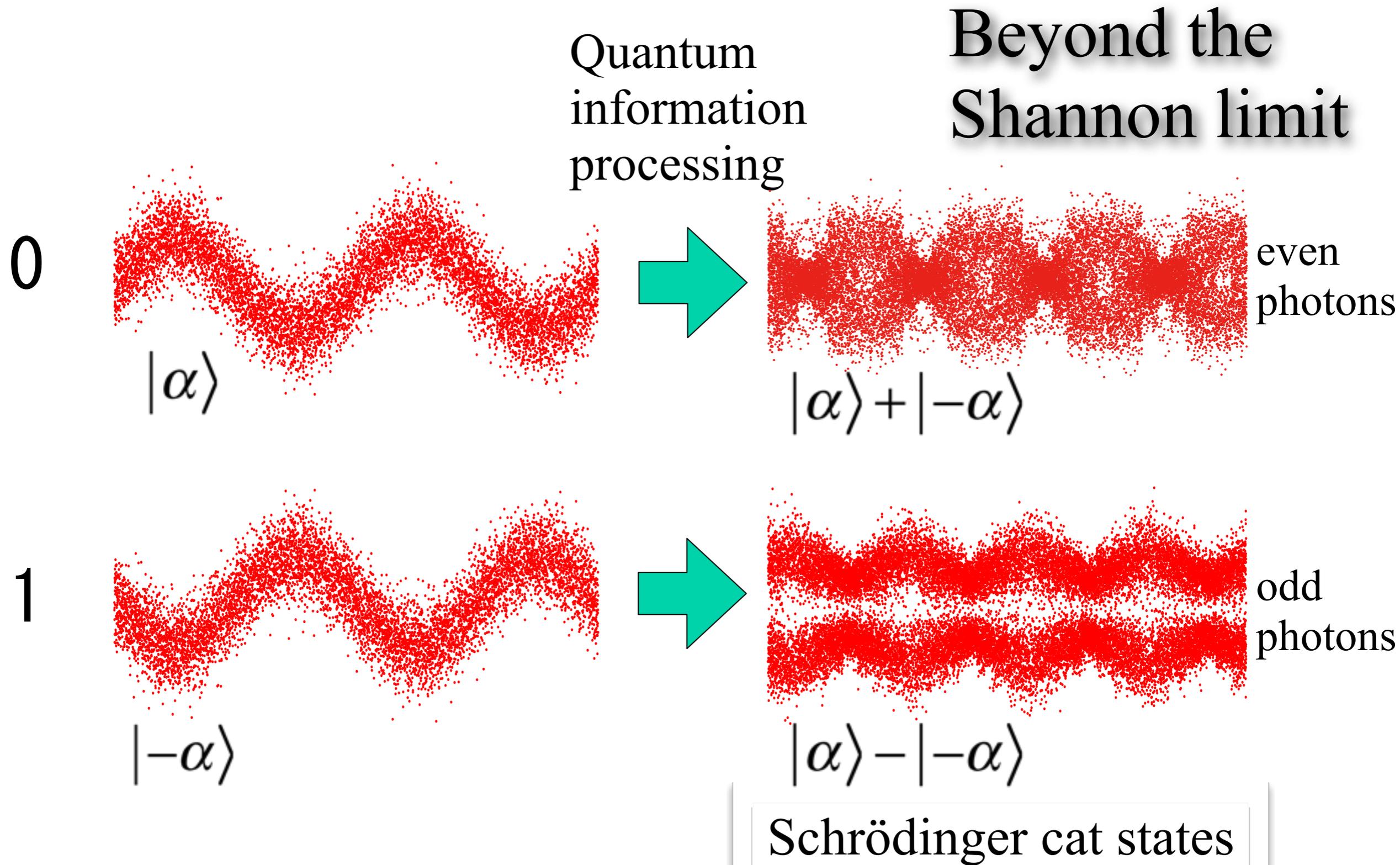
# Coherent communication



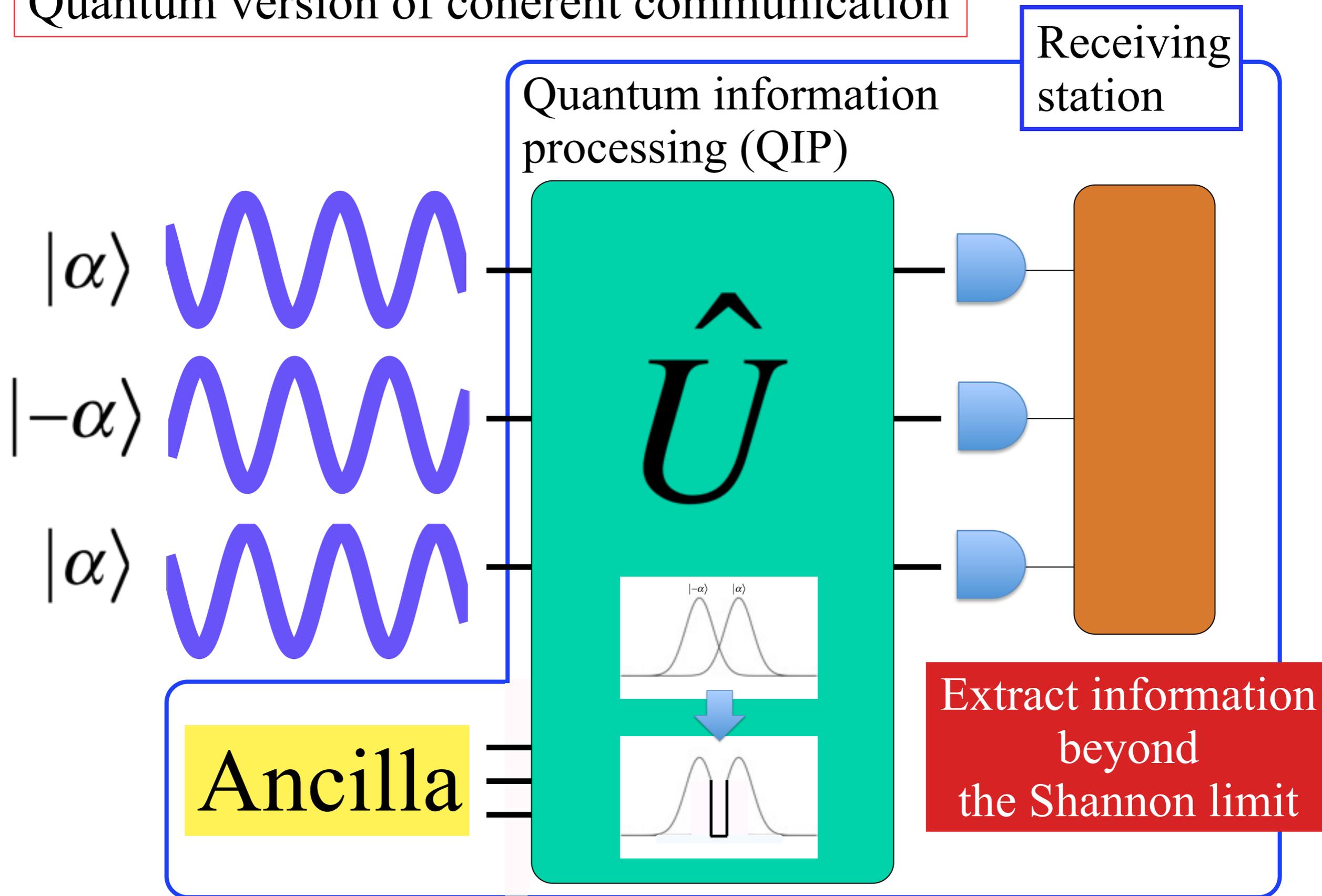
Shannon limit



# Coherent communication with QIP



# Quantum version of coherent communication



We have to handle cat states of light!!

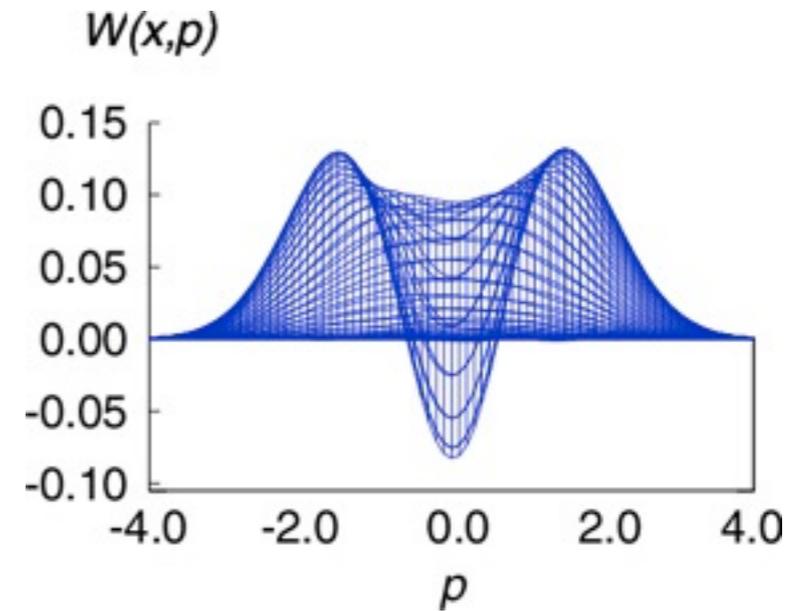
# Schrödinger cat states

$$N_{\alpha} (|\alpha\rangle - |-\alpha\rangle)$$

odd photons

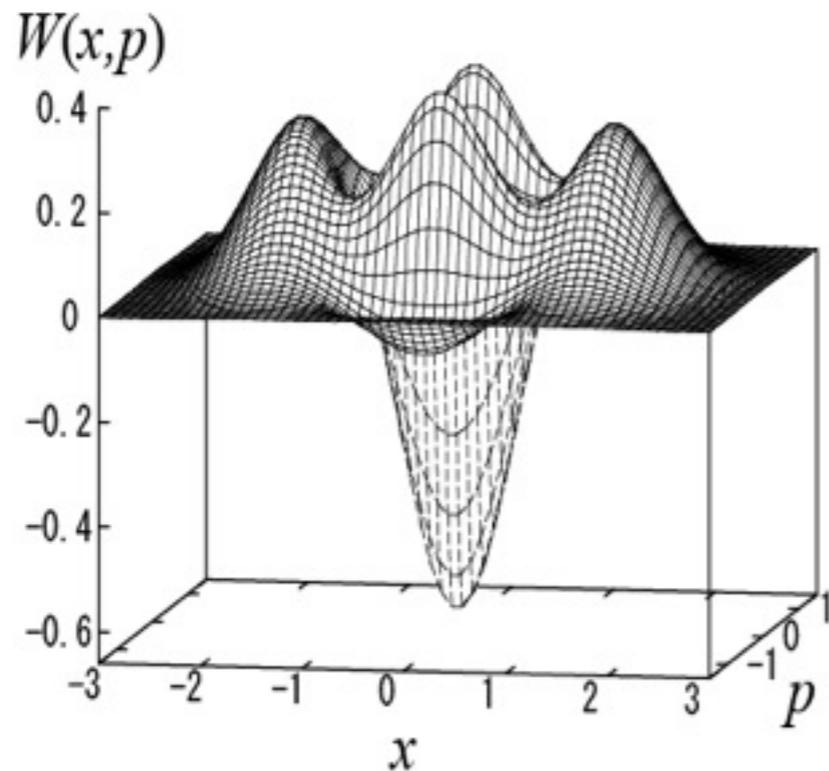
$$N_{\alpha} (|\alpha\rangle + |-\alpha\rangle)$$

even photons



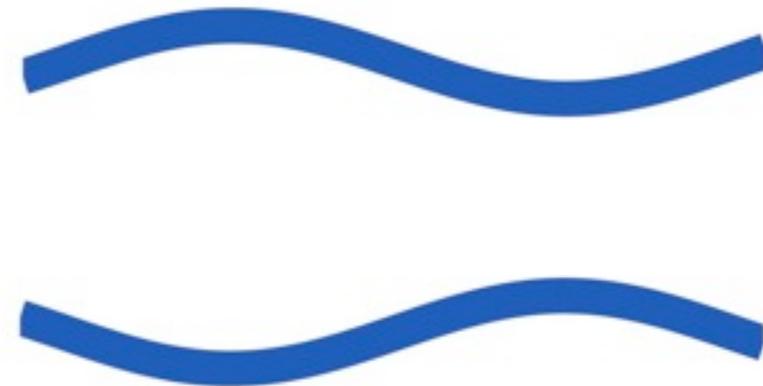
K. Wakui et al.,  
Opt. Exp. 15, 3568 (2007)

H. Takahashi et al.,  
Phys. Rev. Lett. 101, 233605 (2008)

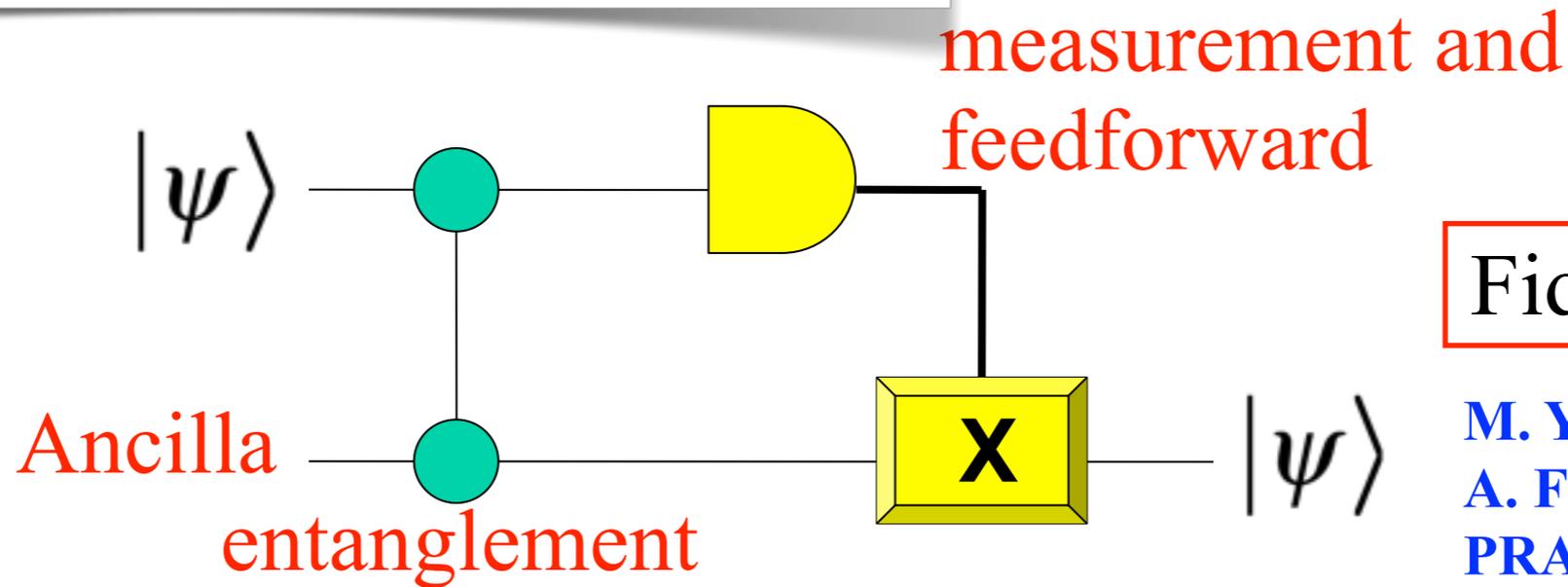


$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$|-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}} |n\rangle$$



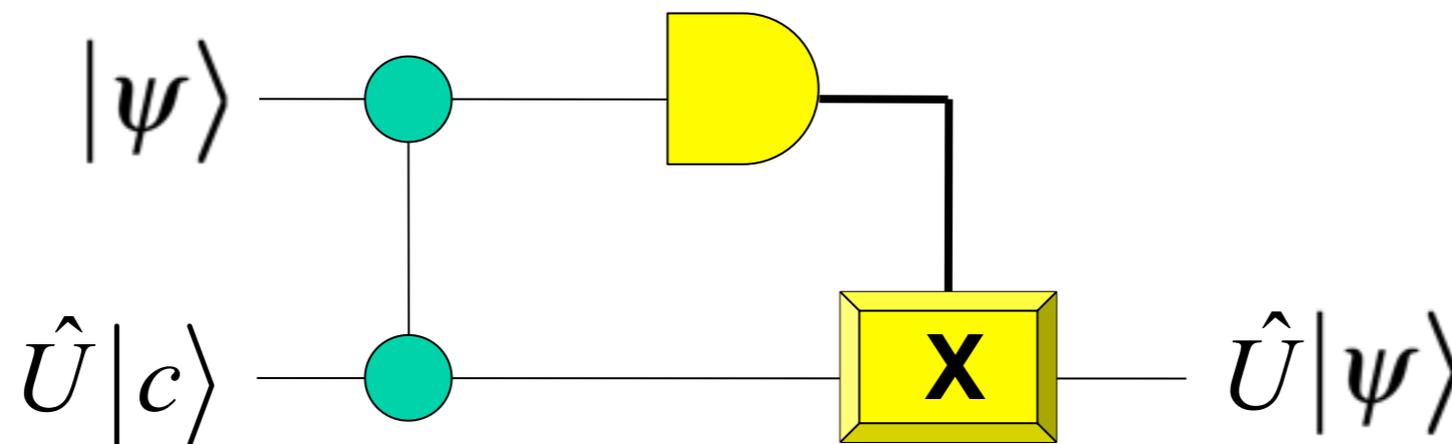
# Generalized teleportation



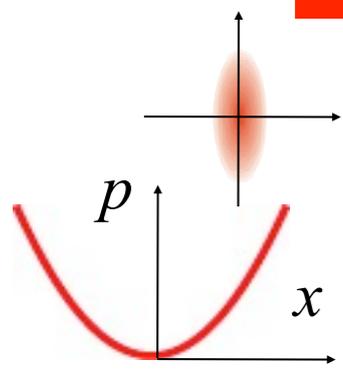
Fidelity = 0.83

M. Yukawa, H. Benichi,  
A. Furusawa  
PRA 77, 022314 (2008)

## Gate teleportation



state preparation



$\hat{S}(r)|0\rangle$  squeezed vacuum

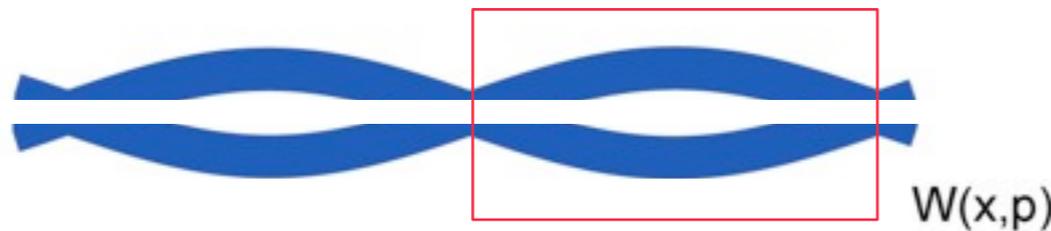
$e^{ik\hat{x}^3}|p=0\rangle$  cubic phase state

Universal squeezer

CV  $\pi/8$  gate

# First step of teleportation based QIP for coherent states

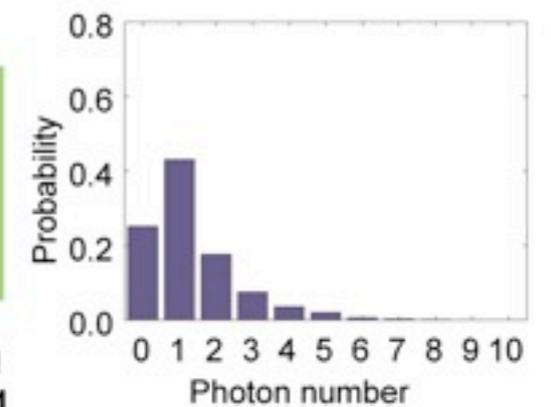
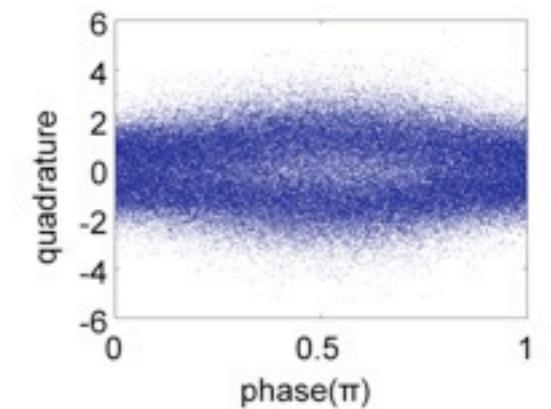
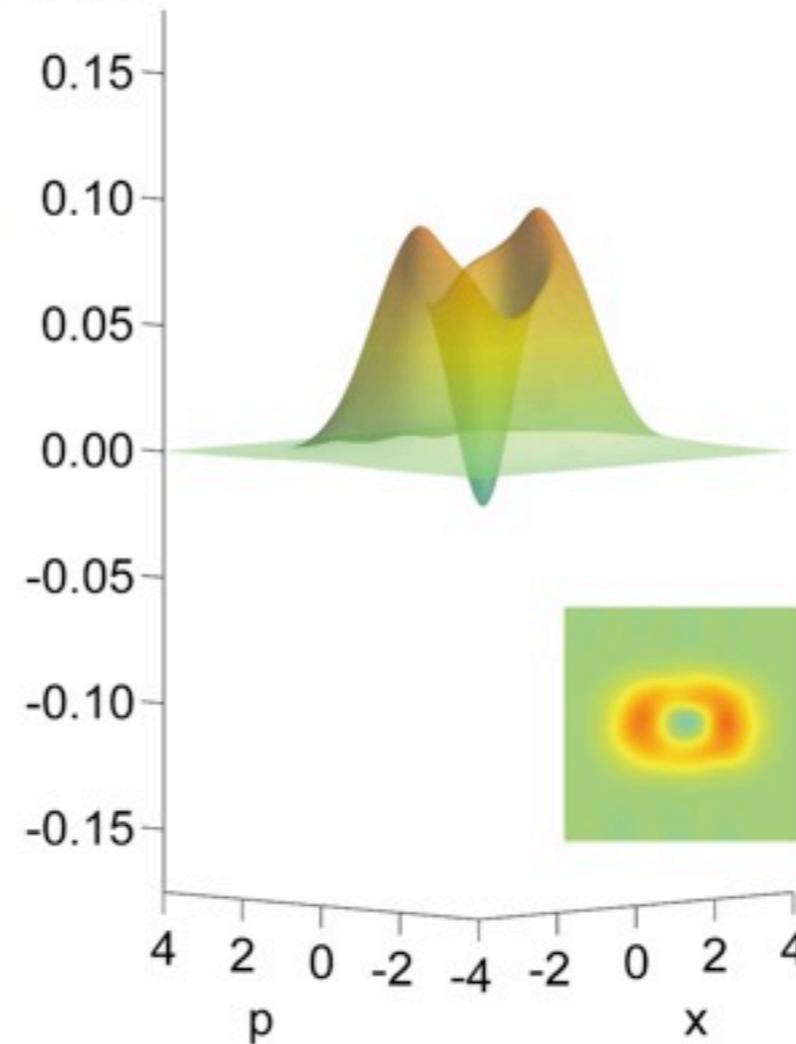
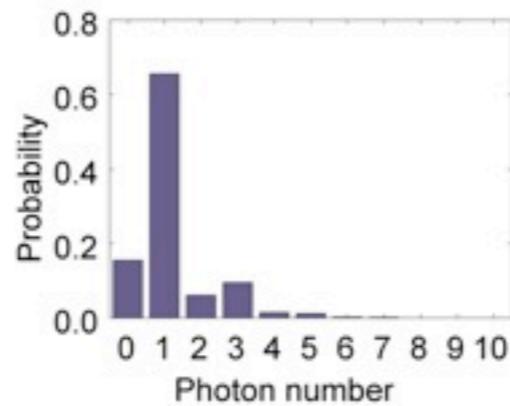
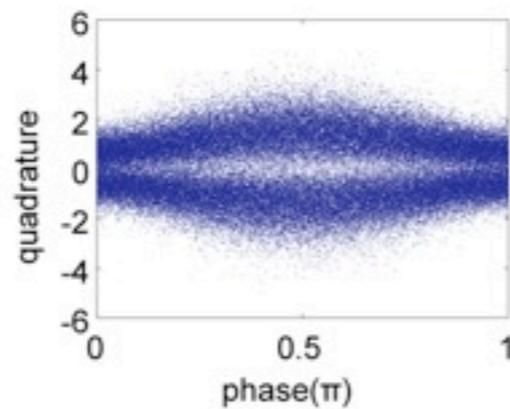
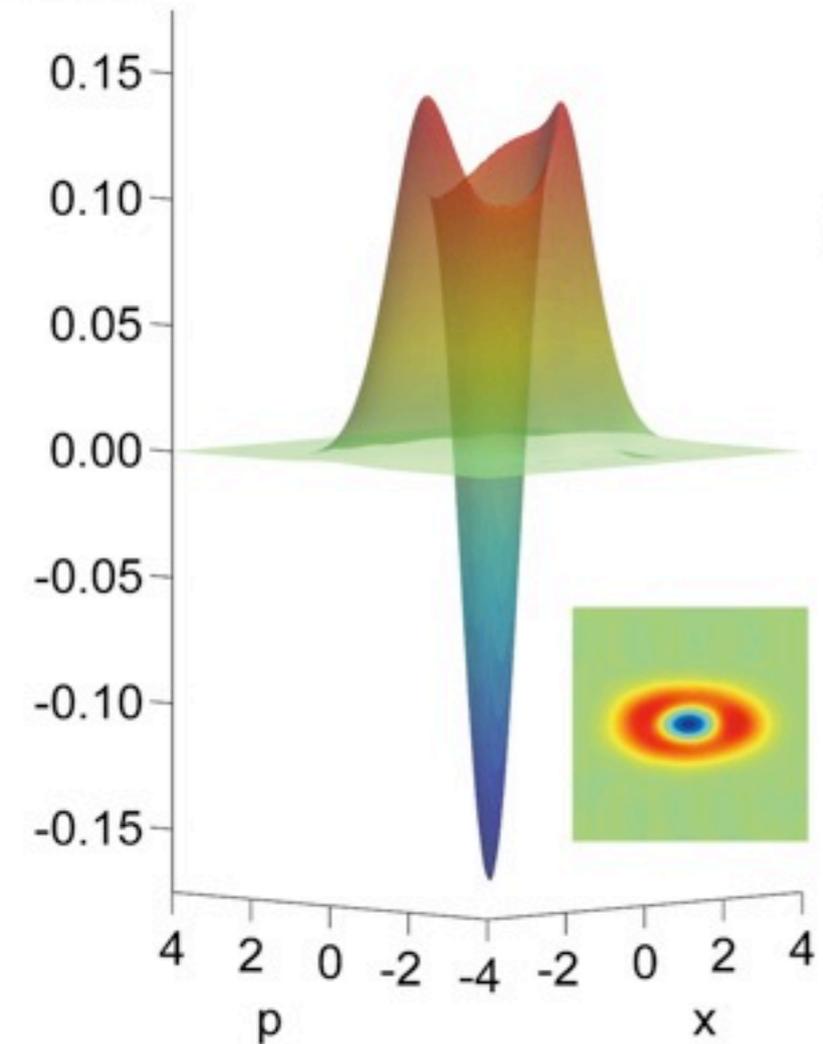
## Teleportation of a Schrödinger cat state of light



$$N_{\alpha} (|\alpha\rangle - |-\alpha\rangle)$$

$W(x,p)$

$W(x,p)$



Input

Output



SCIENCE

# Quantum Leap: Scientists Teleport Bits of Light

By [Clara Moskowitz](#)  
Published April 14, 2011



16.05.2011 20:50

**Ученые из Японии телепортировали запутанный квант**

Автор: Сергей Мингажев



# Scientists teleport Schrodinger's cat

By Carl Holm for ABC Science Online

Updated Fri Apr 15, 2011 12:13pm AEST

**N. Lee, H. Benichi, Y. Takeno, S. Takeda, J. Webb, E. Huntington, & A. Furusawa, Science 332, 330 (2011)**

Physical process of

# Quantum information processing

encoding in  
physical systems

**quantum**



state transformation  
of physical systems

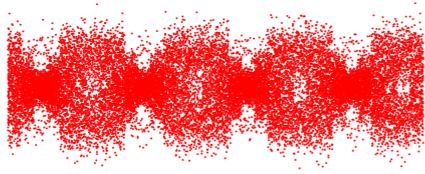
**quantum**



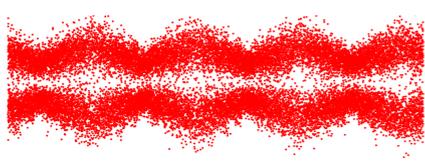
decoding from  
physical systems

**quantum**

$|0_L\rangle$



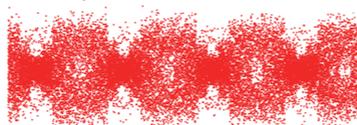
$|1_L\rangle$



Inputs



$|1_L\rangle$



$|0_L\rangle$

Outputs

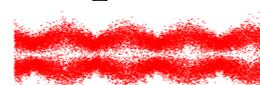


$|1_L\rangle$

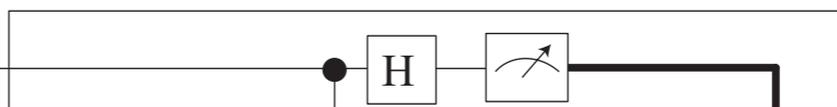


$|1_L\rangle$

Input



**teleporter**

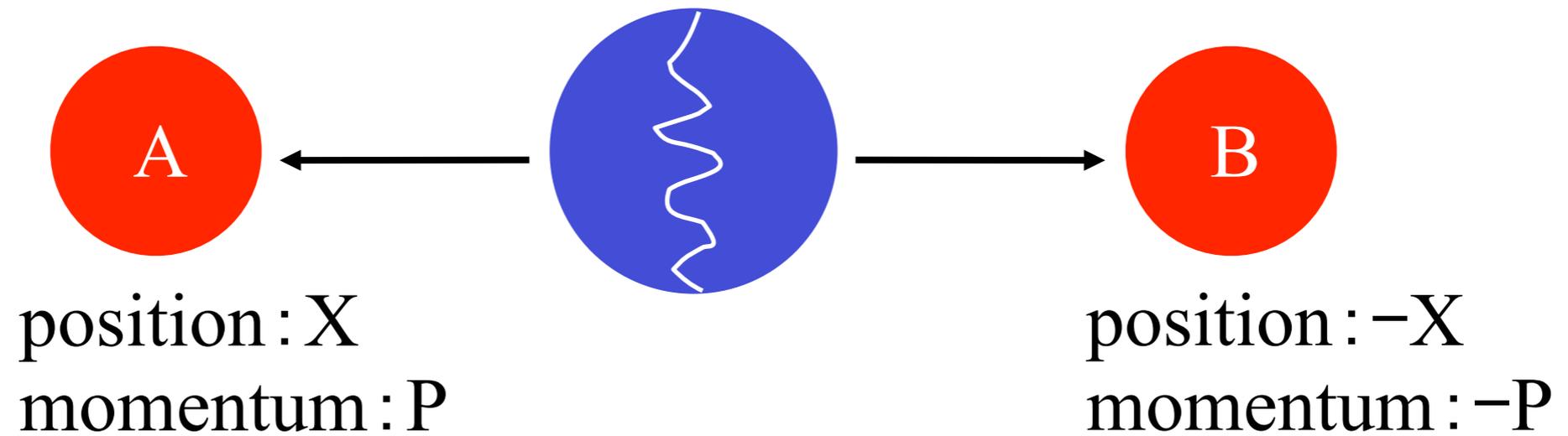


**Gate teleportation** output



**What is quantum teleportation?**

# Quantum entanglement



Einstein-Podolsky-Rosen (EPR) paradox

# Quantum teleportation

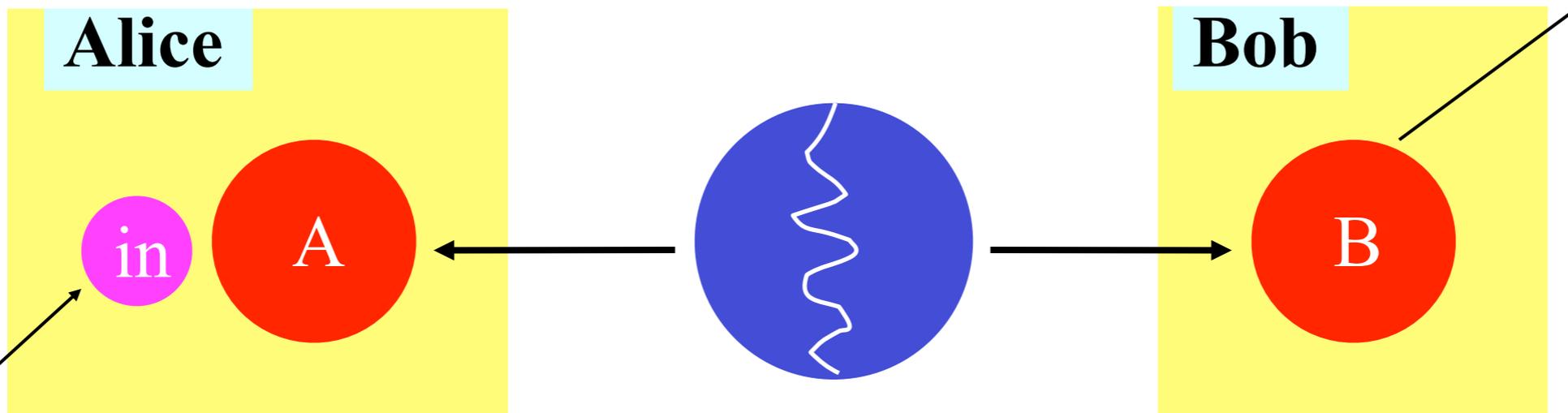
simu  
of A

$$x_{\text{in}} - x_A = X$$

$$p_{\text{in}} + p_A = P$$

$$x_B = x_{\text{in}} - X - (x_A - x_B)$$

$$p_B = p_{\text{in}} - P + (p_A + p_B)$$



Alice

Bob

Victor

$$\hat{x}_A - \hat{x}_B \rightarrow 0$$

$$\hat{p}_A + \hat{p}_B \rightarrow 0$$

$$[\hat{x}_{\text{in}}, \hat{p}_{\text{in}}] = i\hbar$$

$$[\hat{x}_A, \hat{p}_A] = i\hbar$$

$$[\hat{x}_B, \hat{p}_B] = i\hbar$$

Classical information

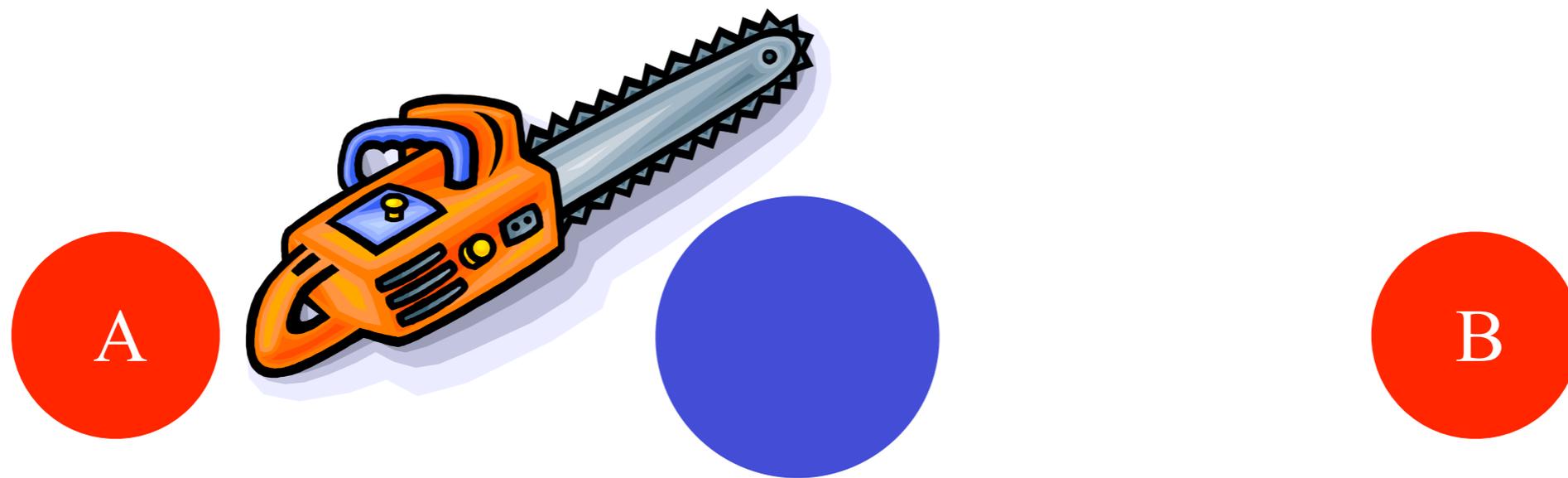
AM signal =  $\hat{x}$

FM signal =  $\hat{p}$

$$[\hat{x}_A - \hat{x}_B, \hat{p}_A + \hat{p}_B] = 0$$

# Creation of optical entanglement

# creation of entanglement



$$\hat{x}_A - \hat{x}_B \rightarrow 0$$

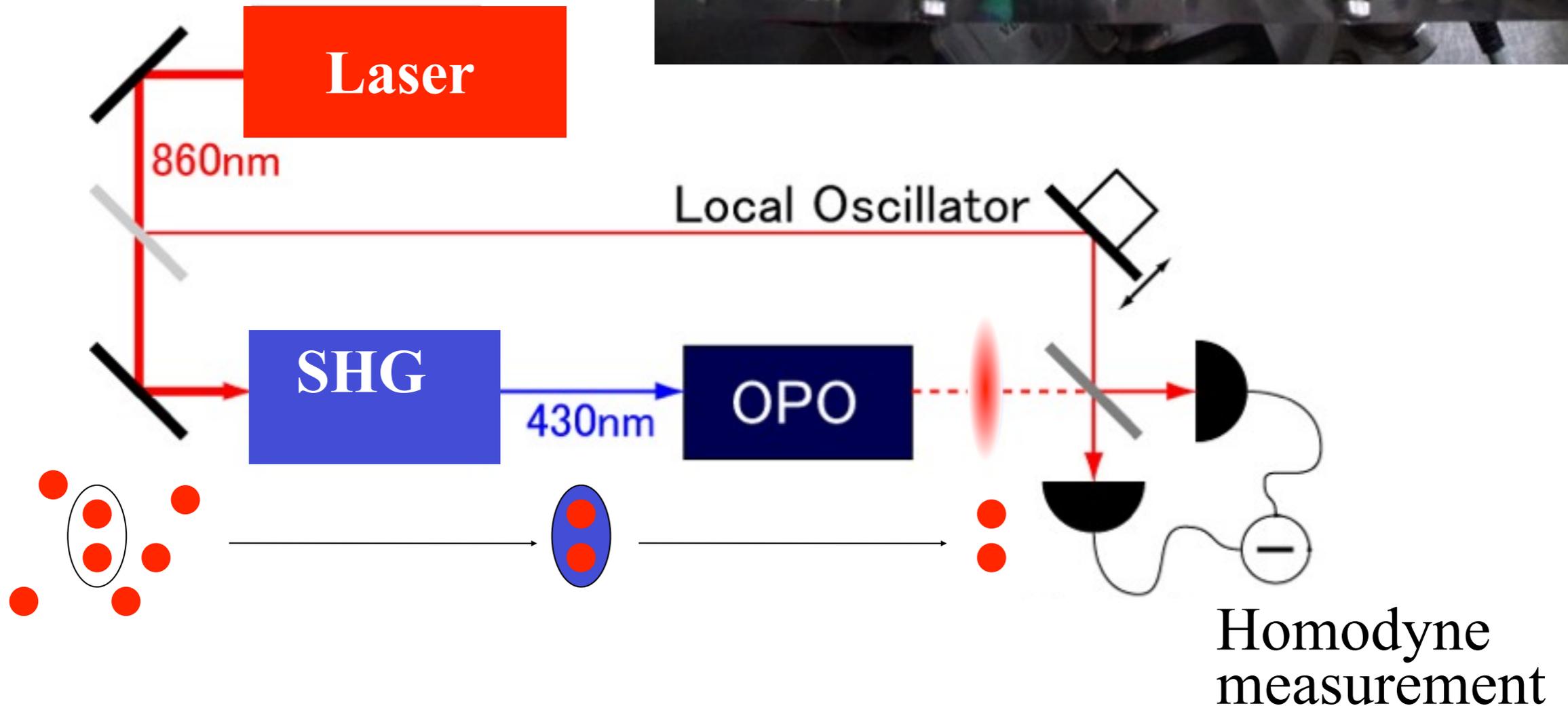
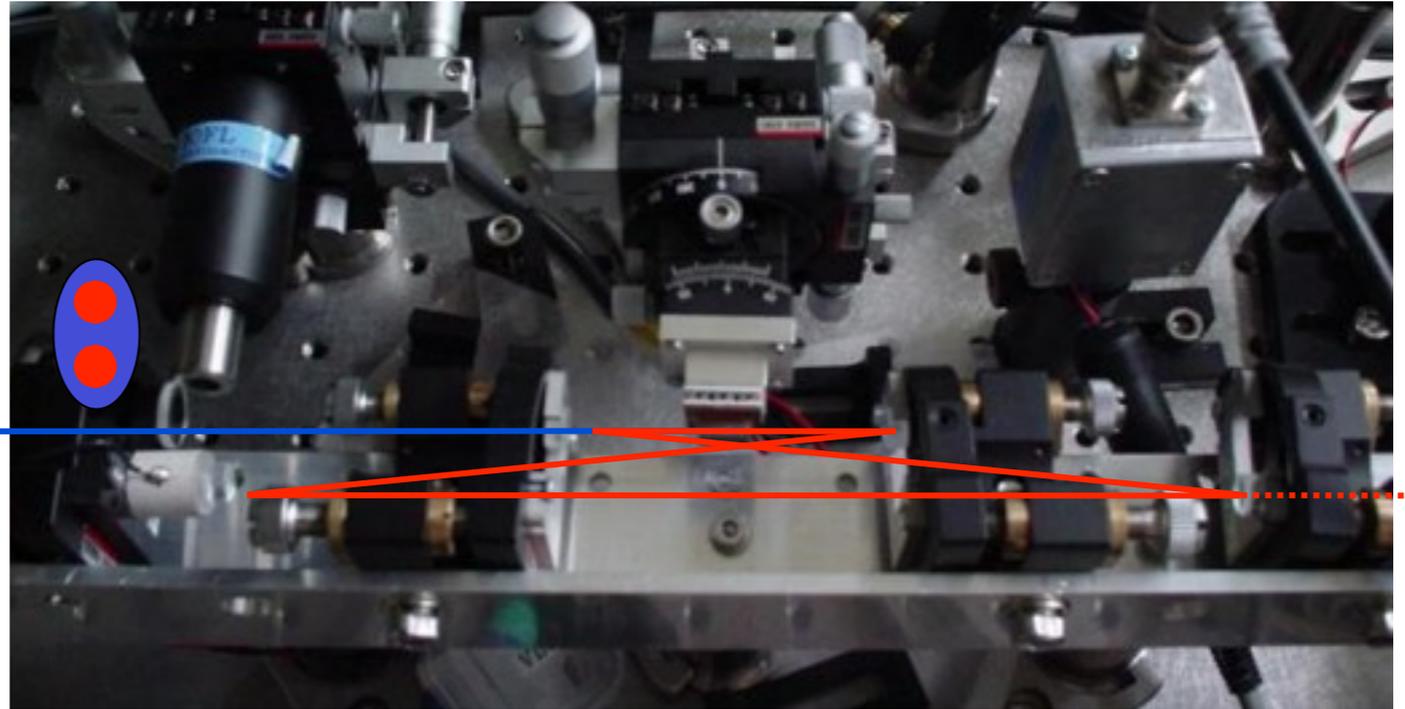
$$\hat{p}_A + \hat{p}_B \rightarrow 0$$

AM signal =  $\hat{x}$

FM signal =  $\hat{p}$

# Optical parametric oscillator (OPO)

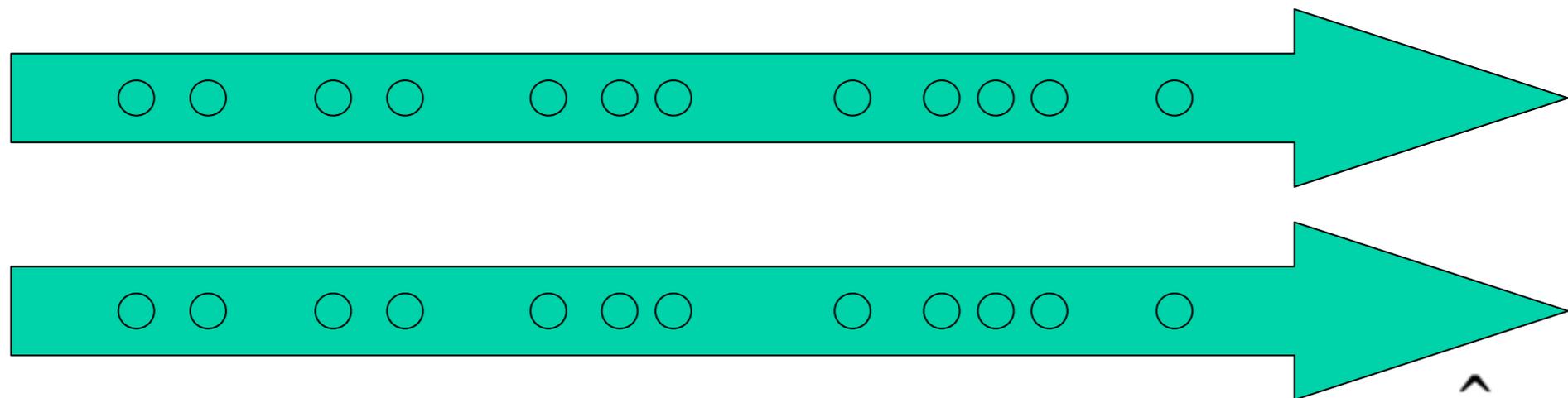
generation of squeezed light



Particle

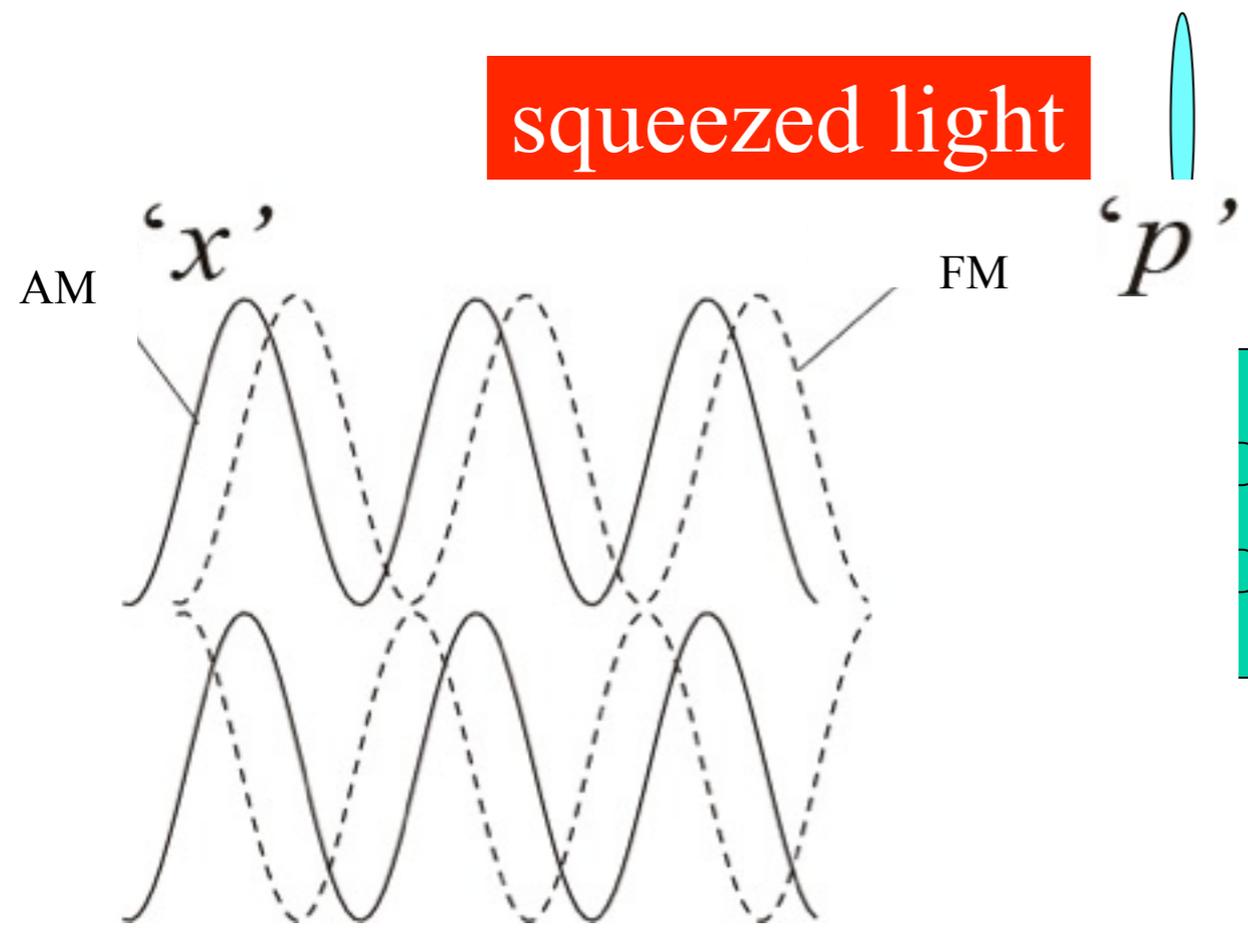
AM signal =  $\hat{x}$   
FM signal =  $\hat{p}$

entangled light beams

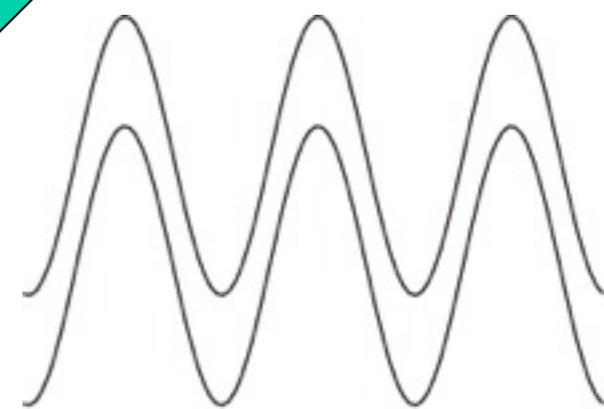


$$\hat{x}_A - \hat{x}_B \rightarrow 0$$
$$\hat{p}_A + \hat{p}_B \rightarrow 0$$

squeezed light



Wave



# Wave

$$\hat{x}_A - \hat{x}_B \rightarrow 0$$

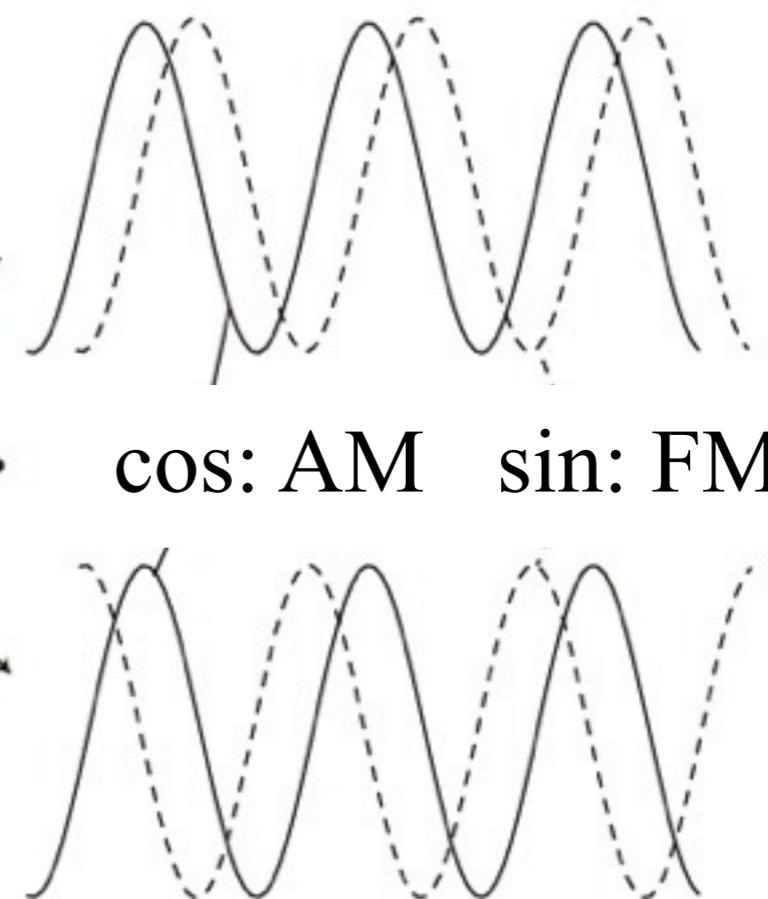
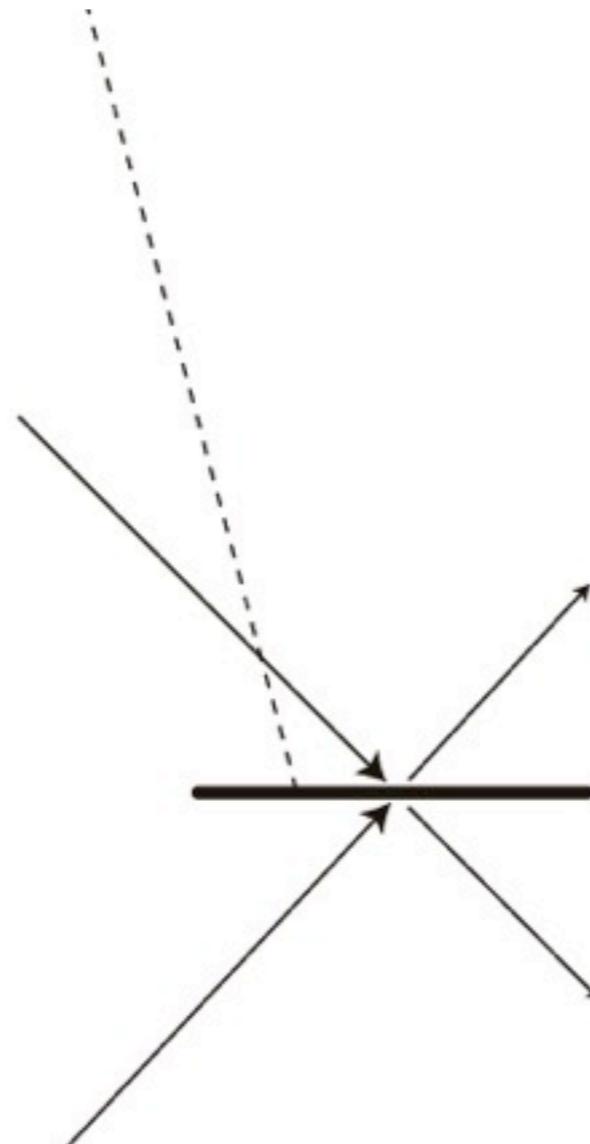
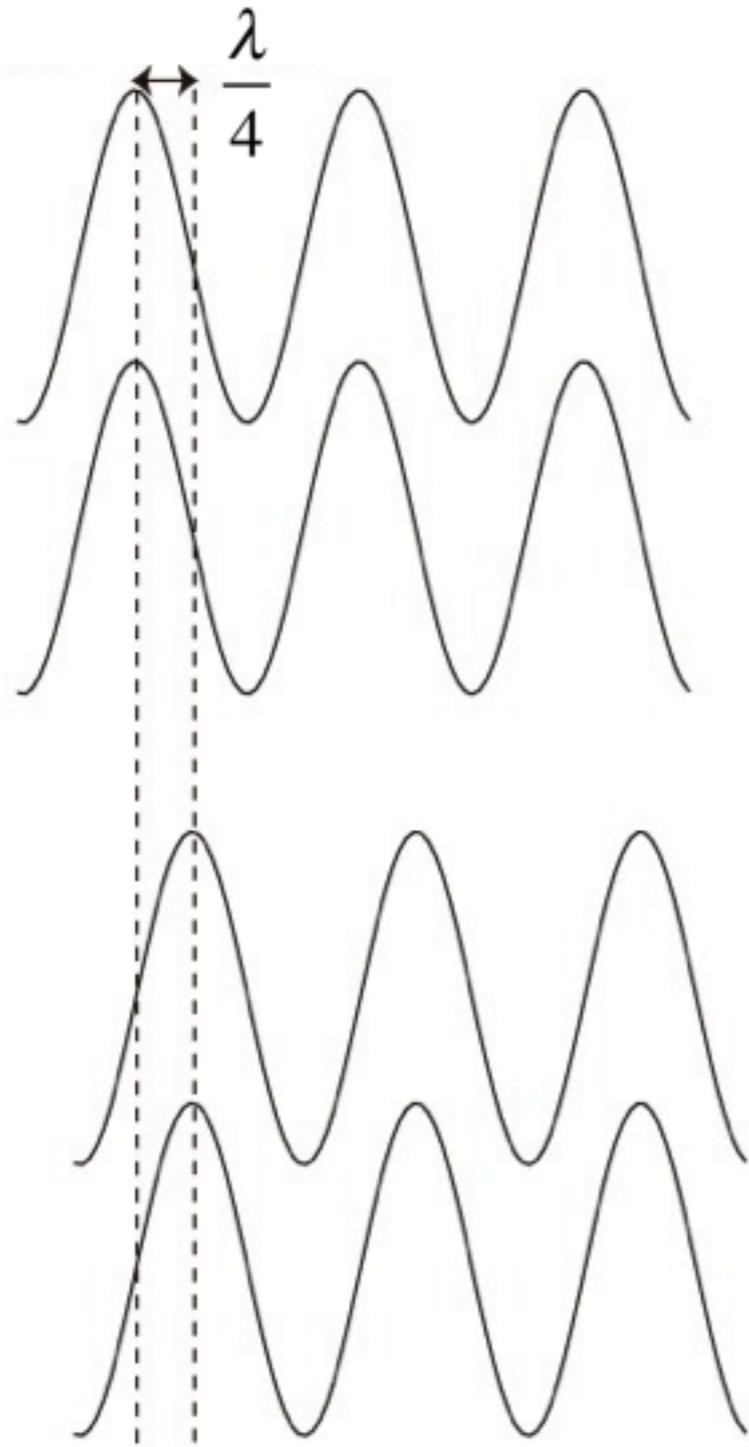
$$\hat{p}_A + \hat{p}_B \rightarrow 0$$

squeezed light

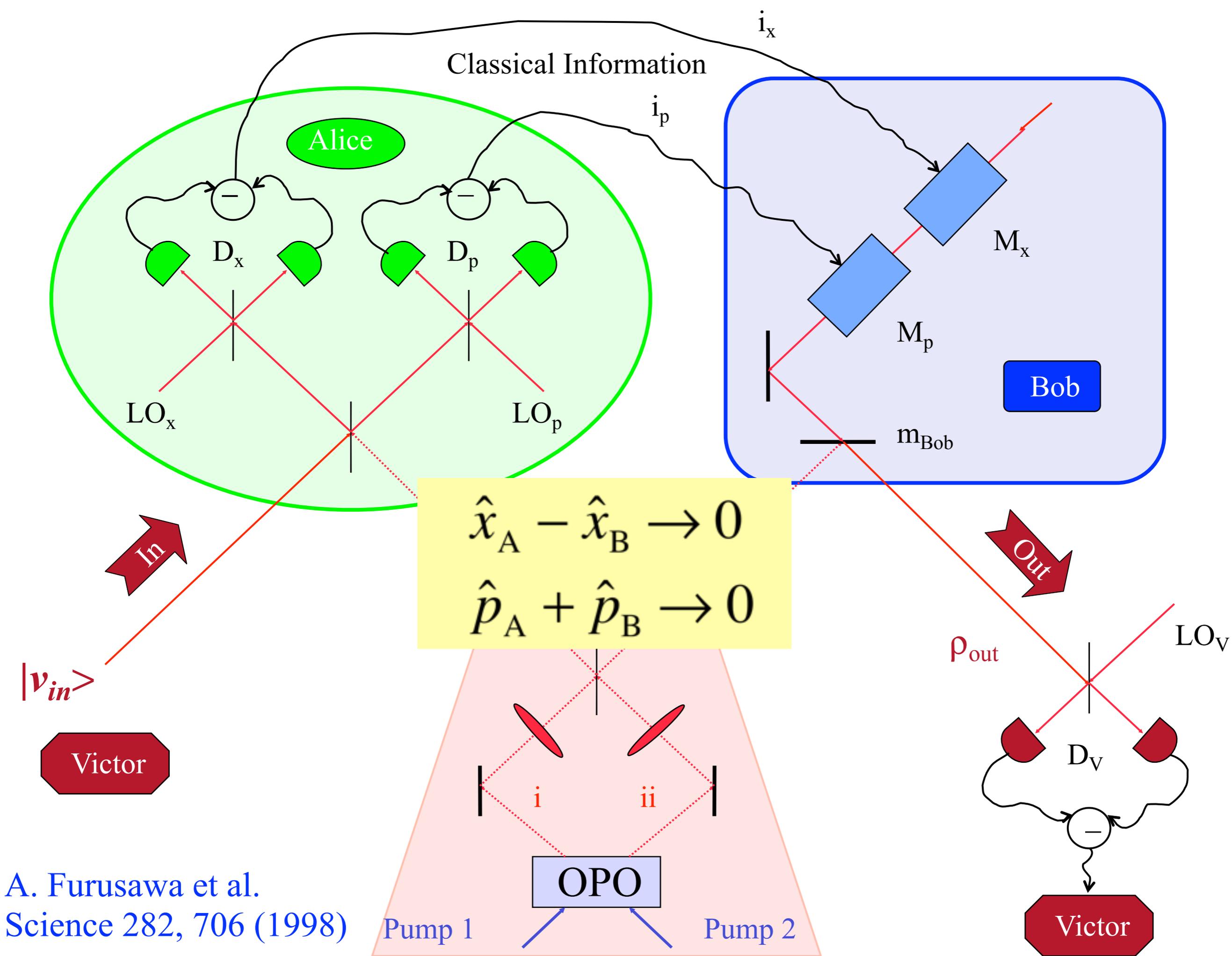
50/50 beam splitter

EPR beams

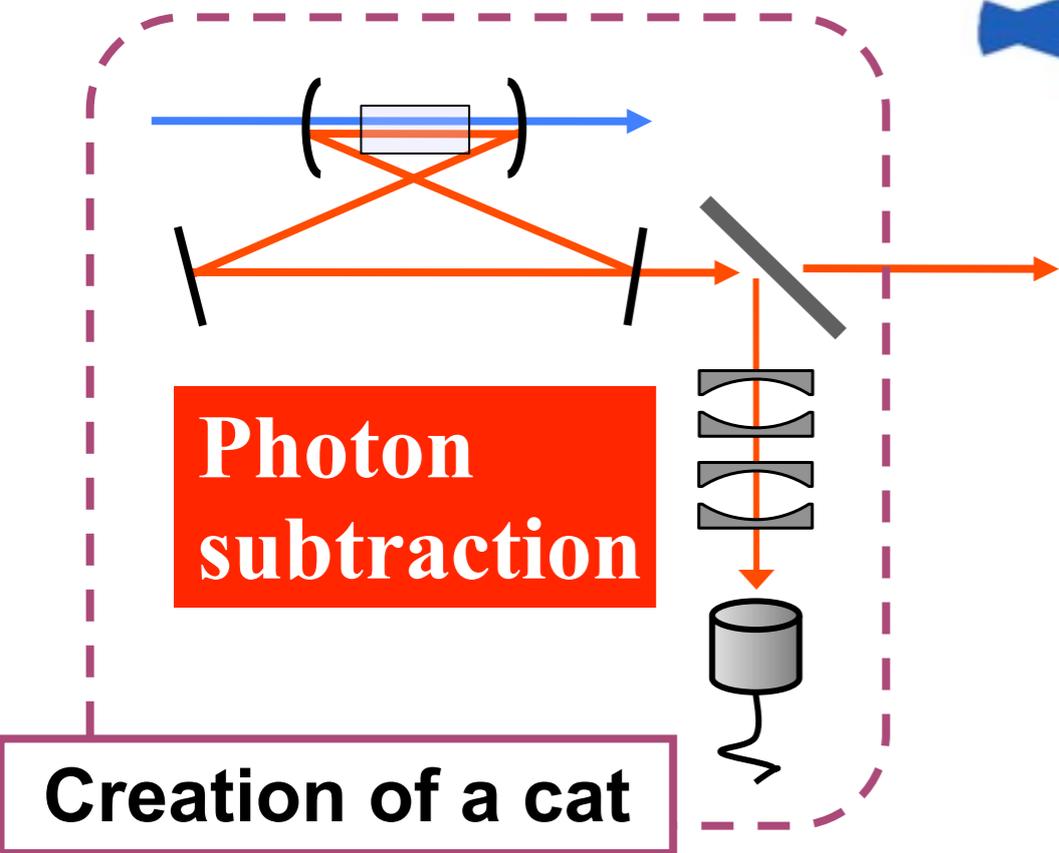
cos: AM sin: FM



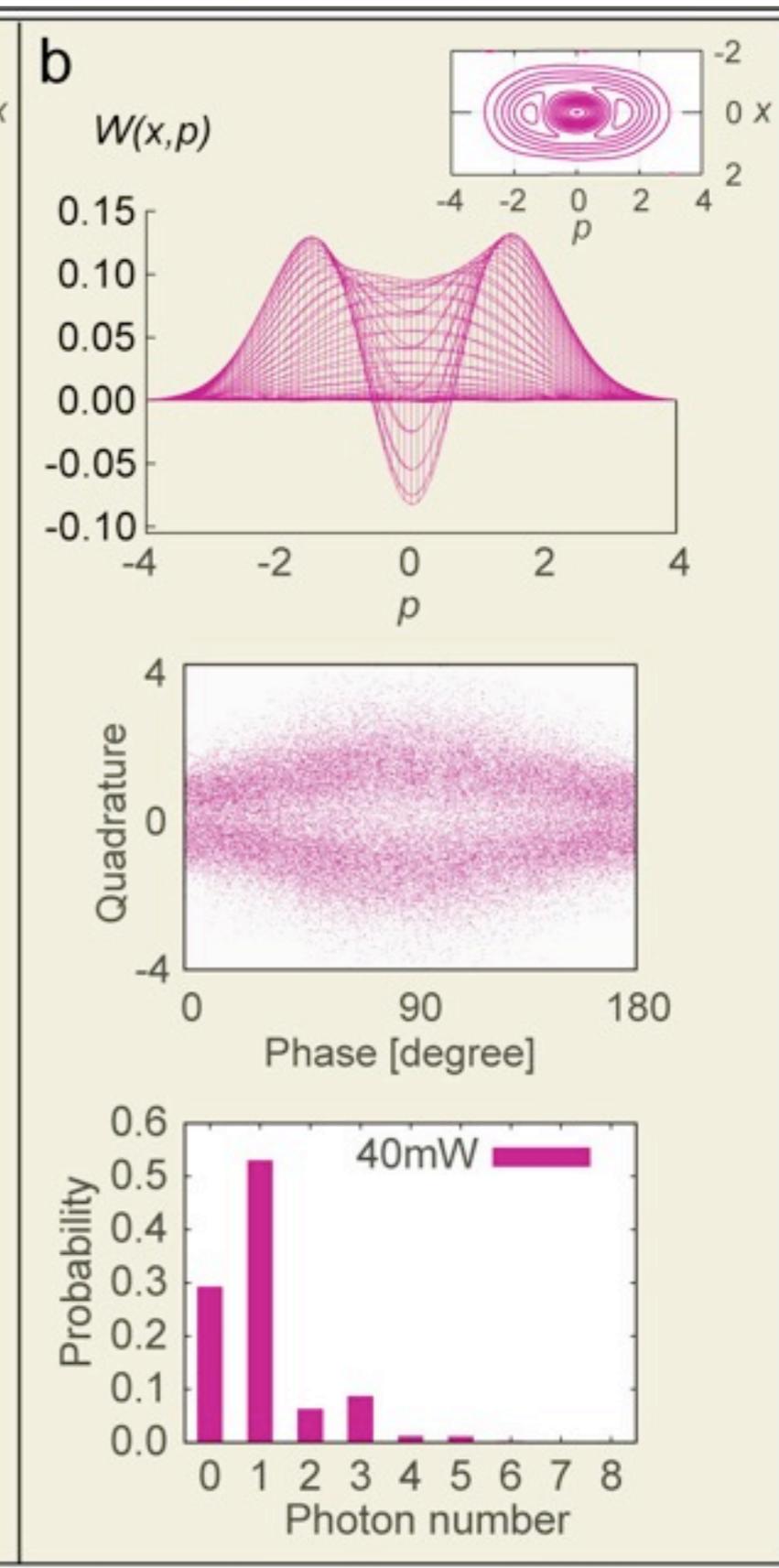
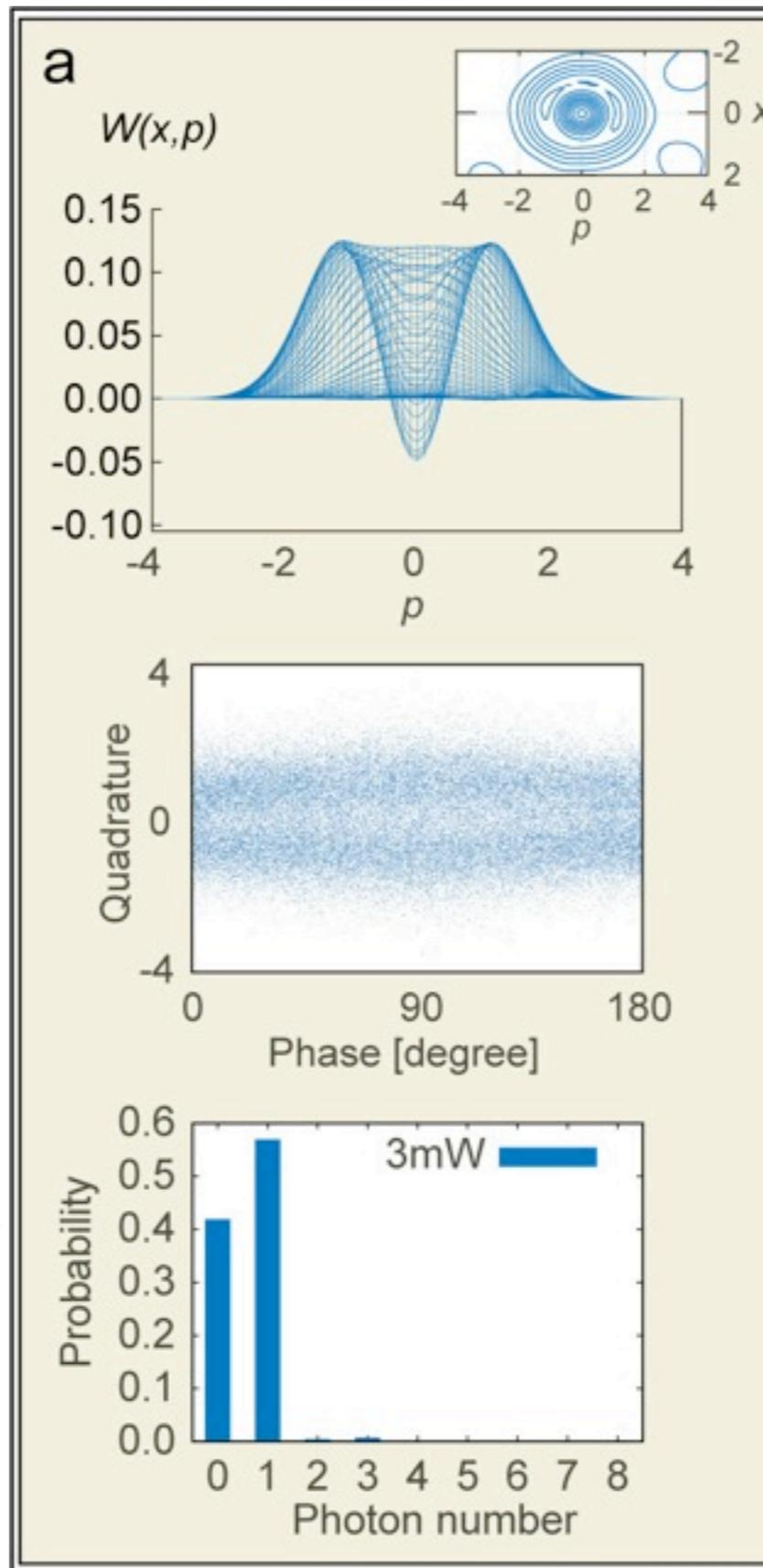
entanglement generation with squeezed light beams



# Teleportation of a Schrödinger cat state of light



**Defined in time-domain**

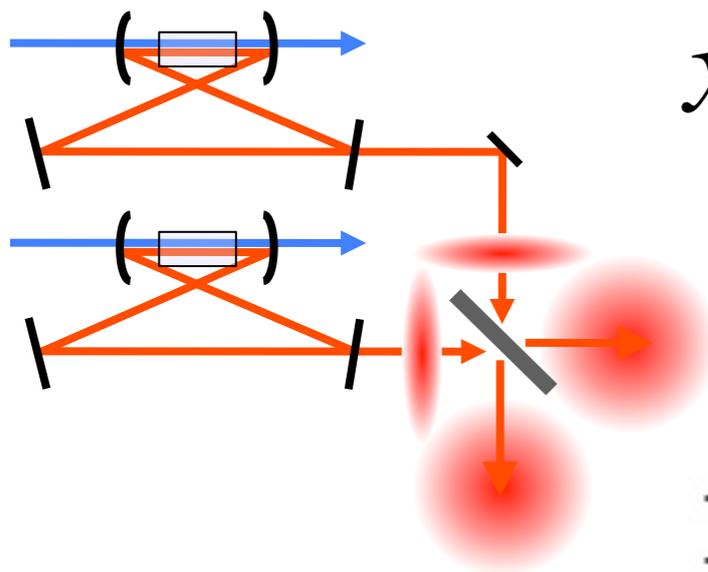


$$\hat{S}(r)|0\rangle = \frac{1}{\sqrt{\cosh r}} \sum_{n=0}^{\infty} \frac{\sqrt{(2n)!}}{2^n n!} \tanh^n r |2n\rangle$$

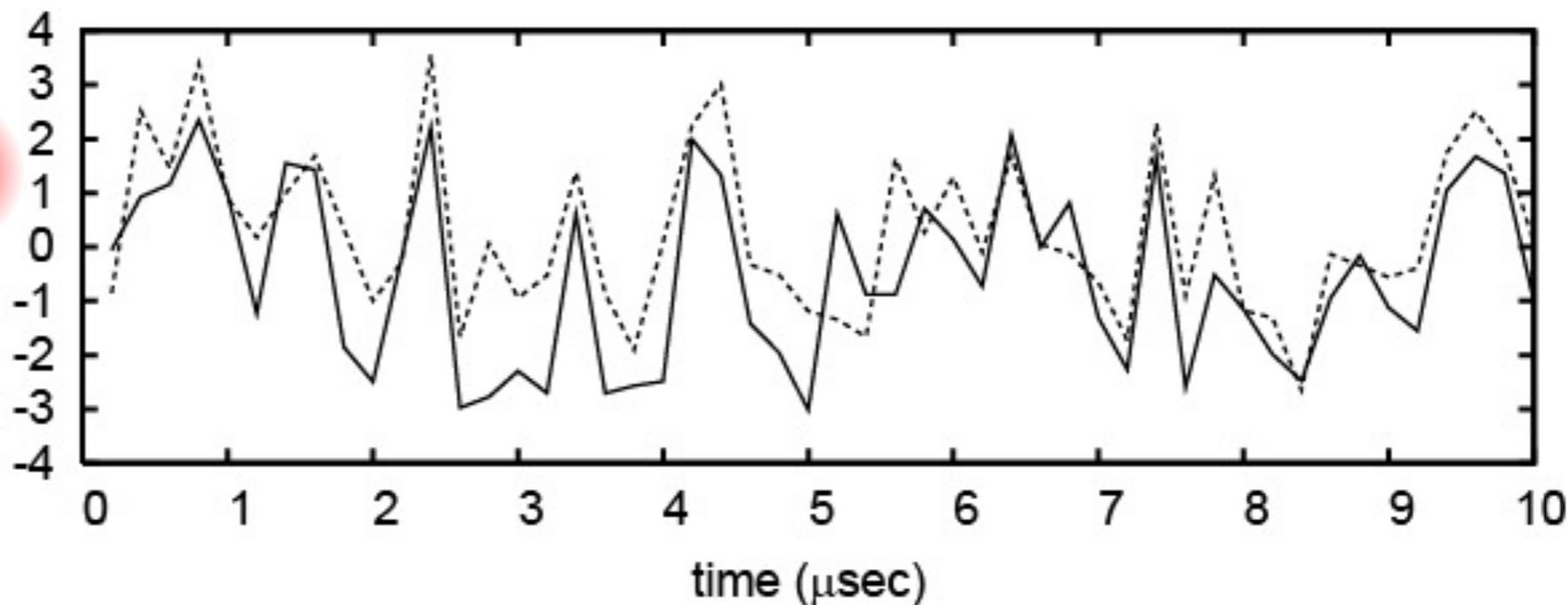
$$|\alpha\rangle - |-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^{2n+1}}{\sqrt{(2n+1)!}} |2n+1\rangle$$

# Time-domain EPR correlation

Alice ———  
Bob ·····

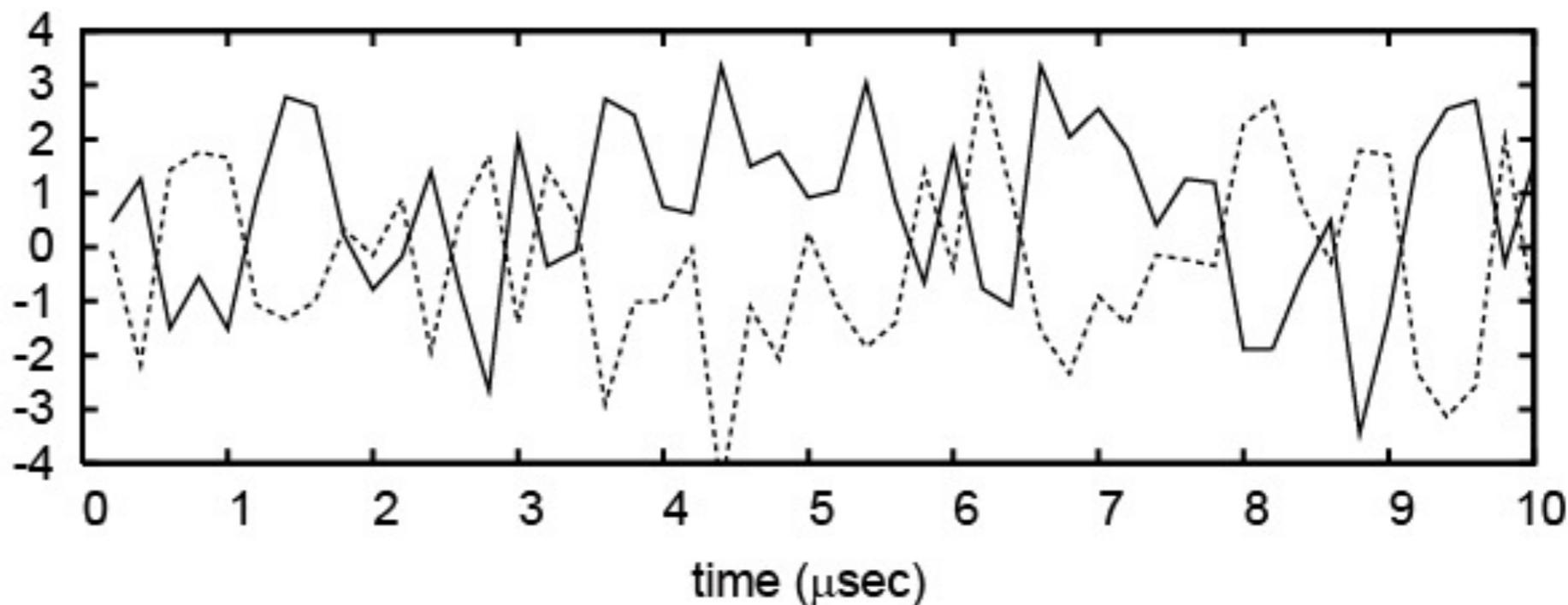


$x$  measurements



$$\begin{cases} \hat{x}_A - \hat{x}_B \rightarrow 0 \\ \hat{p}_A + \hat{p}_B \rightarrow 0 \end{cases}$$

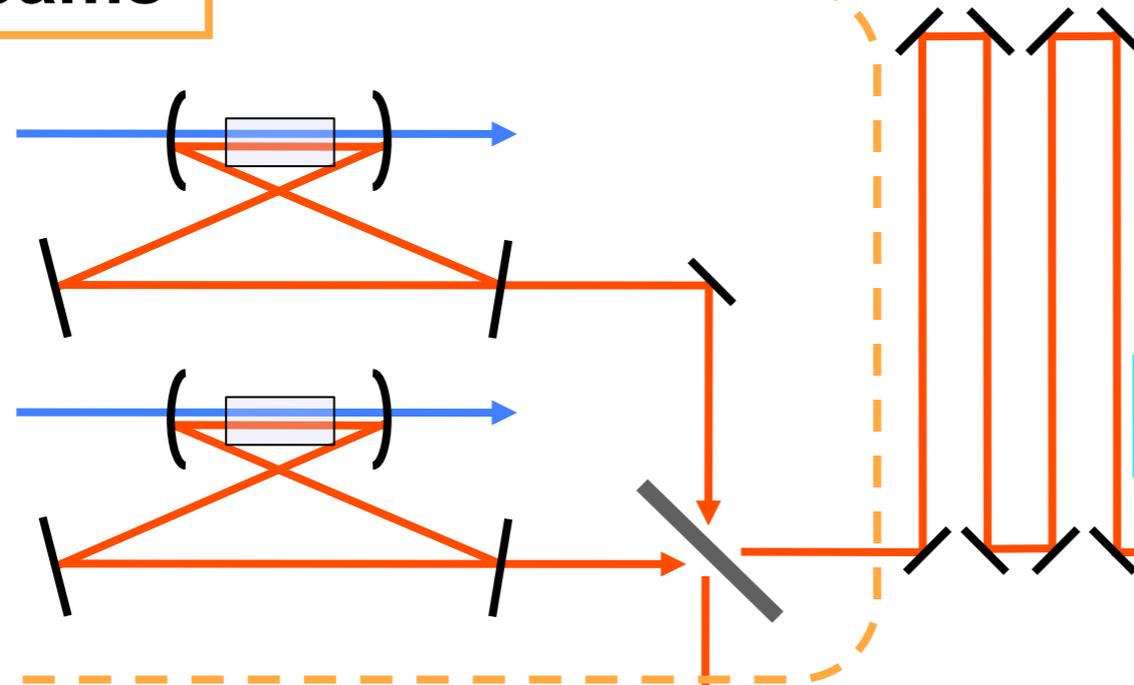
$p$  measurements



$$\begin{aligned} \text{AM signal} &= \hat{x}_{\text{noise}} \\ \text{FM signal} &= \hat{p}_{\text{noise}} \end{aligned}$$

# Teleportation of Schrödinger cats

Creation of EPR beams



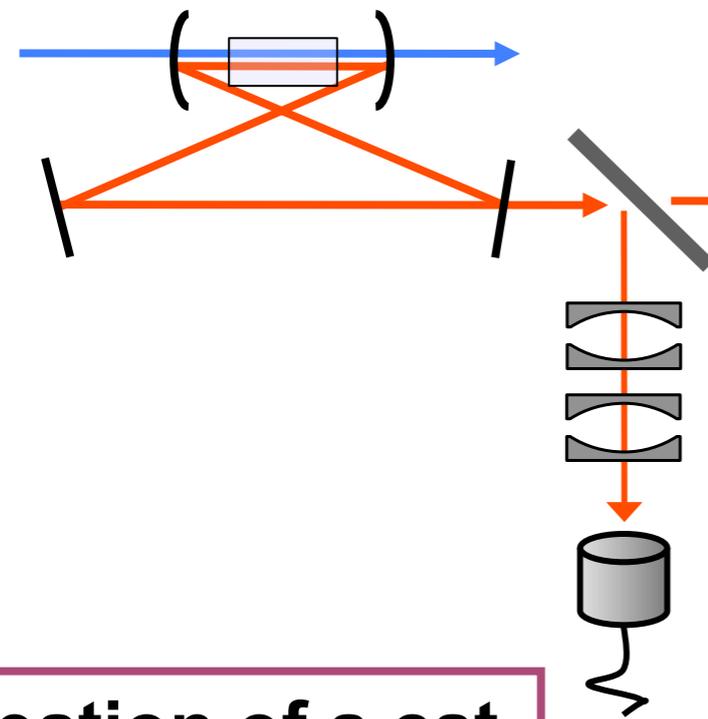
APD  
PD

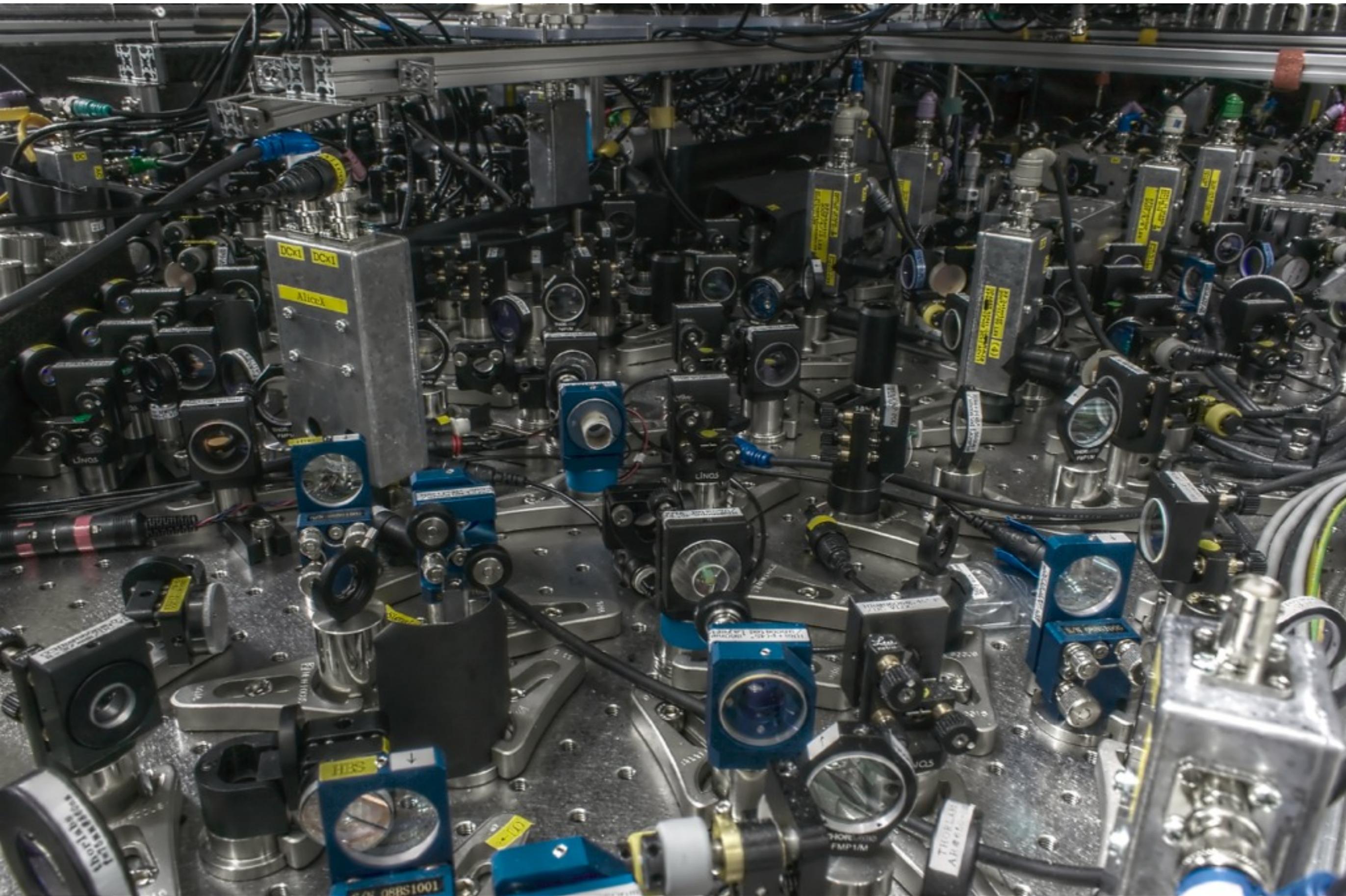
Bob

Alice

Tomography

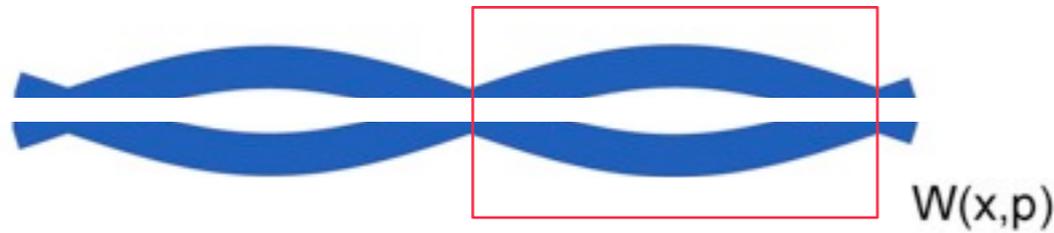
Creation of a cat





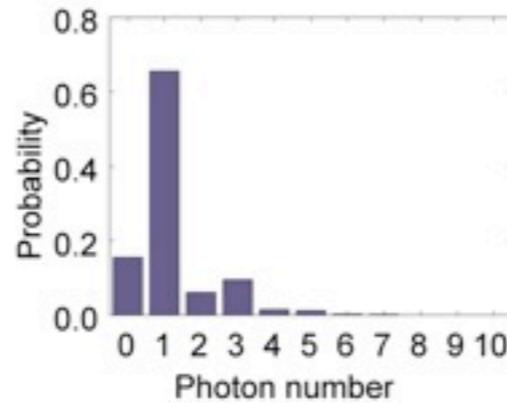
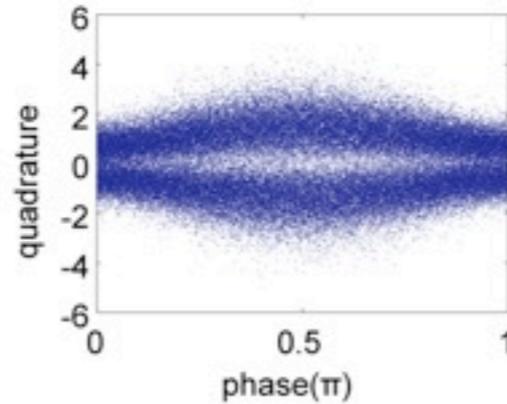
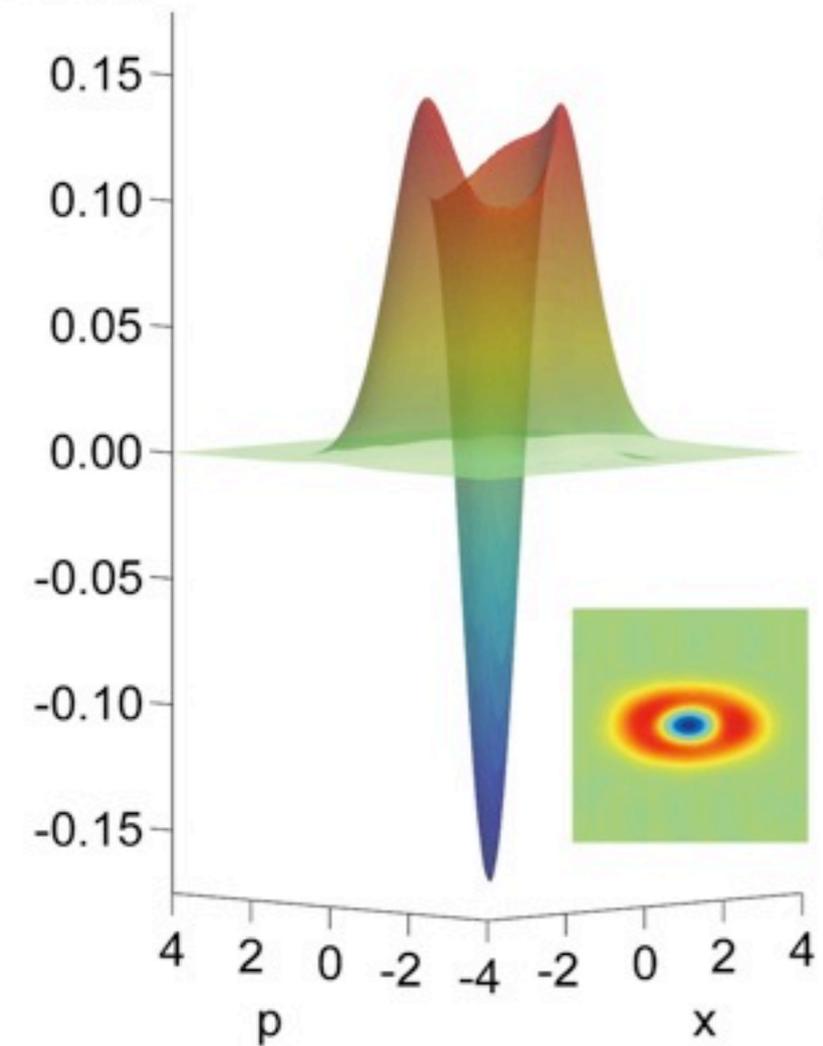
**N. Lee, H. Benichi, Y. Takeno, S. Takeda, J. Webb, E. Huntington, & A. Furusawa, *Science* 332, 330 (2011)**

# Teleportation of a Schrödinger cat state of light

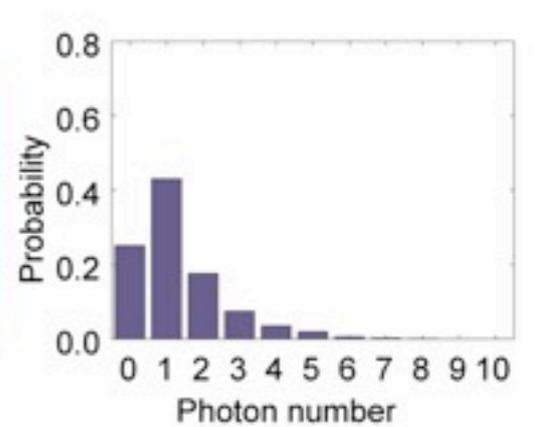
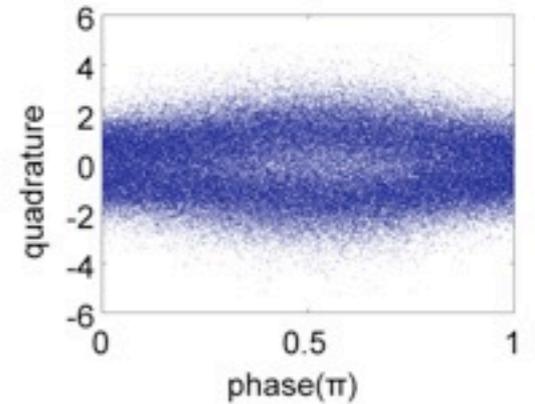
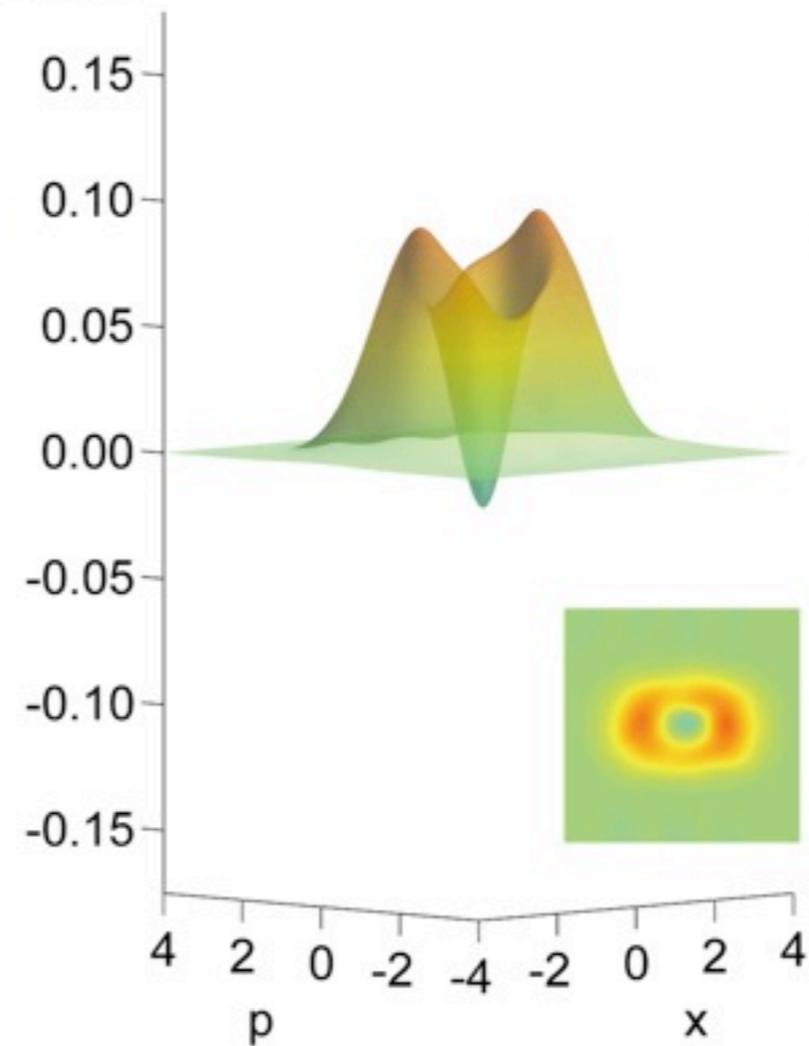


$$N_{\alpha} (|\alpha\rangle - |-\alpha\rangle)$$

$W(x,p)$



$W(x,p)$



Input

Output

High-fidelity universal squeezers

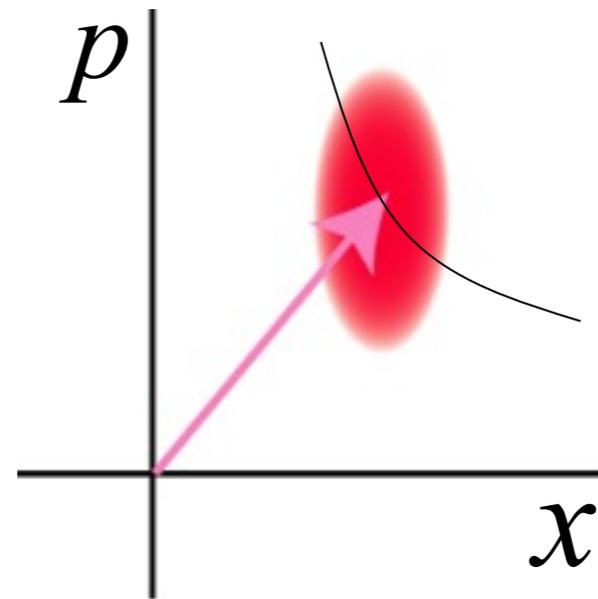
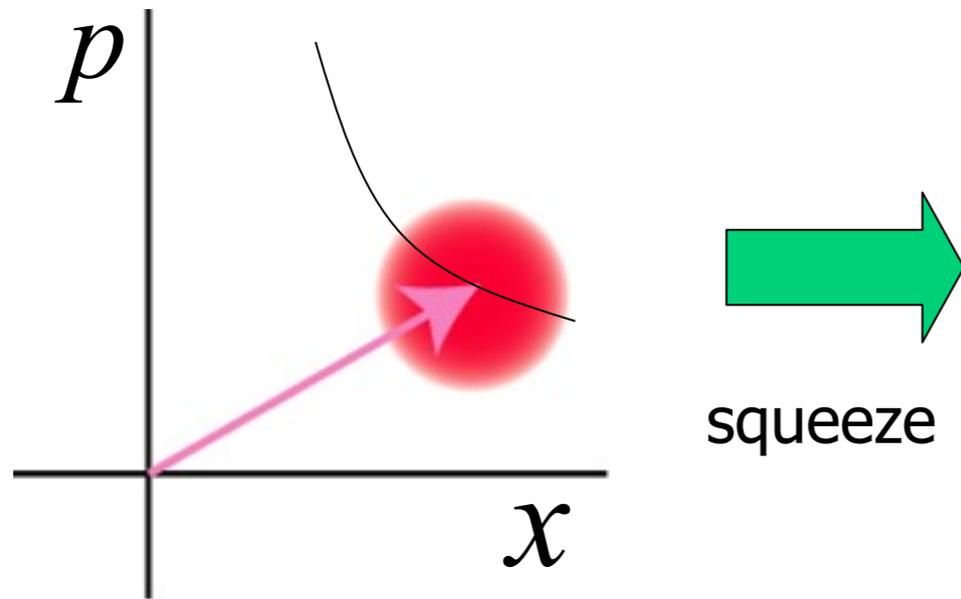
QND gate

One-way quantum information

processing with a QND gate

**Gate teleportation**

# High-fidelity universal squeezer

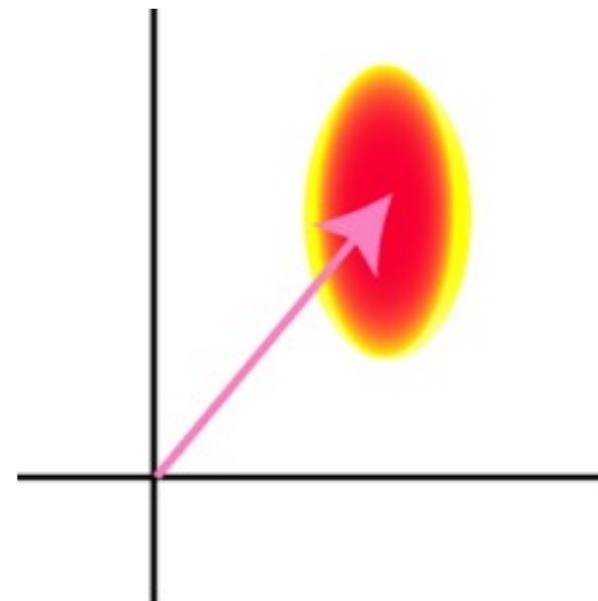


squeezing

$$\hat{x} \Rightarrow \frac{\hat{x}}{c}$$

$$\hat{p} \Rightarrow c\hat{p}$$

In reality



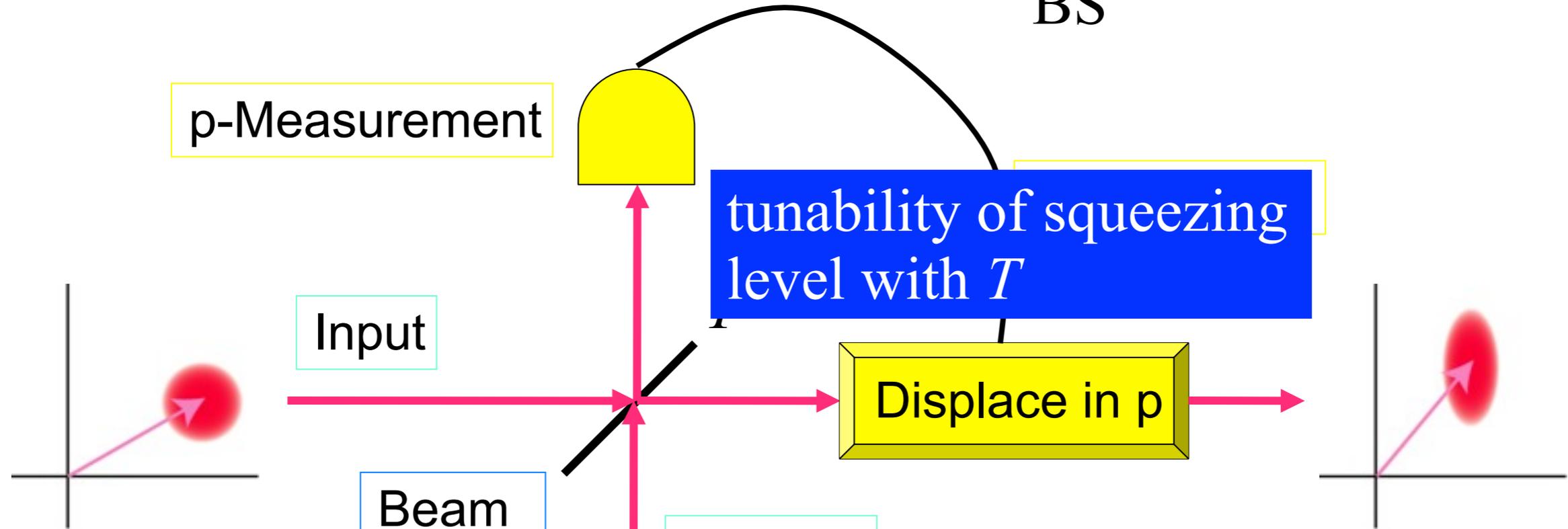
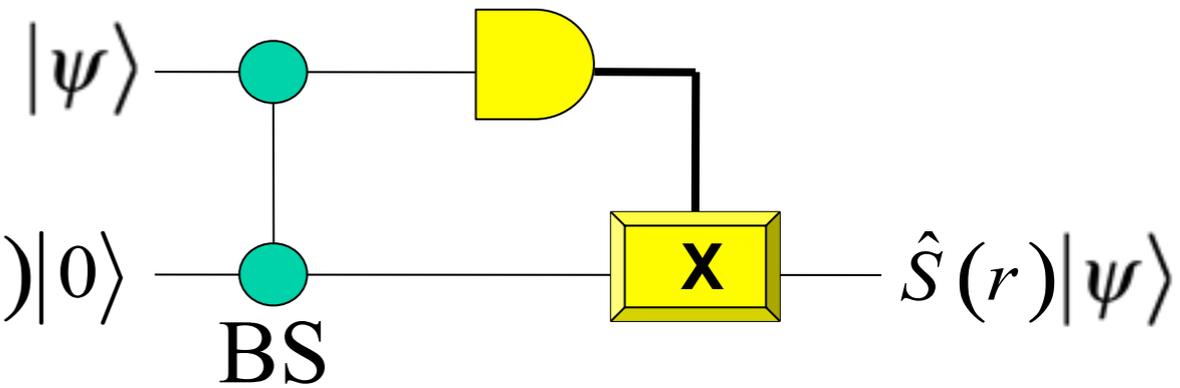
on average

$$\langle \hat{x} \rangle \Rightarrow \left\langle \frac{\hat{x}}{c} \right\rangle$$

$$\langle \hat{p} \rangle \Rightarrow \langle c\hat{p} \rangle$$

# High-fidelity universal squeezer with measurement and feedforward

**gate teleportation**



tunability of squeezing level with  $T$

AM signal =  $\hat{x}$   
FM signal =  $\hat{p}$

$$\hat{x}_1'' = \sqrt{T} \hat{x}_1 + \sqrt{1-T} \hat{x}_A^{(0)} e^{-r}$$

$$\hat{p}_1'' = \frac{1}{\sqrt{T}} \hat{p}_1$$

$$\langle \hat{x}_A^{(0)} \rangle = 0$$

# Output of High-fidelity squeezer

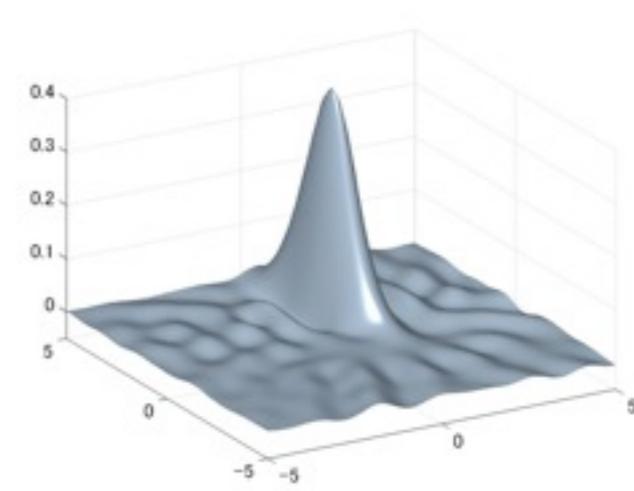
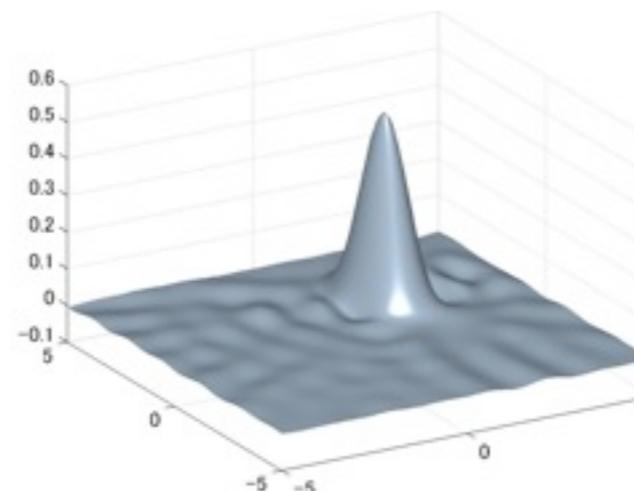
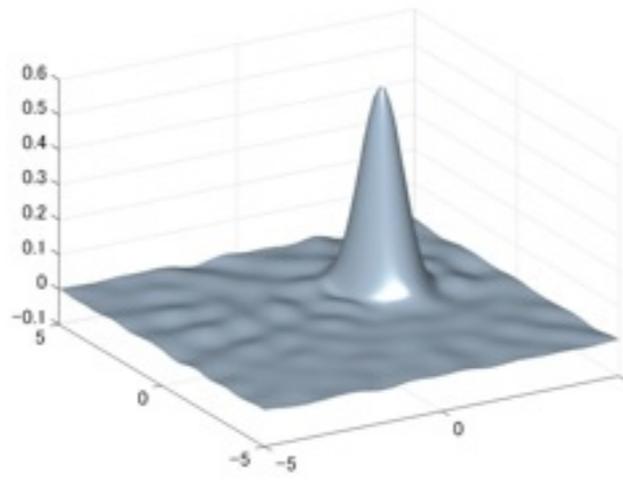
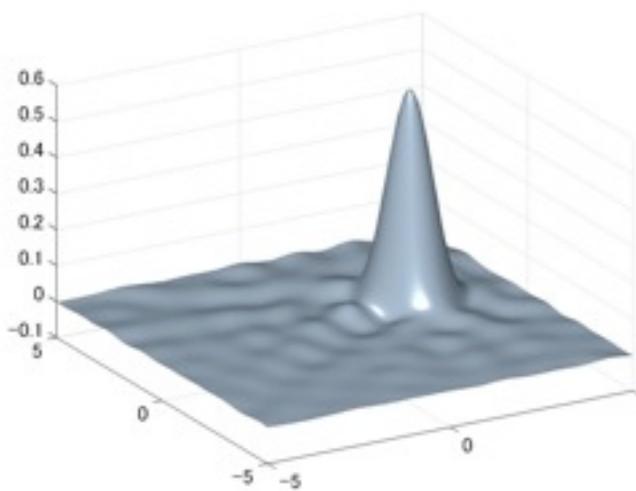
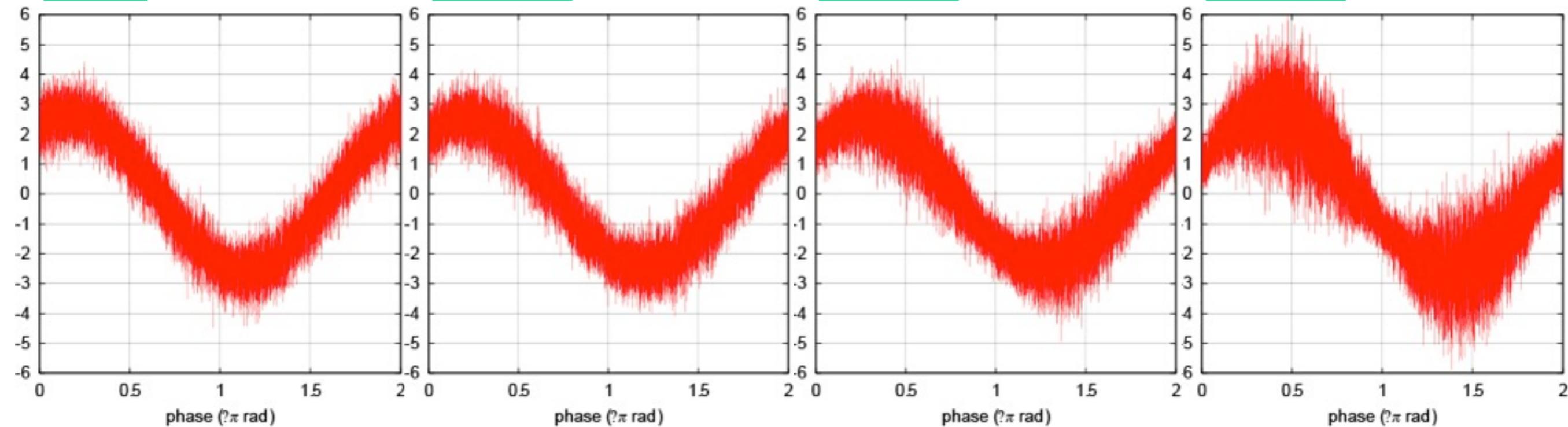
ancilla: -5dB of squeezing

Input

T=75%

T=50%

T=25%



J. Yoshikawa, T. Hayashi, T. Akiyama, N. Takei, A. Huck,  
U. L. Andersen, and A. Furusawa, Phys. Rev. A 76, 060301(R) (2007).

$$\hat{U}_{\text{QND}} = e^{-i2G\hat{x}_1\hat{p}_2}$$

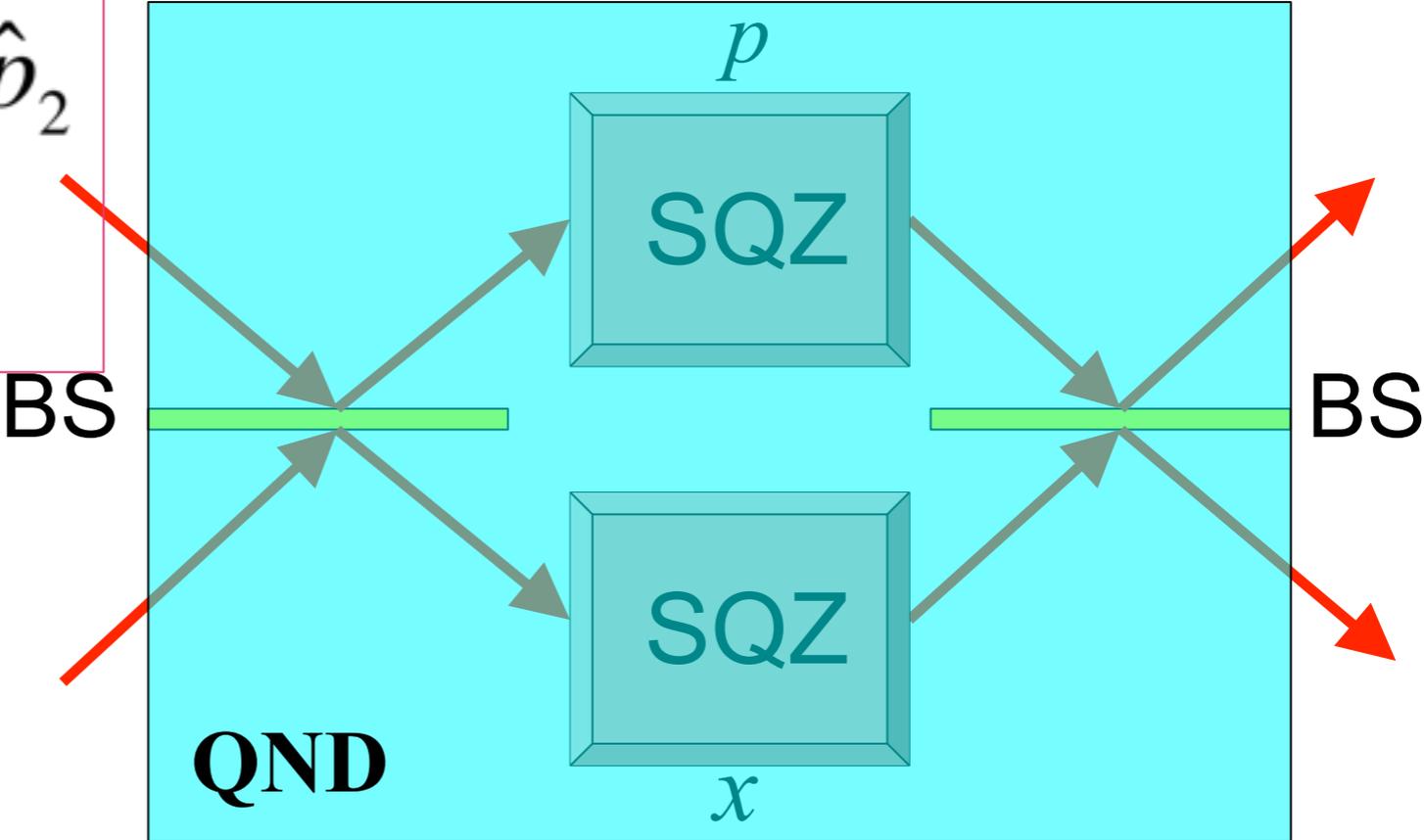
Quantum Non-Demolition (QND) interaction

**QND gate**

$$\begin{aligned} \hat{U}_{\text{QND}}^{-1}\hat{x}_1\hat{U}_{\text{QND}} &= \hat{x}_1 \\ \hat{U}_{\text{QND}}^{-1}\hat{x}_2\hat{U}_{\text{QND}} &= \hat{x}_2 + G\hat{x}_1 \\ \hat{U}_{\text{QND}}^{-1}\hat{p}_1\hat{U}_{\text{QND}} &= \hat{p}_1 - G\hat{p}_2 \\ \hat{U}_{\text{QND}}^{-1}\hat{p}_2\hat{U}_{\text{QND}} &= \hat{p}_2 \end{aligned}$$

$$e^{-2i\hat{x}_1\hat{p}_2} |x_1\rangle \otimes |x_2\rangle = |x_1\rangle \otimes |x_1 + x_2\rangle$$

CV-CNOT gate ( $G=1$ )



# QND interaction with universal squeezers

$$\hat{x}'_1 = \hat{x}_1 - \sqrt{\frac{1-T}{1+T}} \hat{x}_A^{(0)} e^{-r}$$

$$\hat{x}'_2 = \hat{x}_2 + \left( \frac{1}{\sqrt{T}} - \sqrt{T} \right) \hat{x}_1 + \sqrt{T \frac{1-T}{1+T}} \hat{x}_A^{(0)} e^{-r}$$

$$\hat{p}'_1 = \hat{p}_1 - \left( \frac{1}{\sqrt{T}} - \sqrt{T} \right) \hat{p}_2 + \sqrt{T \frac{1-T}{1+T}} \hat{x}_B^{(0)} e^{-r}$$

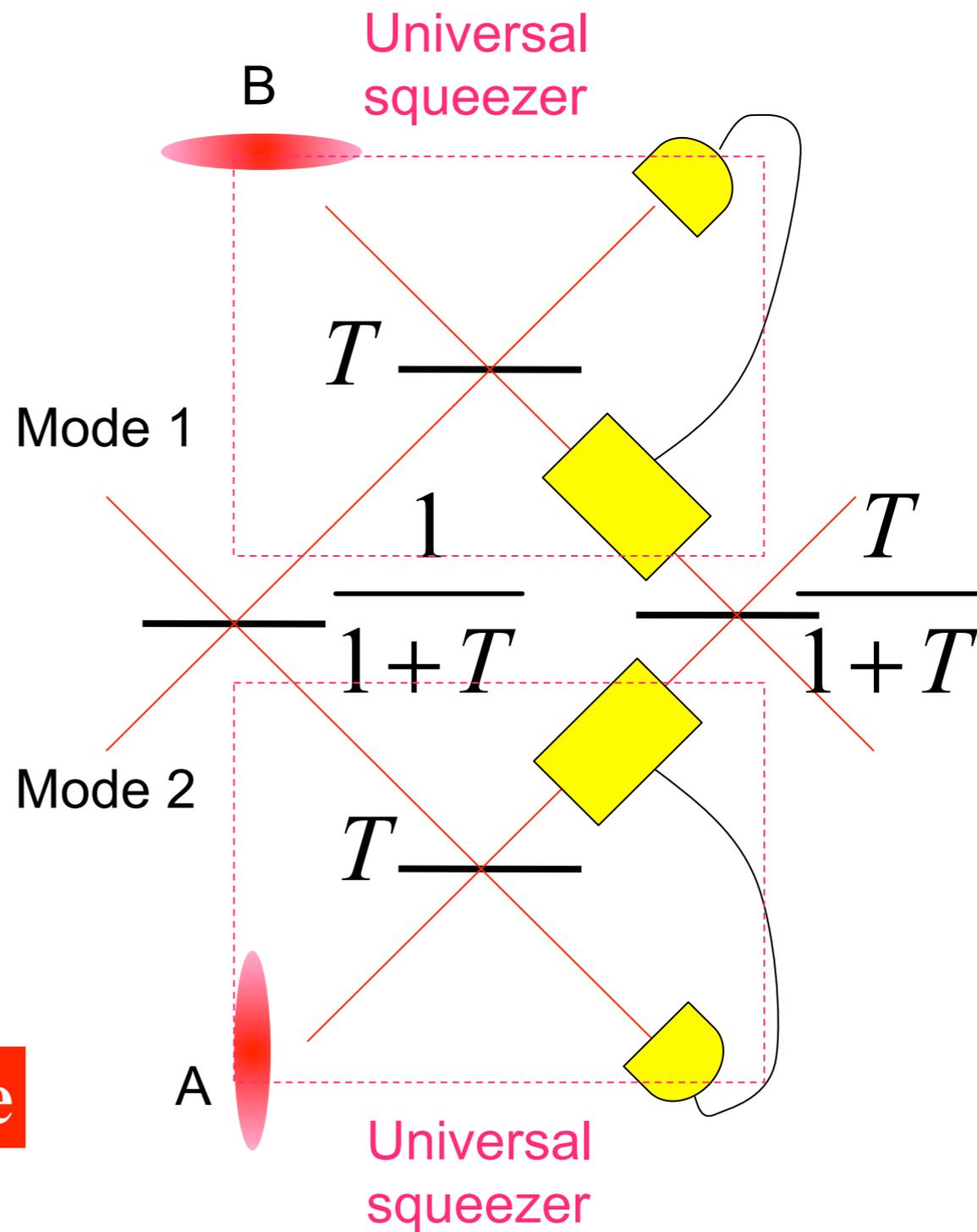
$$\hat{p}'_2 = \hat{p}_2 + \sqrt{\frac{1-T}{1+T}} \hat{p}_B^{(0)} e^{-r}$$

$$G = \frac{1}{\sqrt{T}} - \sqrt{T}$$

$G = 1$  **QND gate**

$T = 0.38$

-4.2 dB of squeezing



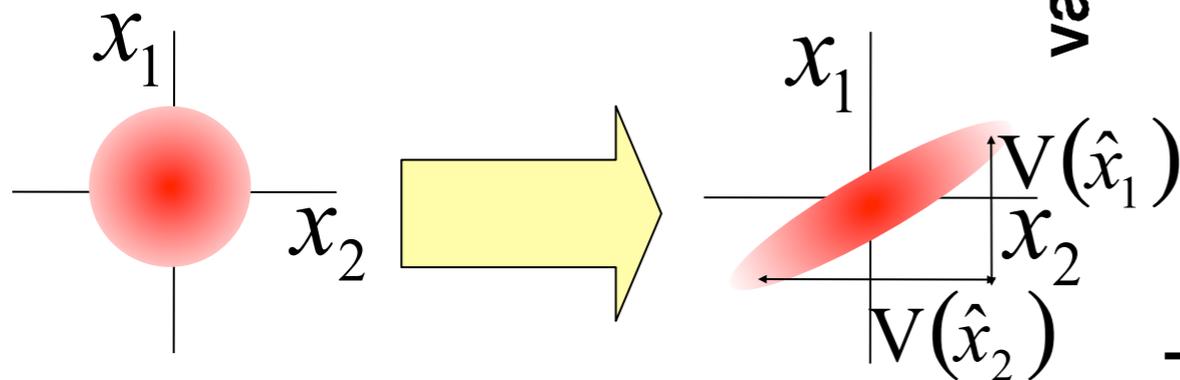
# Experimental results

$$\hat{x}_1^{\text{out}} = \hat{x}_1^{\text{in}} - 0.67 \hat{x}_A^{(0)} e^{-r}$$

$$\hat{x}_2^{\text{out}} = \hat{x}_2^{\text{in}} + \hat{x}_1^{\text{in}} + 0.41 \hat{x}_A^{(0)} e^{-r}$$

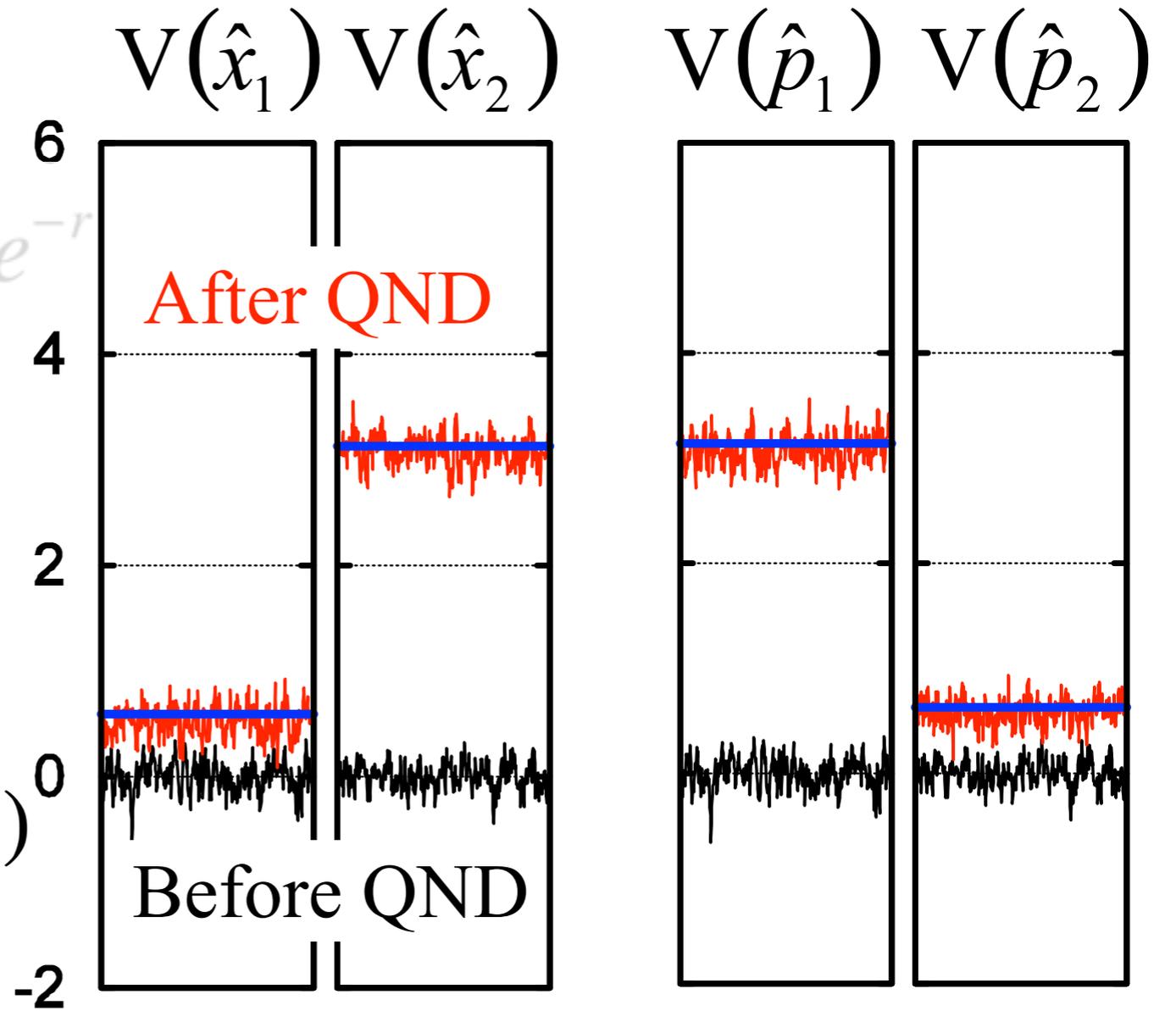
$$\hat{p}_1^{\text{out}} = \hat{p}_1^{\text{in}} - \hat{p}_2^{\text{in}}$$

$$\hat{p}_2^{\text{out}} = \hat{p}_2^{\text{in}}$$



Inputs

Outputs

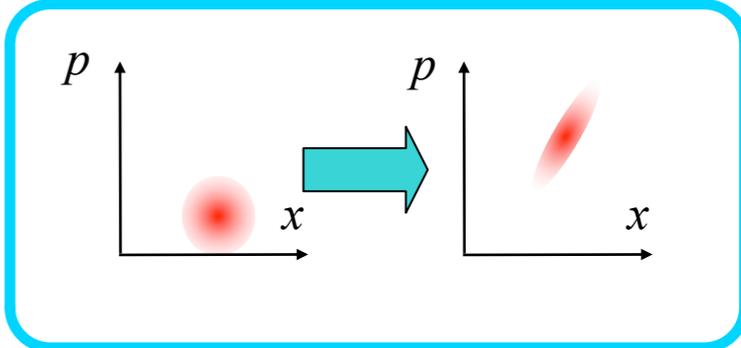
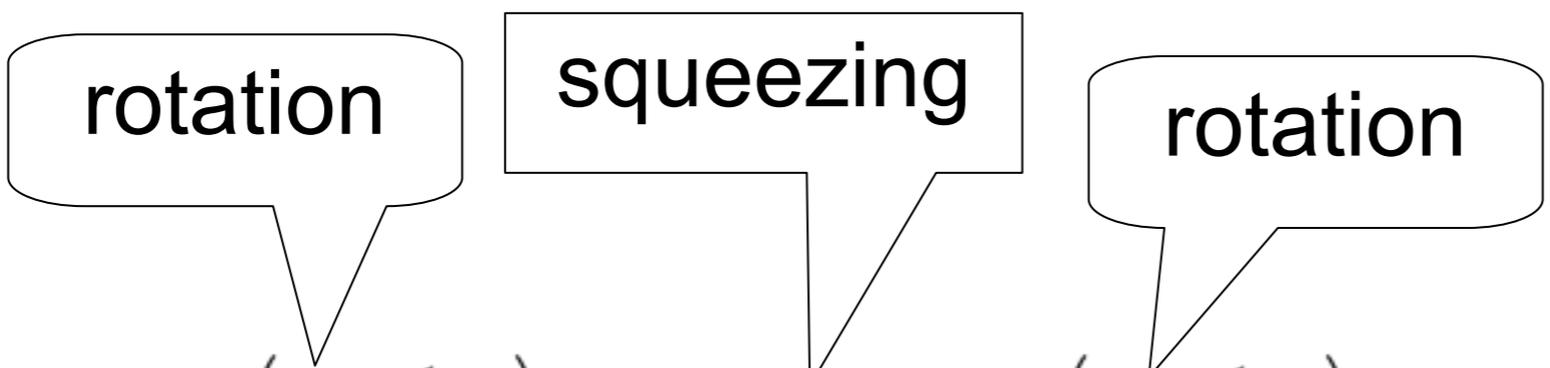
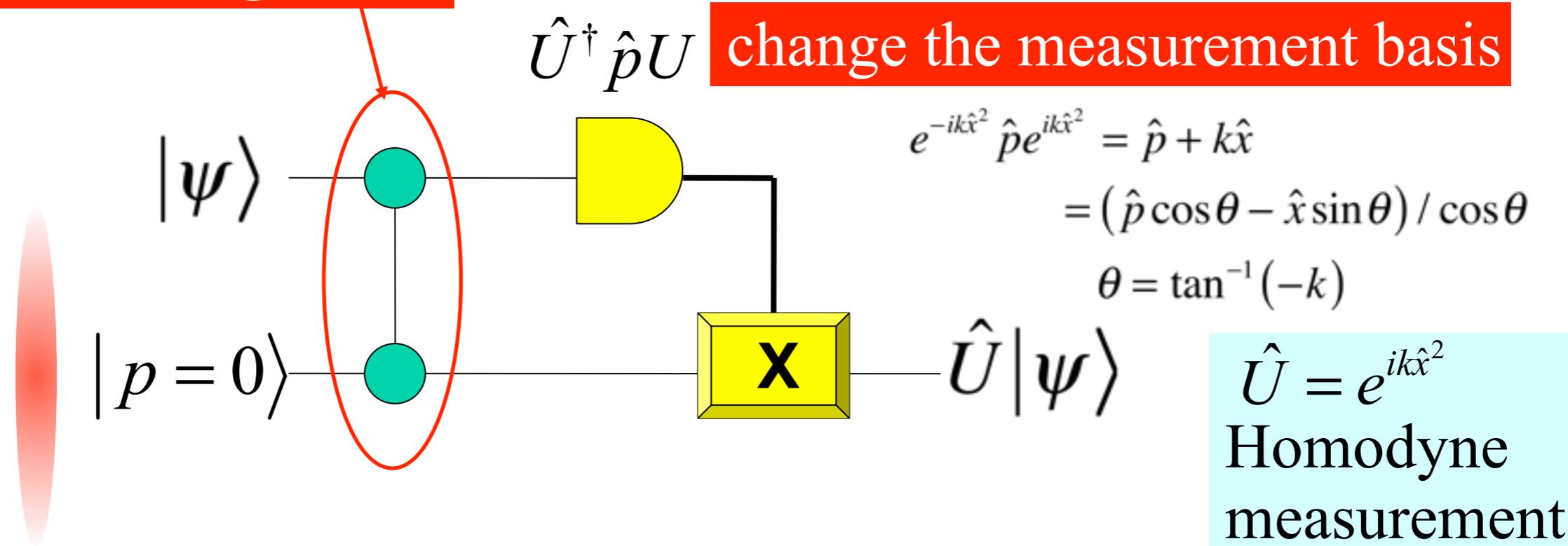


— Theoretical values  
with finite squeezing of ancillae: -4.9dB

J. Yoshikawa et al., Phys. Rev. Lett. 101, 250501 (2008)

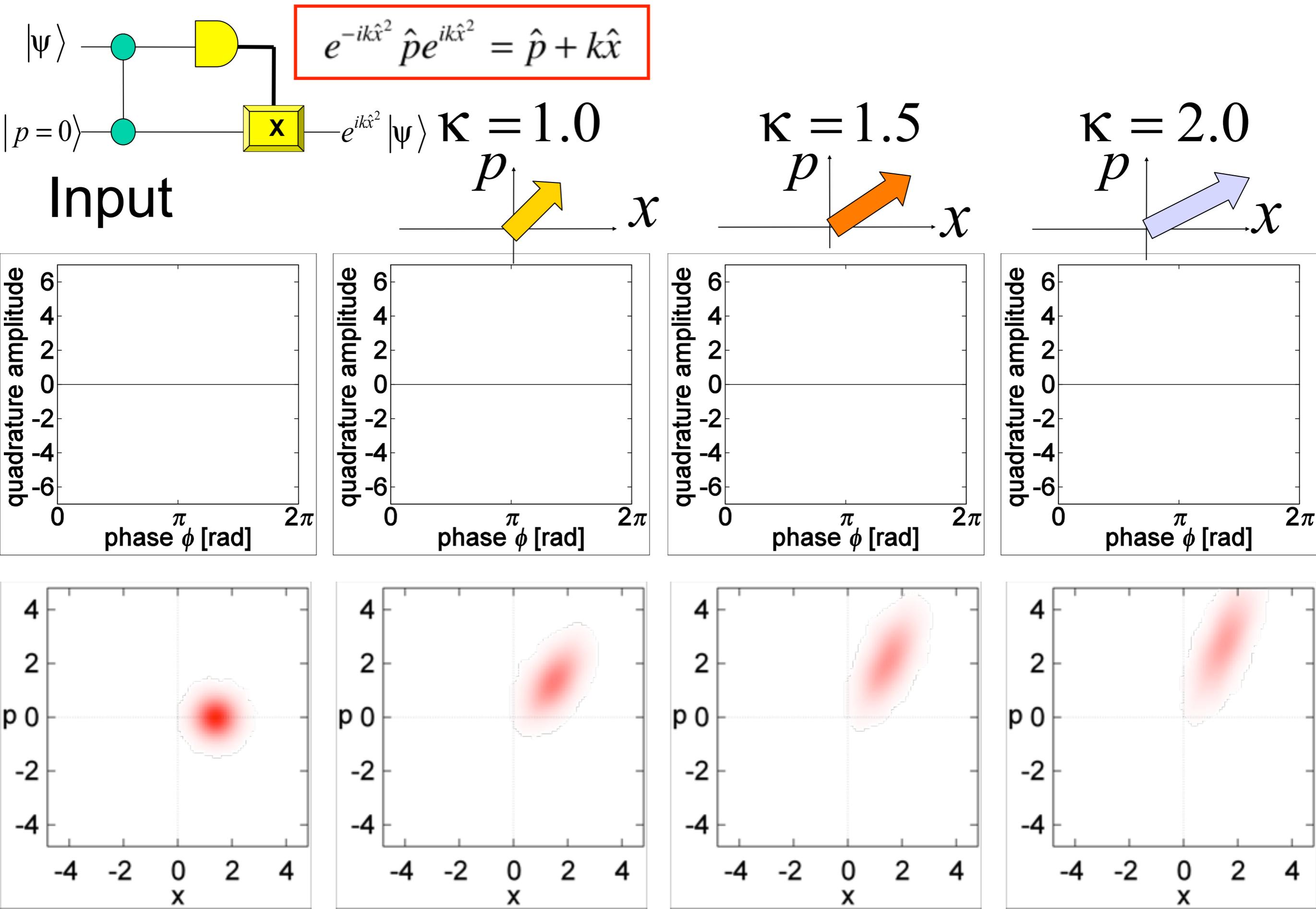
# One-way QIP with an entangling QND gate

## QND gate



$$e^{ik\hat{x}^2} = e^{-i(\cot^{-1} \eta)\hat{n}} e^{(\ln \eta)2\hat{x}\hat{p}} e^{i(\cot^{-1} \eta)\hat{n}} \quad \eta = (\sqrt{k^2 + 4} + |k|) / 2$$

# One-way quantum computing with an entangling QND gate



# Teleportation of time-bin qubits with a CV teleporter

**Deterministic teleportation of optical qubits**

**Hybrid quantum information processing**

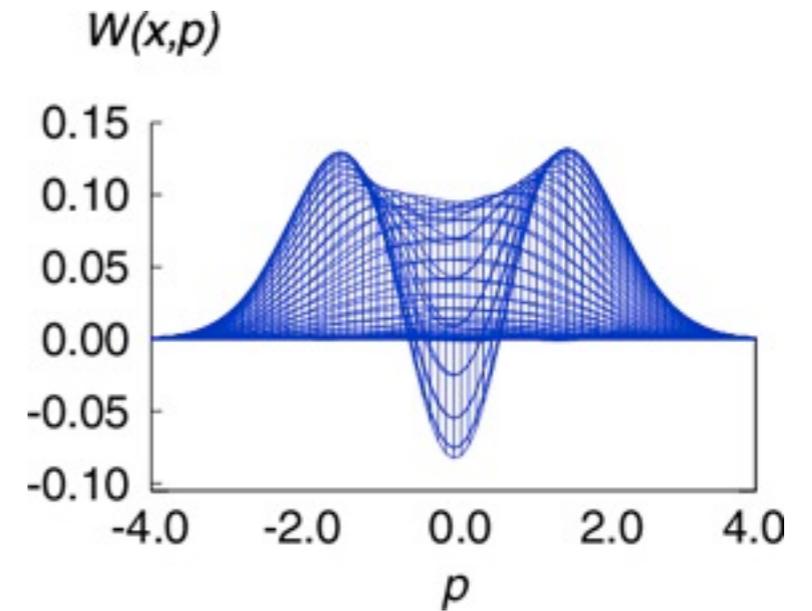
# Schrödinger cat states

$$|0\rangle : N_{\alpha} (|\alpha\rangle - |-\alpha\rangle)$$

odd photons

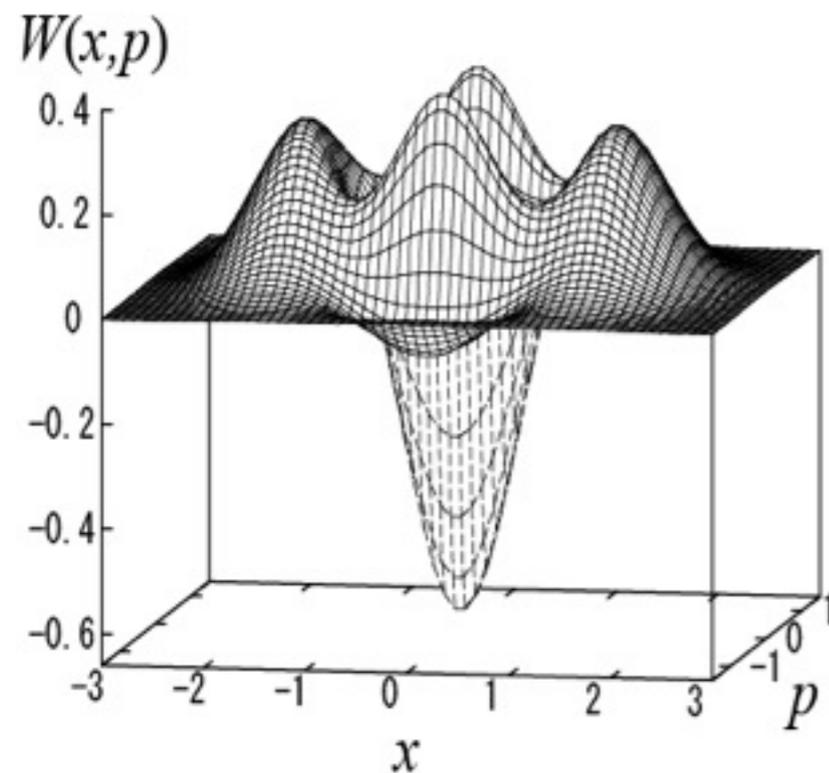
$$|1\rangle : N_{\alpha} (|\alpha\rangle + |-\alpha\rangle)$$

even photons



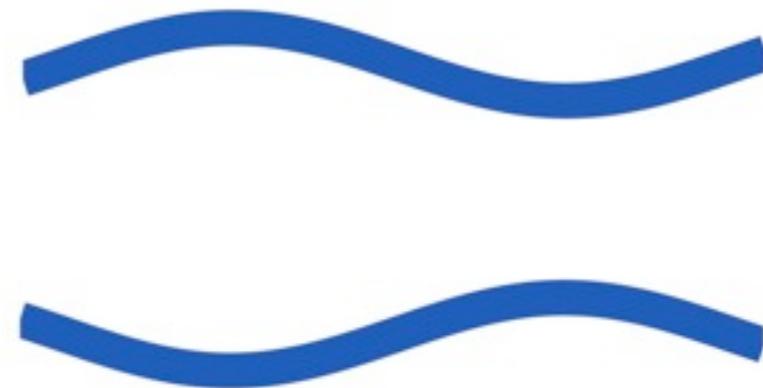
K. Wakui et al.,  
Opt. Exp. 15, 3568 (2007)

H. Takahashi et al.,  
Phys. Rev. Lett. 101, 233605 (2008)



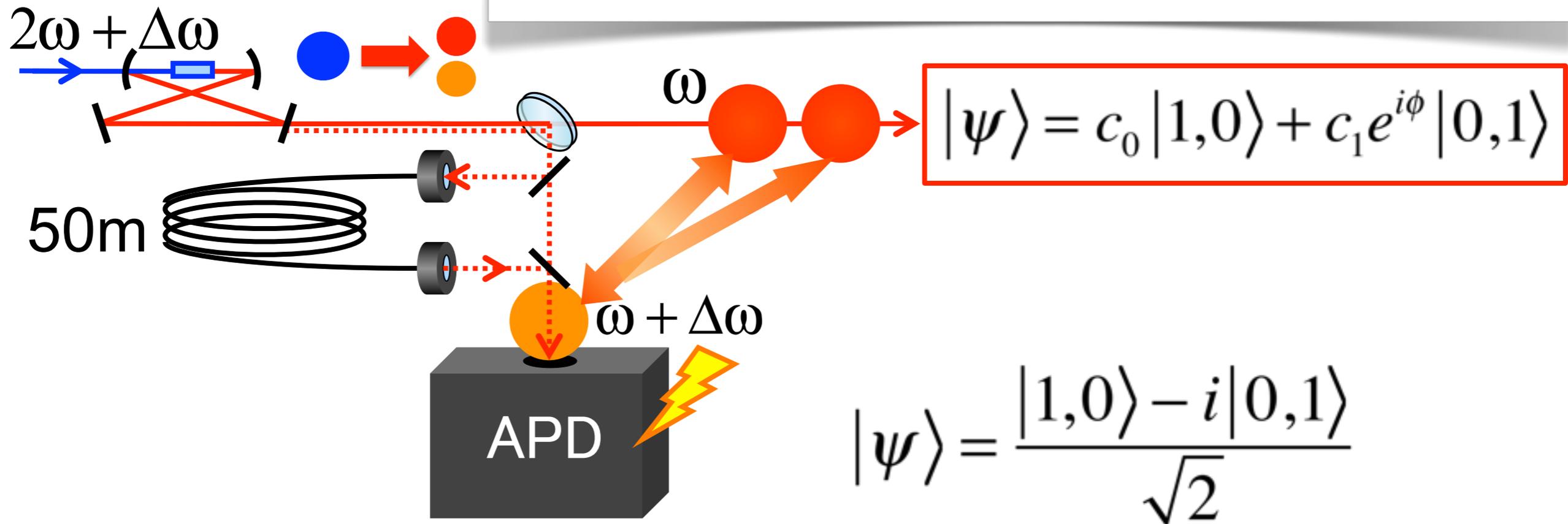
$$|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$|-\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_{n=0}^{\infty} \frac{(-\alpha)^n}{\sqrt{n!}} |n\rangle$$



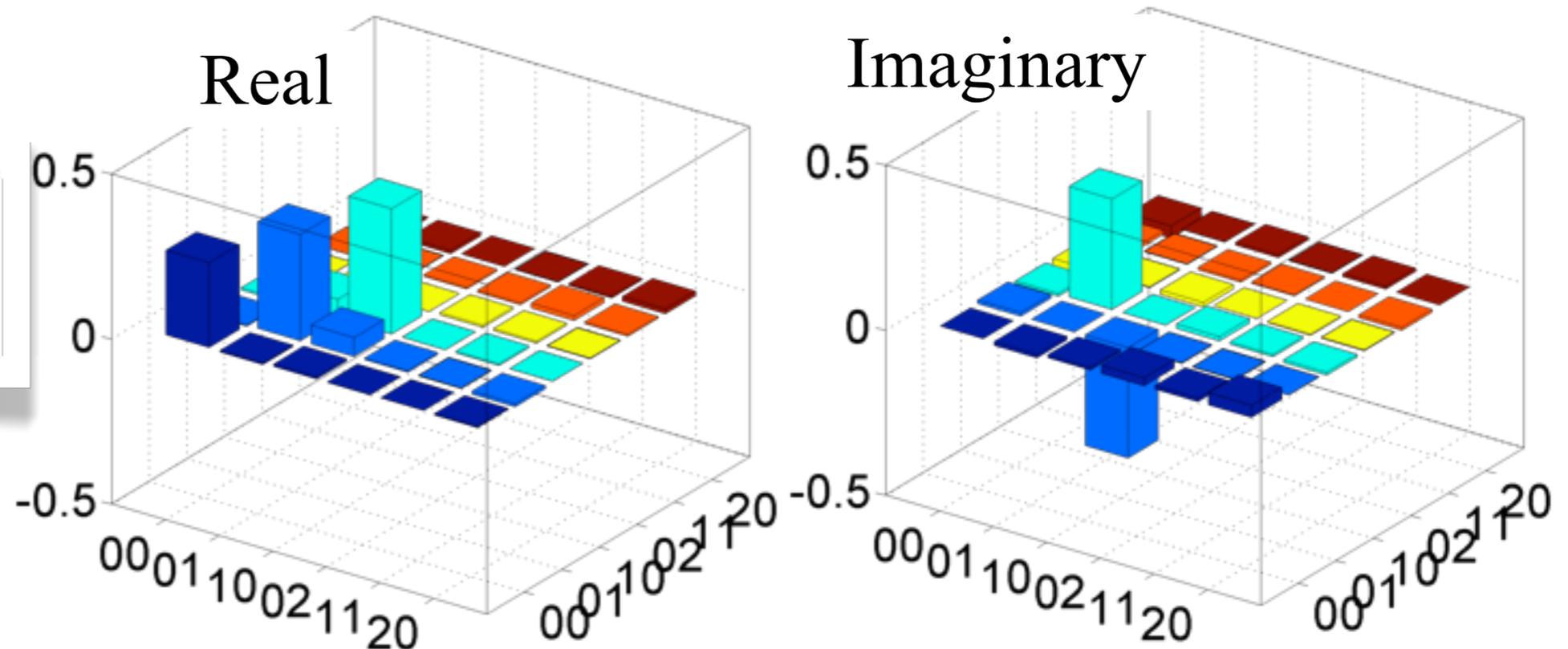
*Now working on*

# Teleportation of time-bin qubits



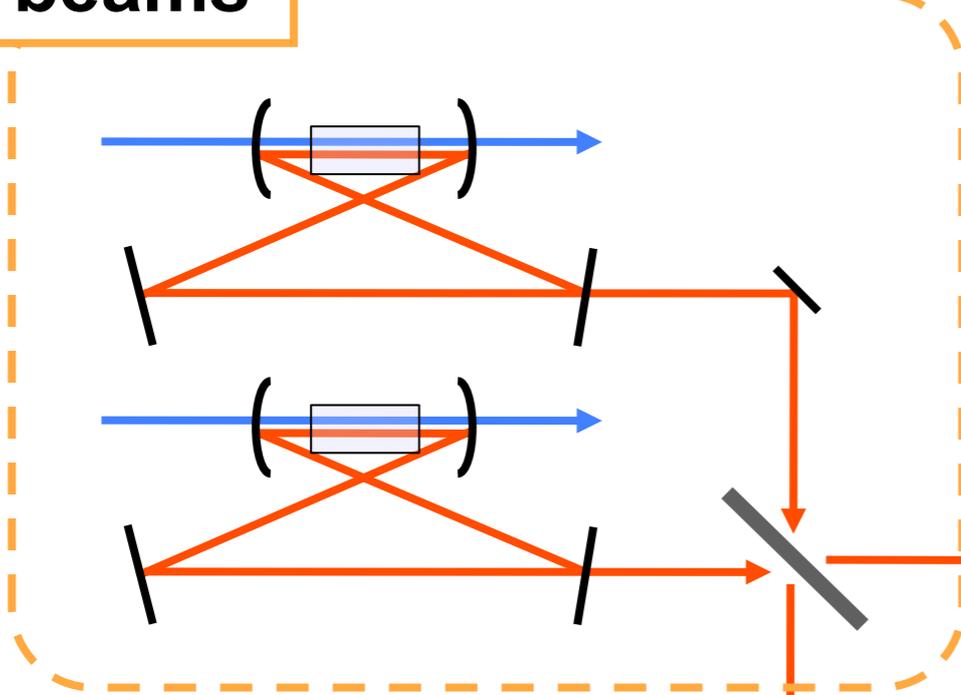
$$|\psi\rangle = \frac{|1,0\rangle - i|0,1\rangle}{\sqrt{2}}$$

Dual-homodyne  
Tomography



# Teleportation of time-bin qubits

Creation of EPR beams

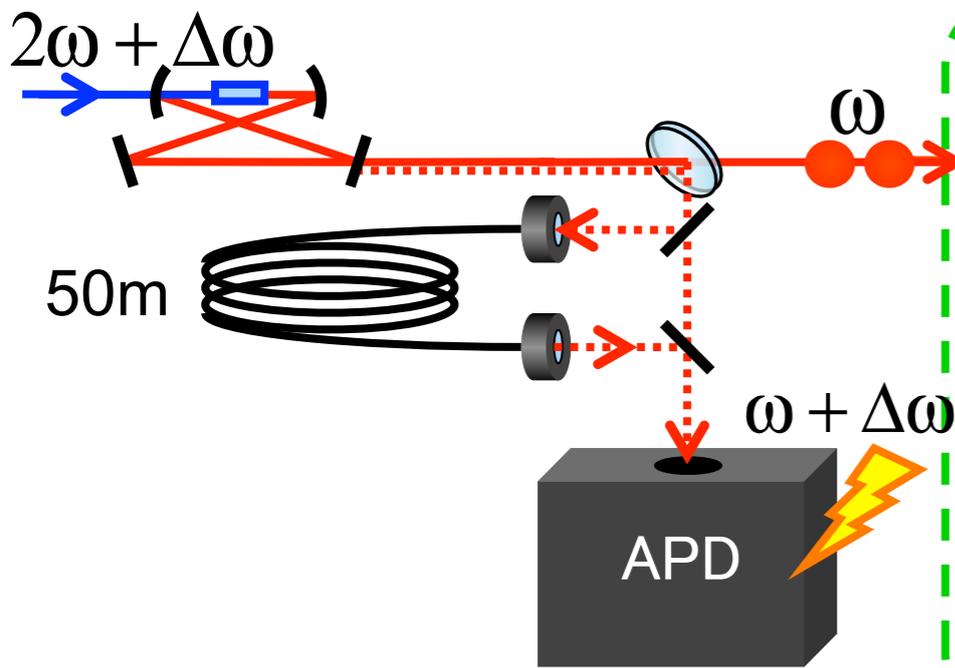


APD  
PD

Bob

Alice

Tomography



Time-bin qubit

# Realization of on-demand single-photon source

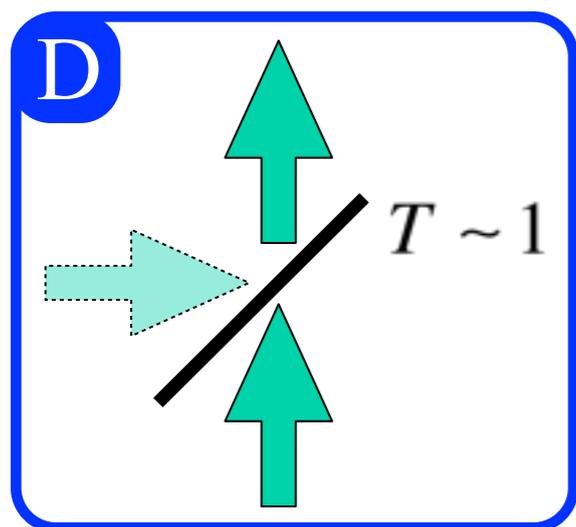
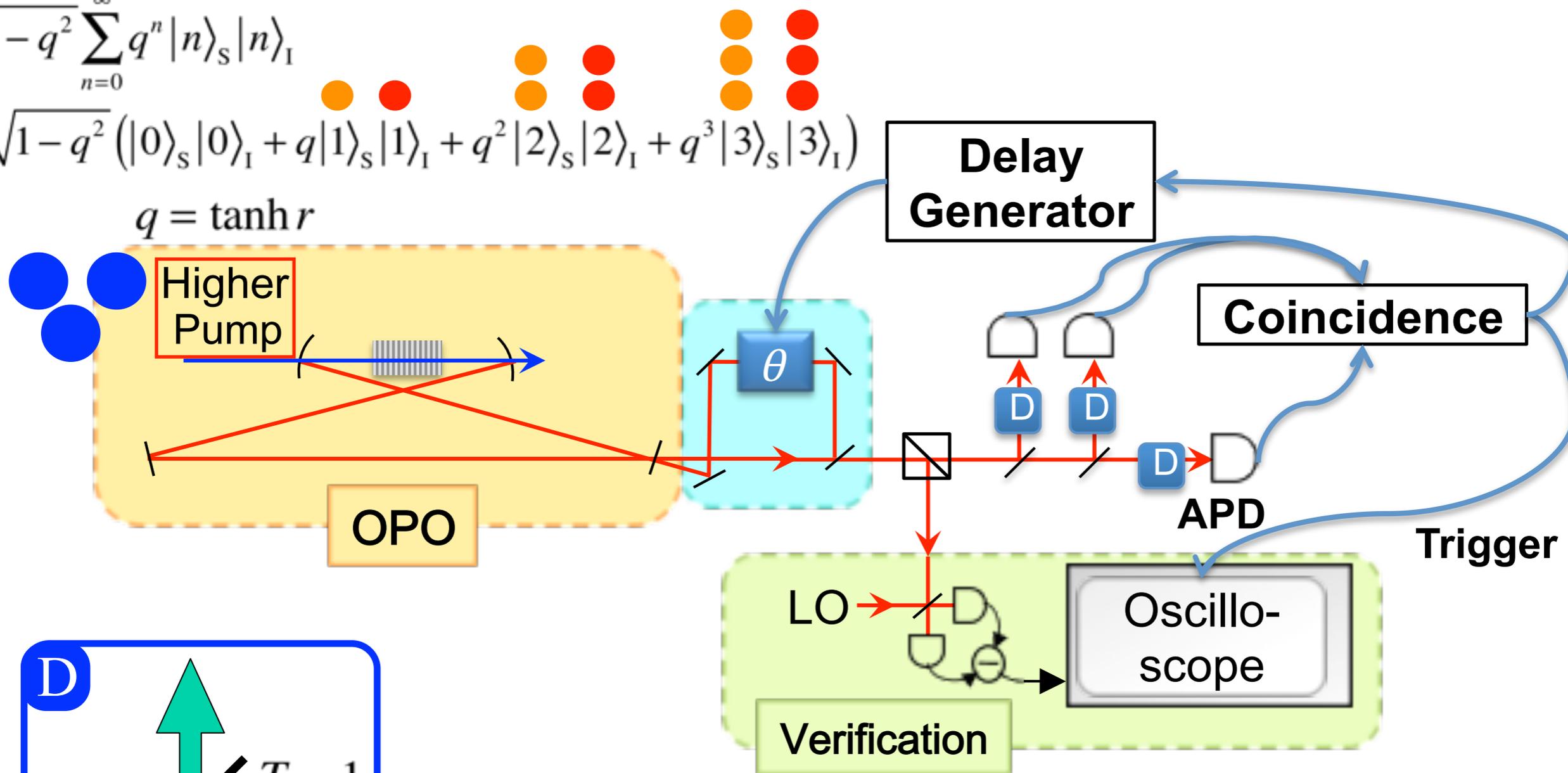
**Quantum memory**

# State generation and quantum memory

$$\sqrt{1-q^2} \sum_{n=0}^{\infty} q^n |n\rangle_S |n\rangle_I$$

$$\approx \sqrt{1-q^2} (|0\rangle_S |0\rangle_I + q|1\rangle_S |1\rangle_I + q^2|2\rangle_S |2\rangle_I + q^3|3\rangle_S |3\rangle_I)$$

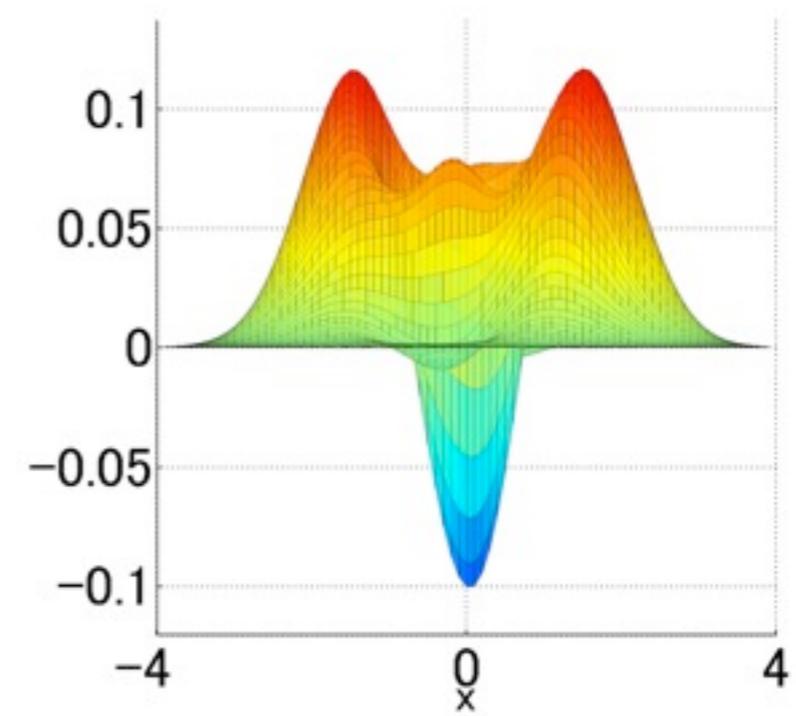
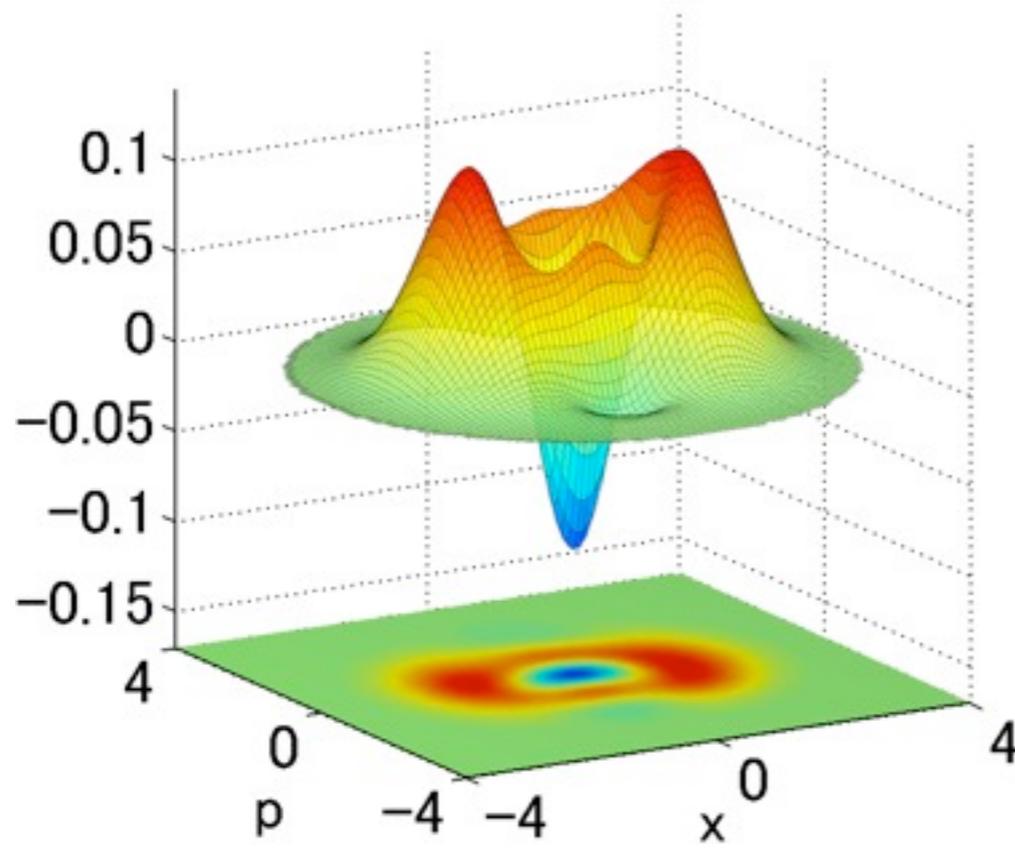
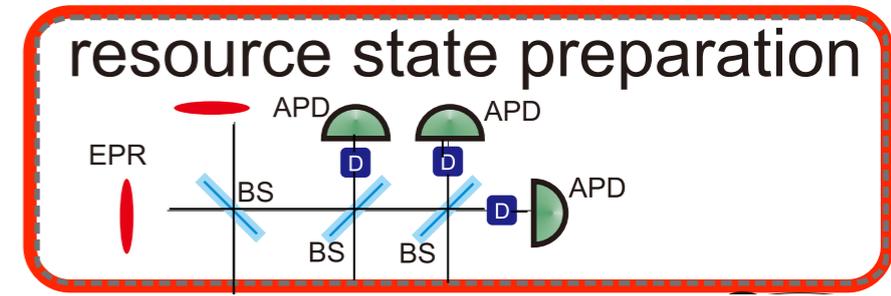
$$q = \tanh r$$



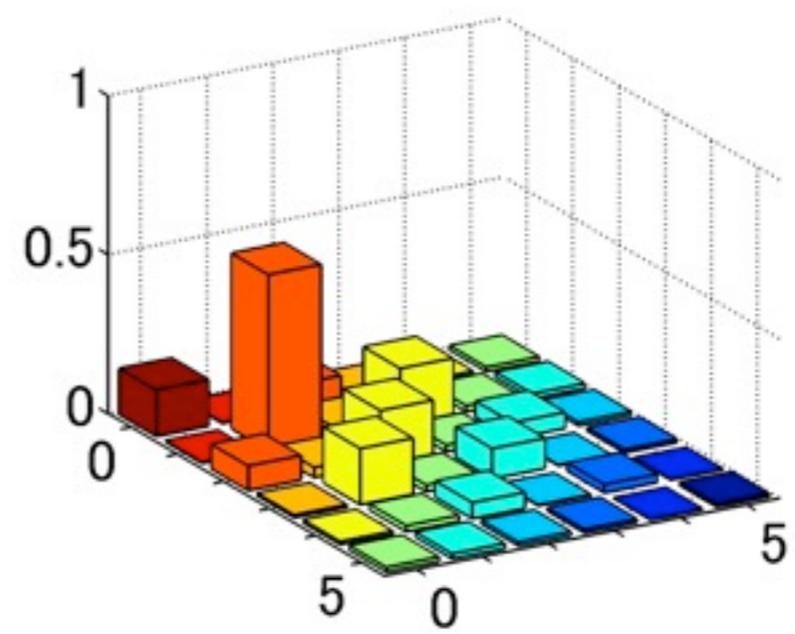
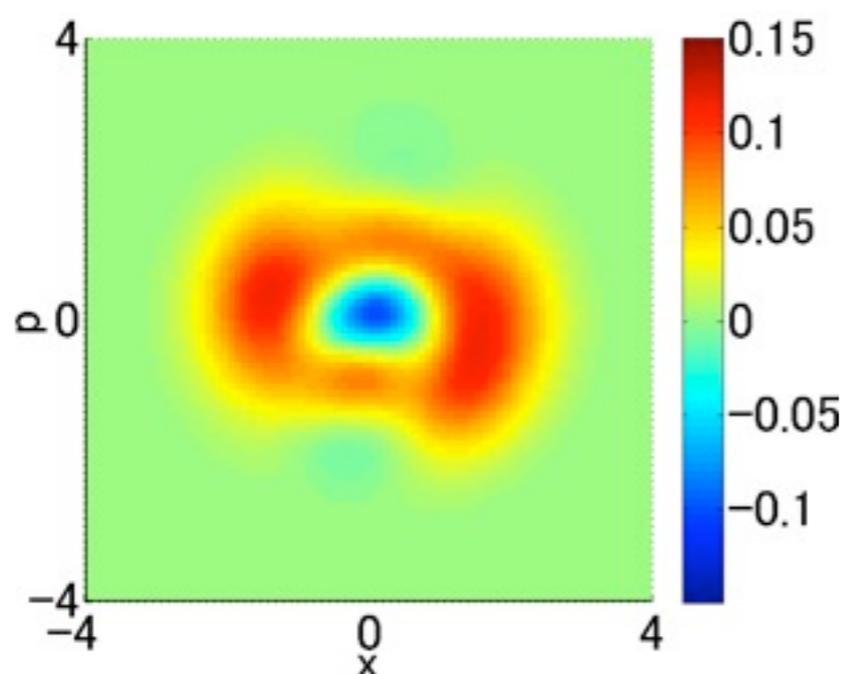
$$|\psi\rangle = a|0\rangle + b|1\rangle + c|2\rangle + d|3\rangle$$

$$|1\rangle + \frac{\alpha^2}{\sqrt{6}} |3\rangle$$

Schrödinger cat state  
A bigger cat!



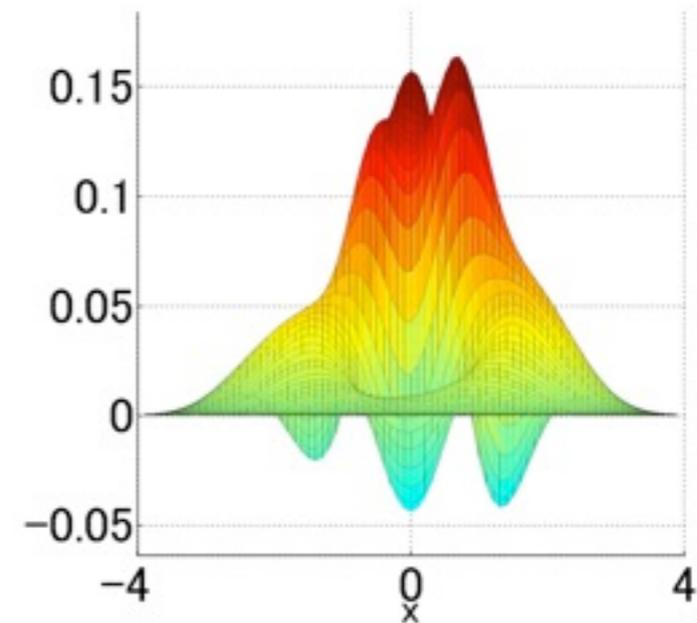
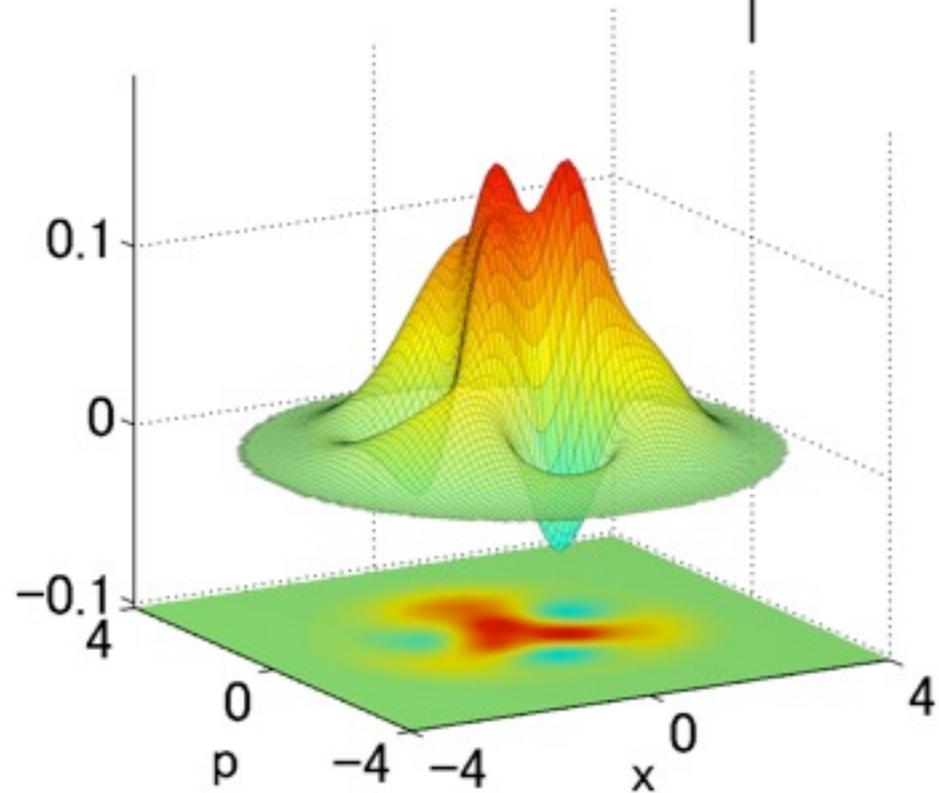
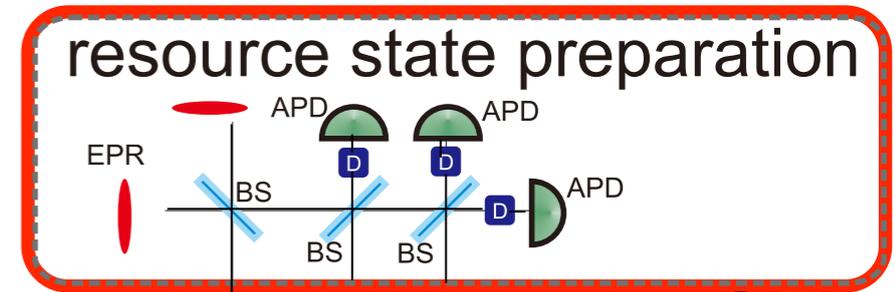
without any correction!!



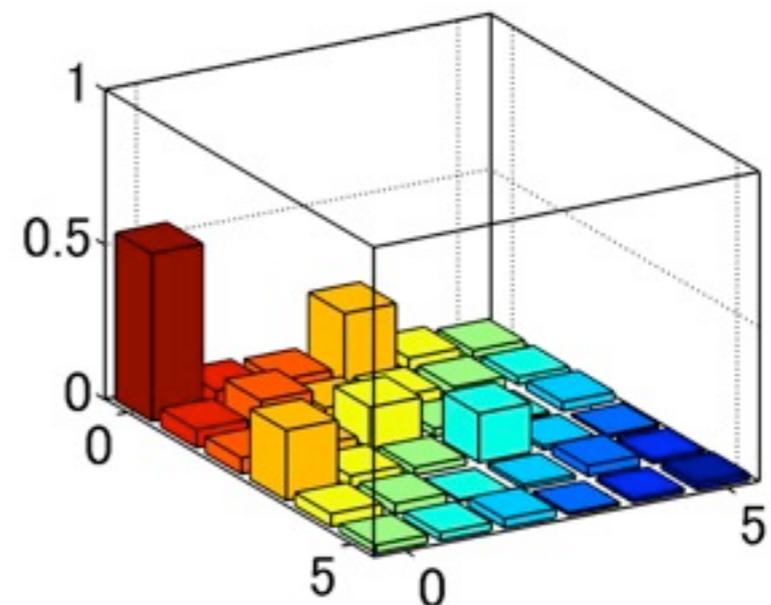
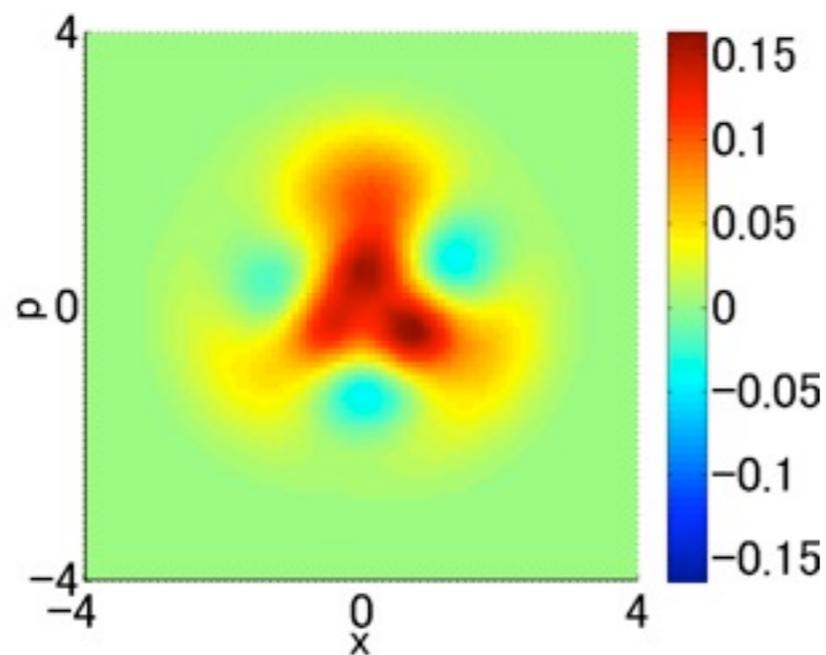
$$|0\rangle + \frac{\alpha^3}{\sqrt{6}}|3\rangle$$

Three-headed cat state

$$|\alpha\rangle + \left|\alpha e^{i\frac{2\pi}{3}}\right\rangle + \left|\alpha e^{-i\frac{2\pi}{3}}\right\rangle$$



without any correction!!



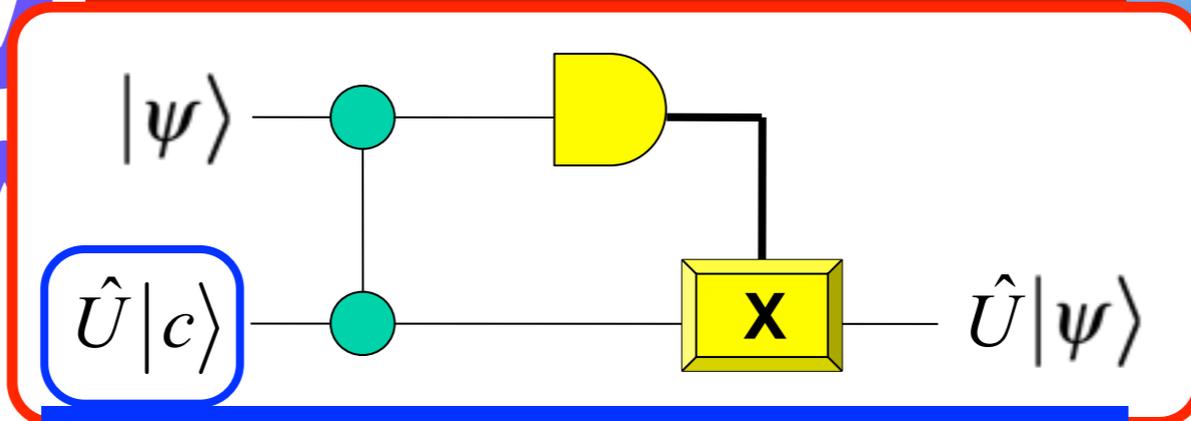
# Quantum version of coherent communication

***Ultimate goal***

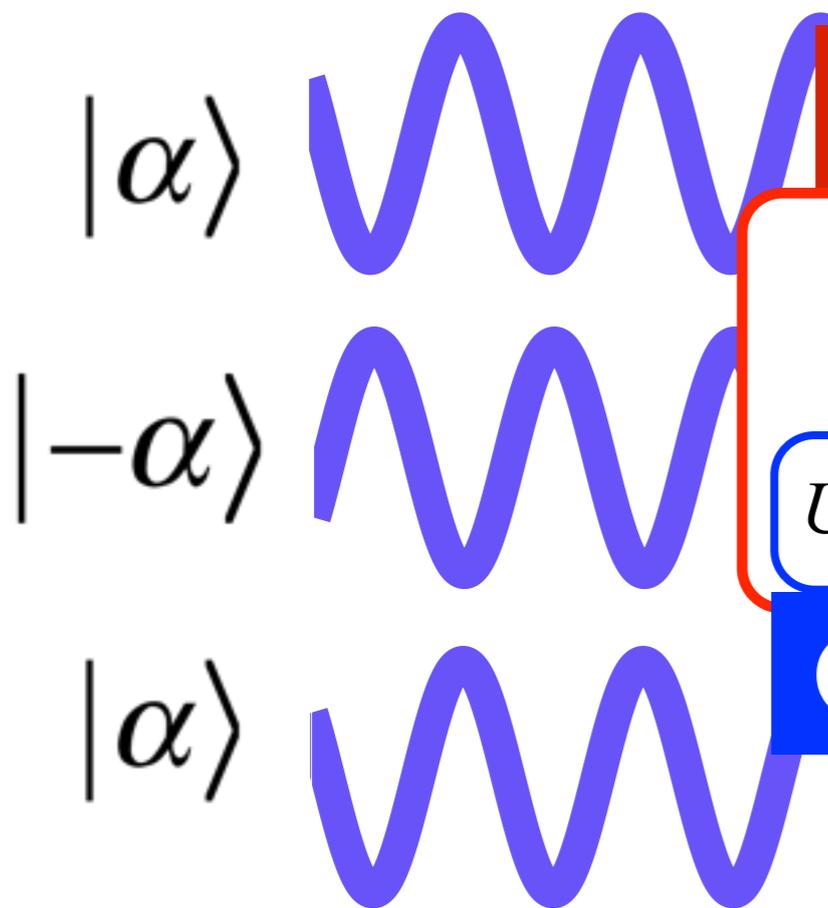
Quantum information processing (QIP)

Receiving station

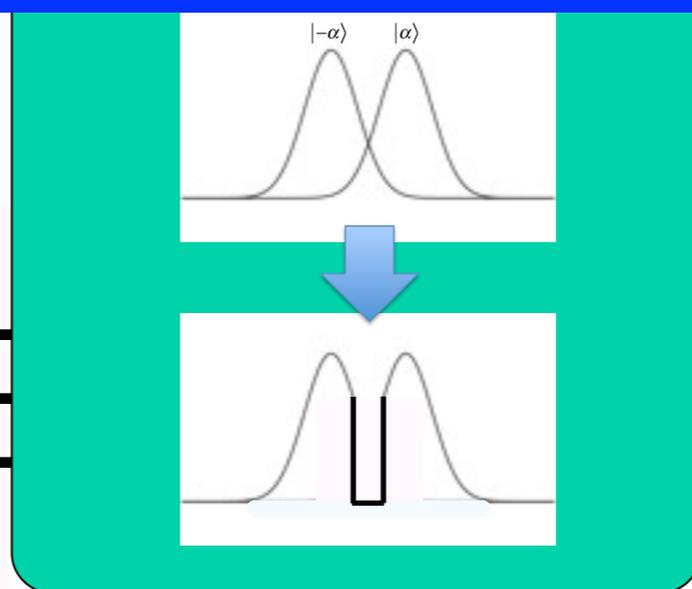
**Gate teleportation**



**Quantum memory**



**Ancilla**



Extract information beyond the Shannon limit

**We have to handle cat states of light!!**

