Junction of 5d CFT, 5-branes and AGT relation

Masato Taki RIKEN, Hashimoto Lab

based on
[L.Bao-V.Mitev-E.Pomoni-M.T-F.Yagi, arXiv:1310.3841]
[M.T, to appear]

2013. 10/24
@ Tokyo Univ.

Question:

Why 5d gauge theory?

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Why 5d gauge theory?

- non-renormalizable
- only cut-off theory?

well-defined QFT ?

well-defined QFT ?

One answer: UV fixed point

well-defined QFT ??

well-defined QFT ??

asymptotic safety

QED ?

scalar φ^4 theory ?

quantum gravity ??

castle in the air ?

castle in the air ?

No! Many 5d thys have non-Gaussian fixed points!

1. 5D SCFTs

5D gauge theories with 8 SUSY

↑

minimal SUSY in 5D

5D minimal SUSY gauge theory

→ non-renormalizable, IR free

5D minimal SUSY gauge theory

→ **non-renormalizable, IR free**

mere cut-off theory ?

theory is not defined ?

5D minimal SUSY gauge theory

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NO! Some of them have UV fixed points

5D minimal SUSY gauge theory

[Seiberg, ’96]

SU(2) gauge theory with 0, 1, 2, … , 7 quarks has UV fixed point.

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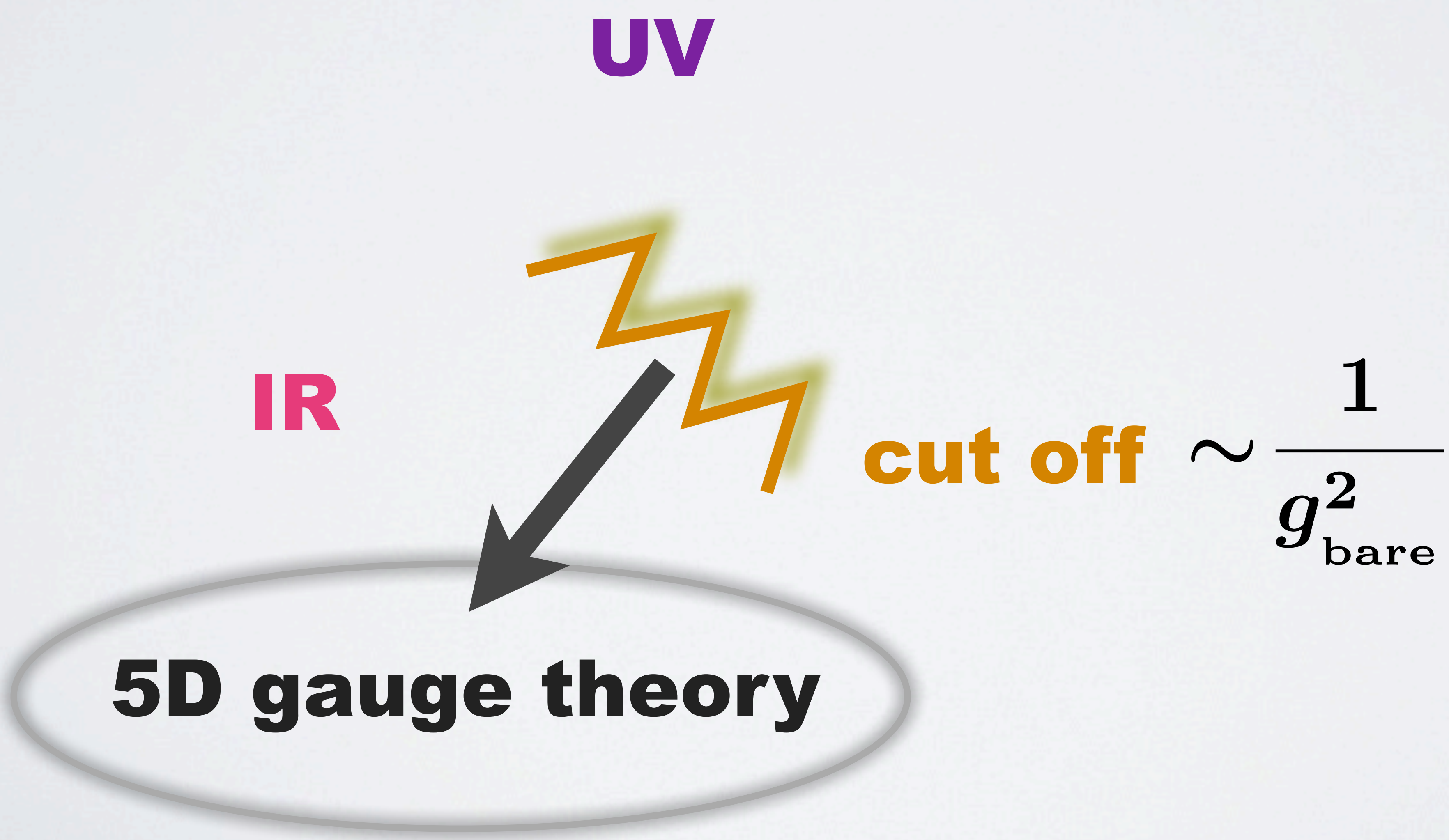

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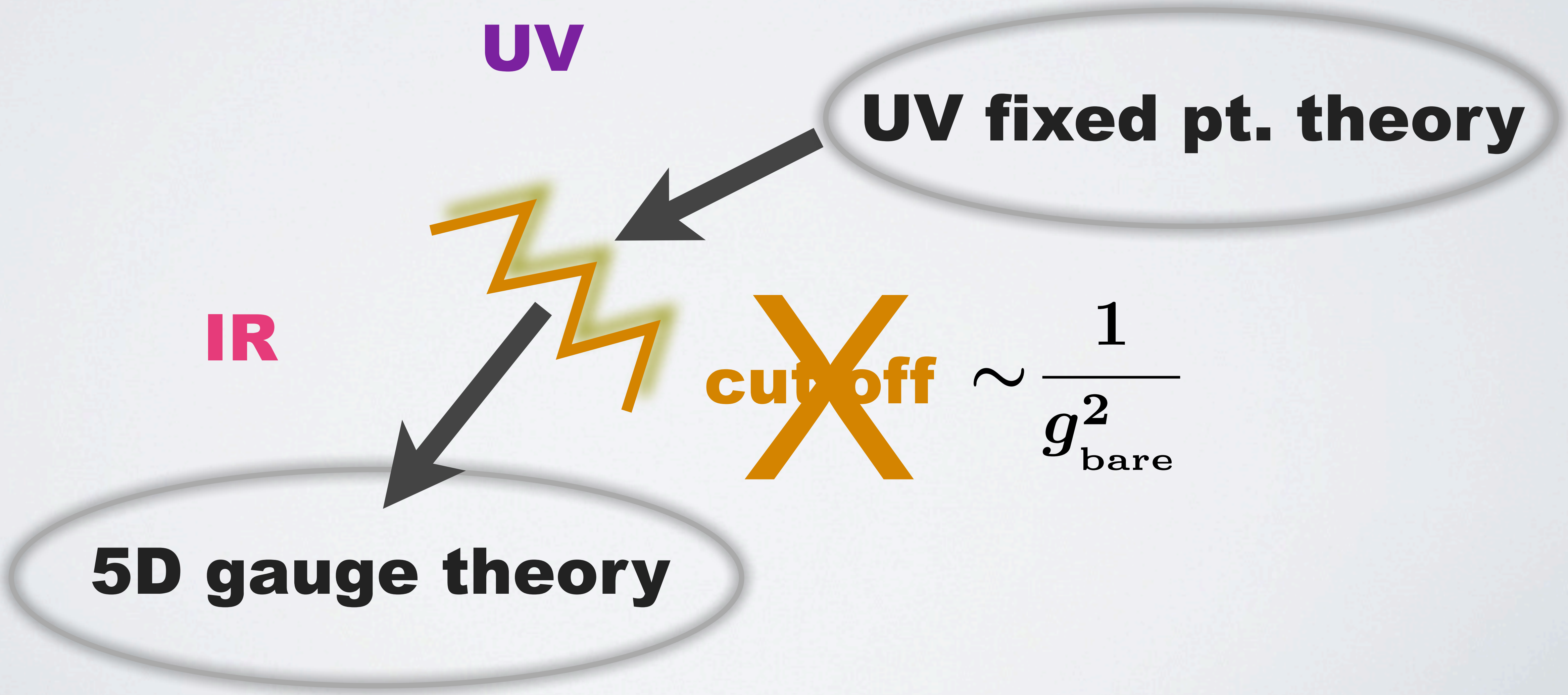

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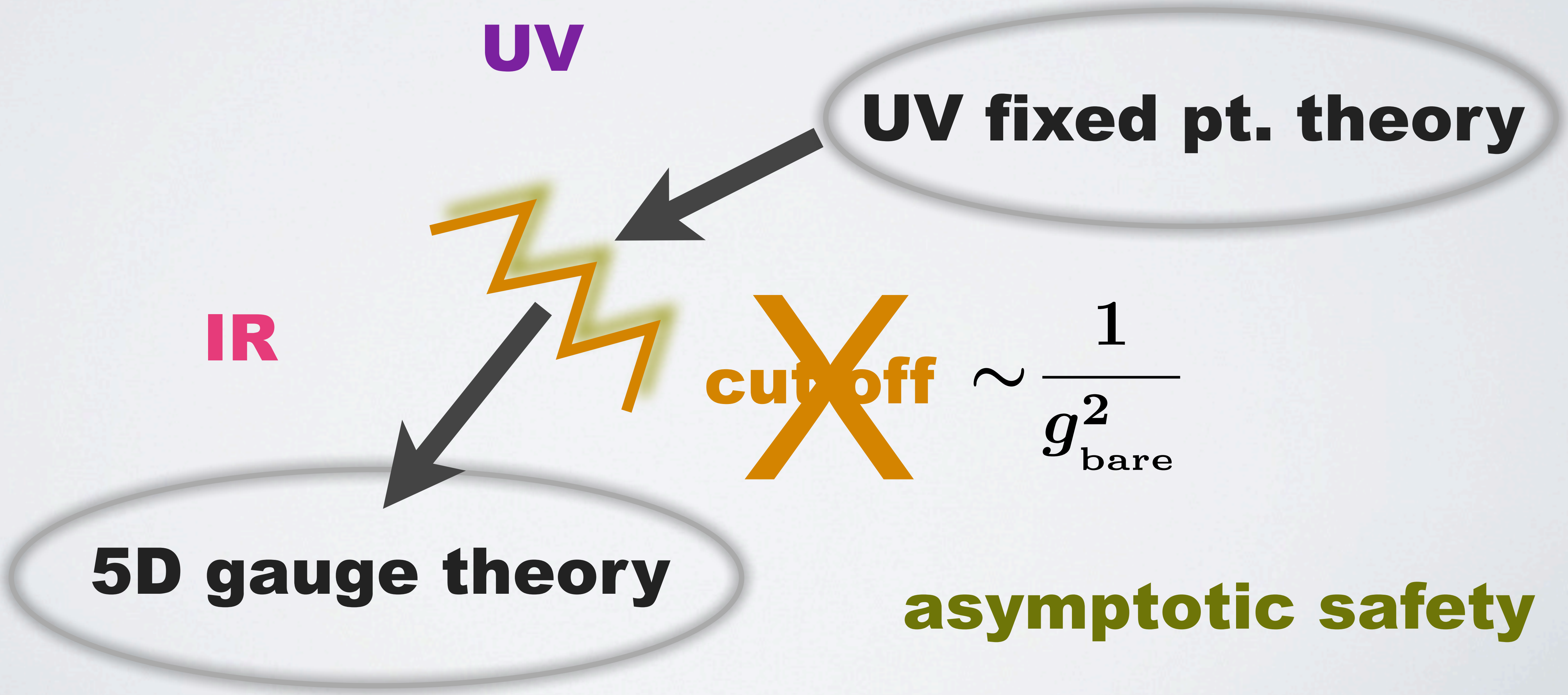
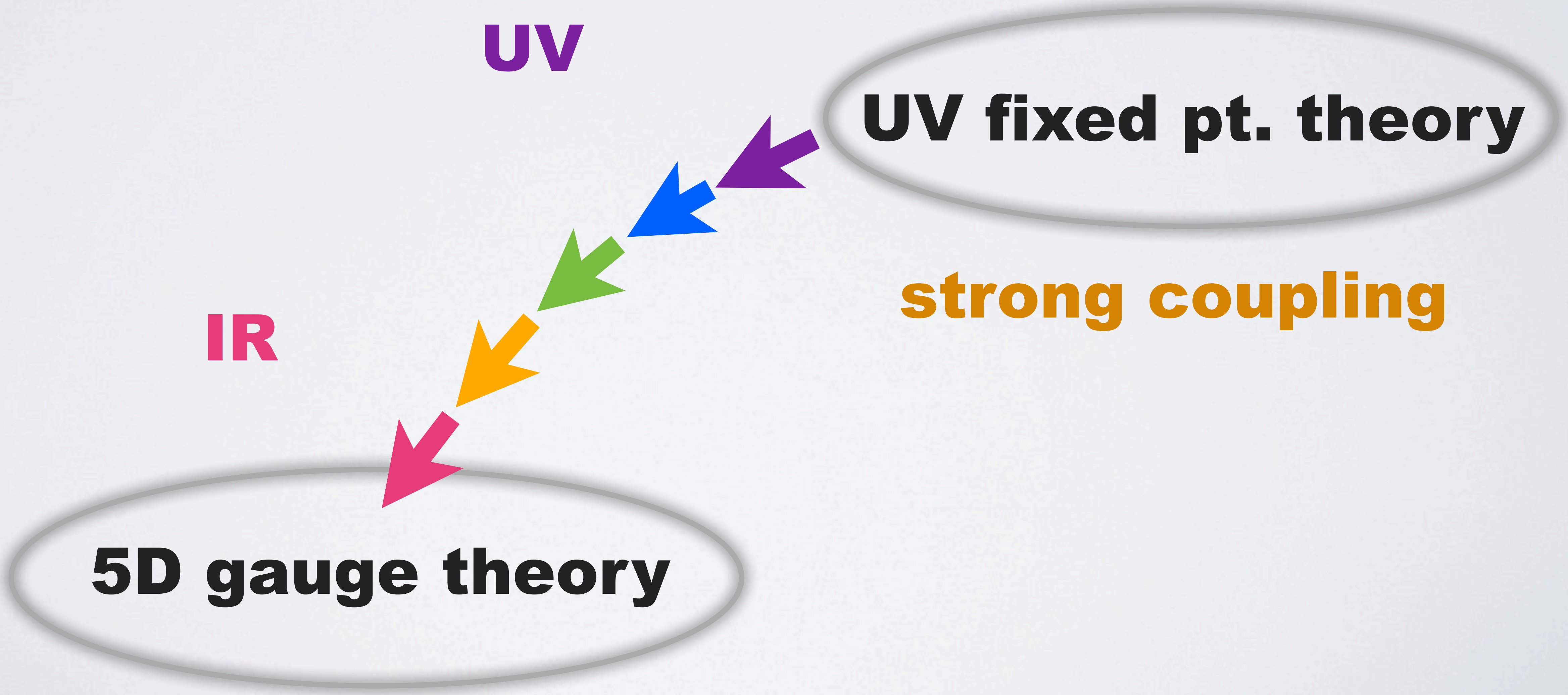


5D minimal SUSY gauge theory

we will study these UV fixed pt theories



5D SU(2) theory & flavor sym. of UV SCFT

$$N_f = 0 \qquad E_1 = SU(2)$$

$$N_f = 1 \qquad E_2 = SU(2) \times U(1)$$

$$N_f = 2 \qquad E_3 = SU(3) \times SU(2)$$

$$N_f = 3 \qquad E_4 = SU(5)$$

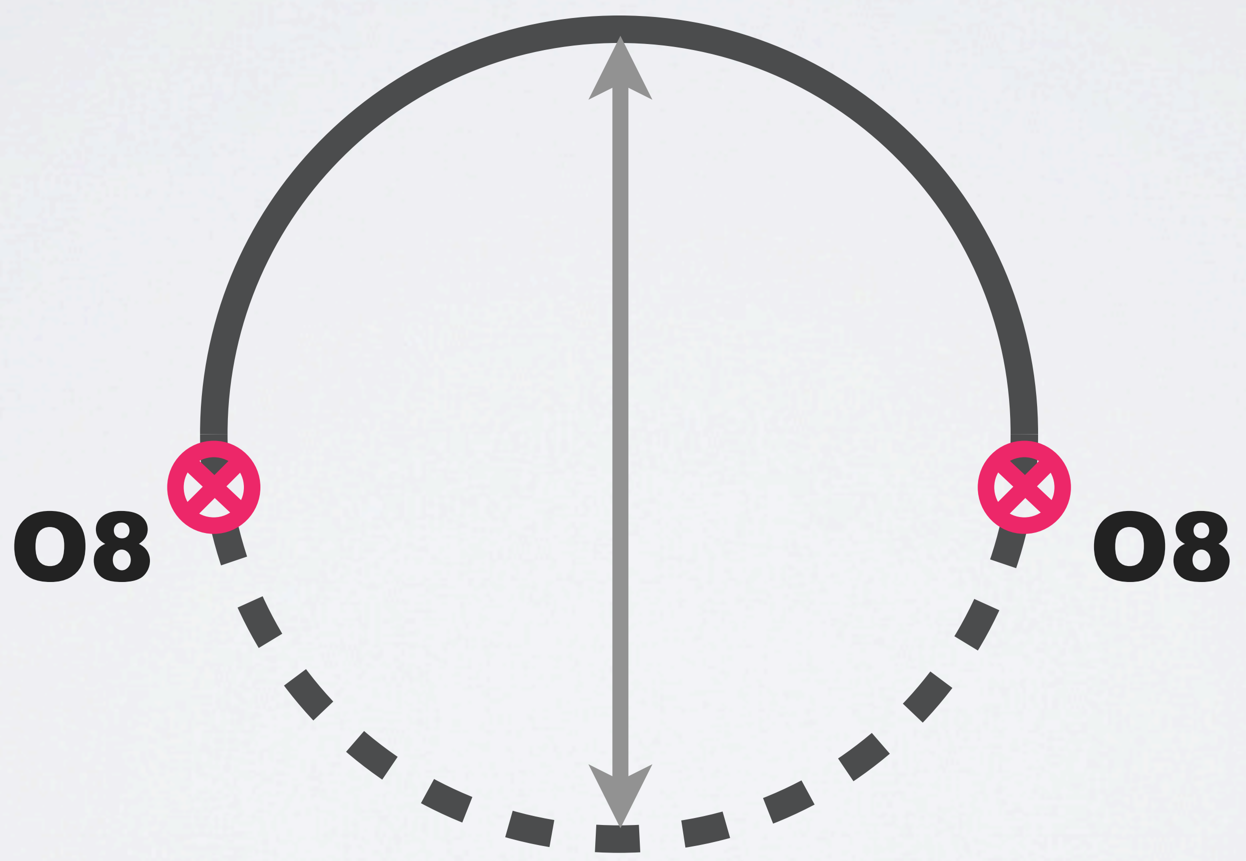
$$N_f = 4 \qquad E_5 = SO(10)$$

$$N_f = 5 \qquad E_6$$

$$N_f = 6 \qquad E_7$$

$$N_f = 7 \qquad E_8$$

E_n via TypeII on orbifold [Seiberg, '96]



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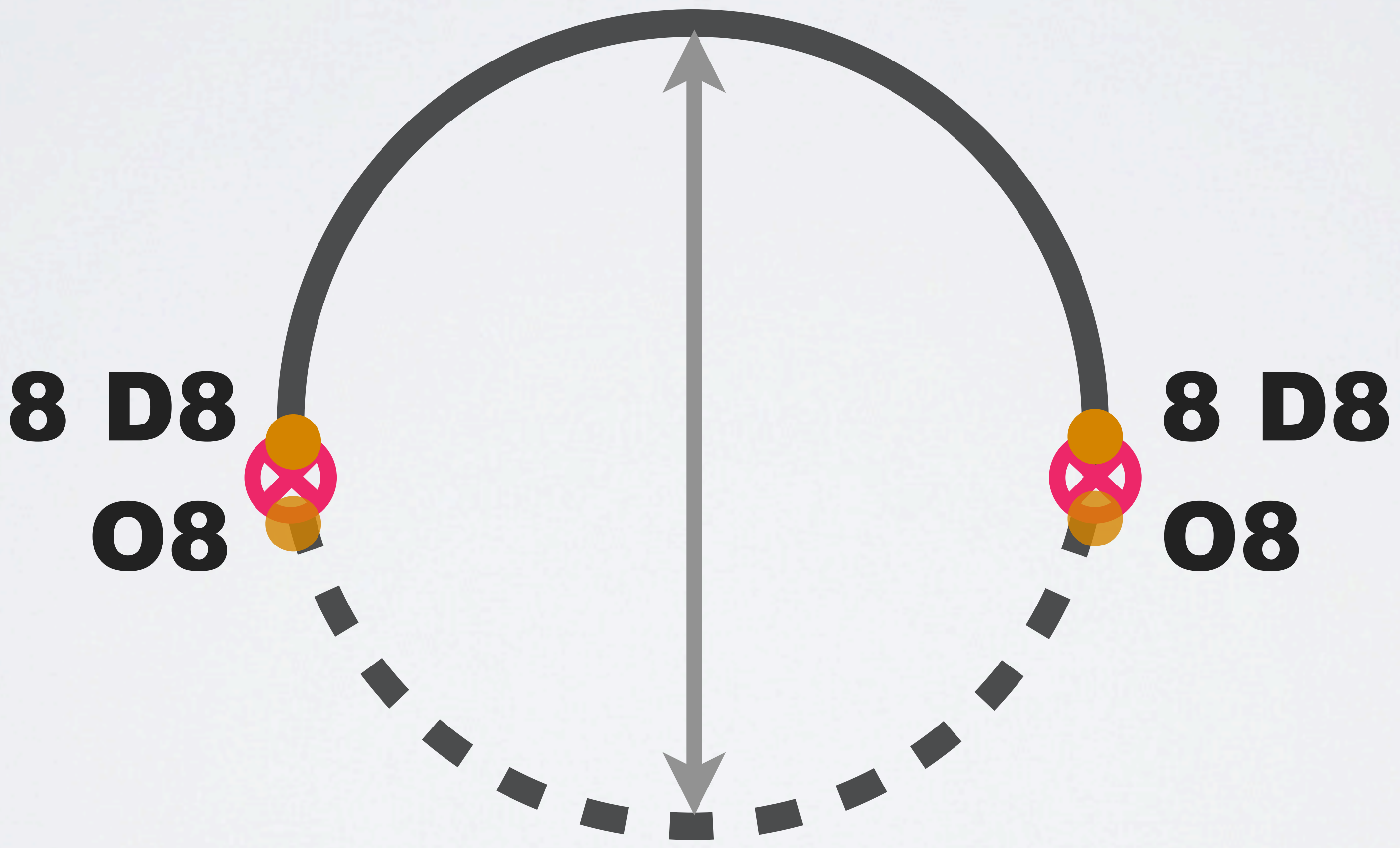

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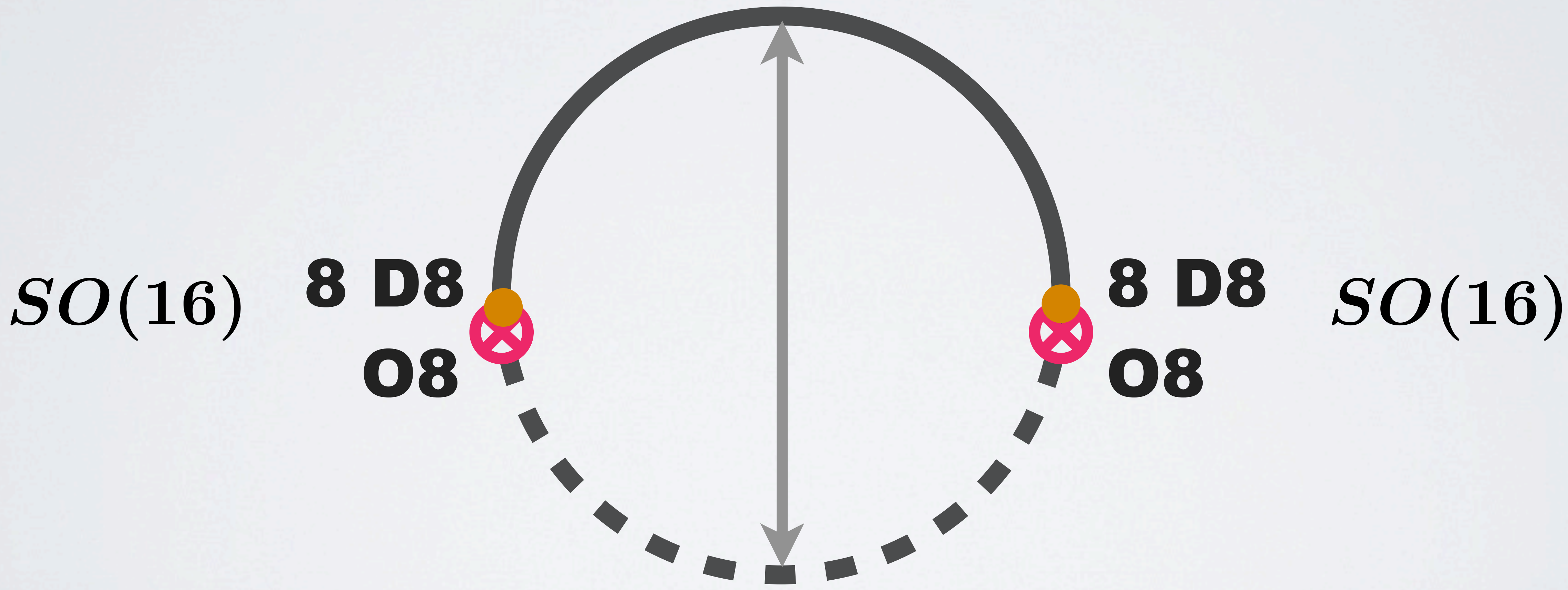

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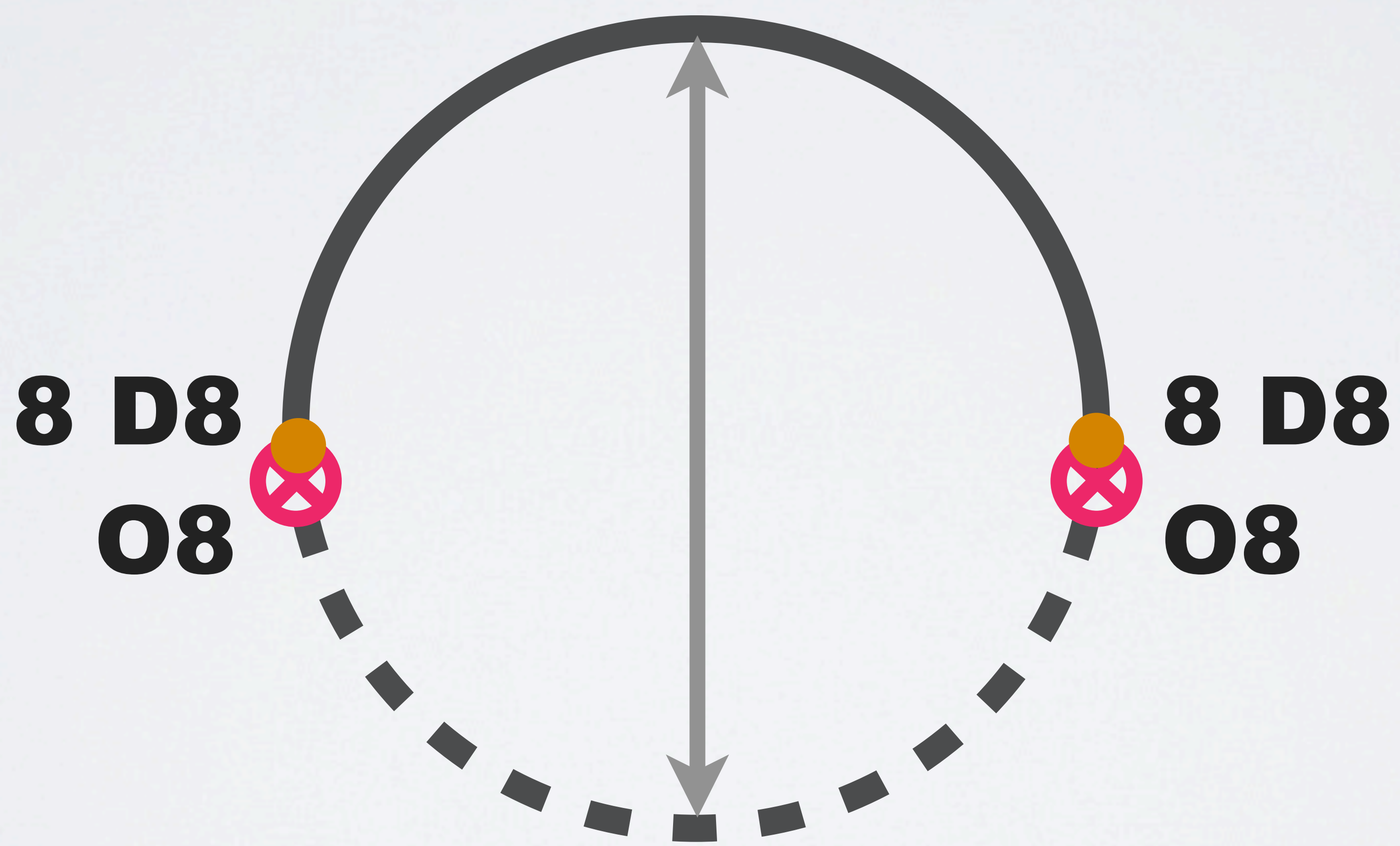
$SO(16)$

$\mathbf{120}$



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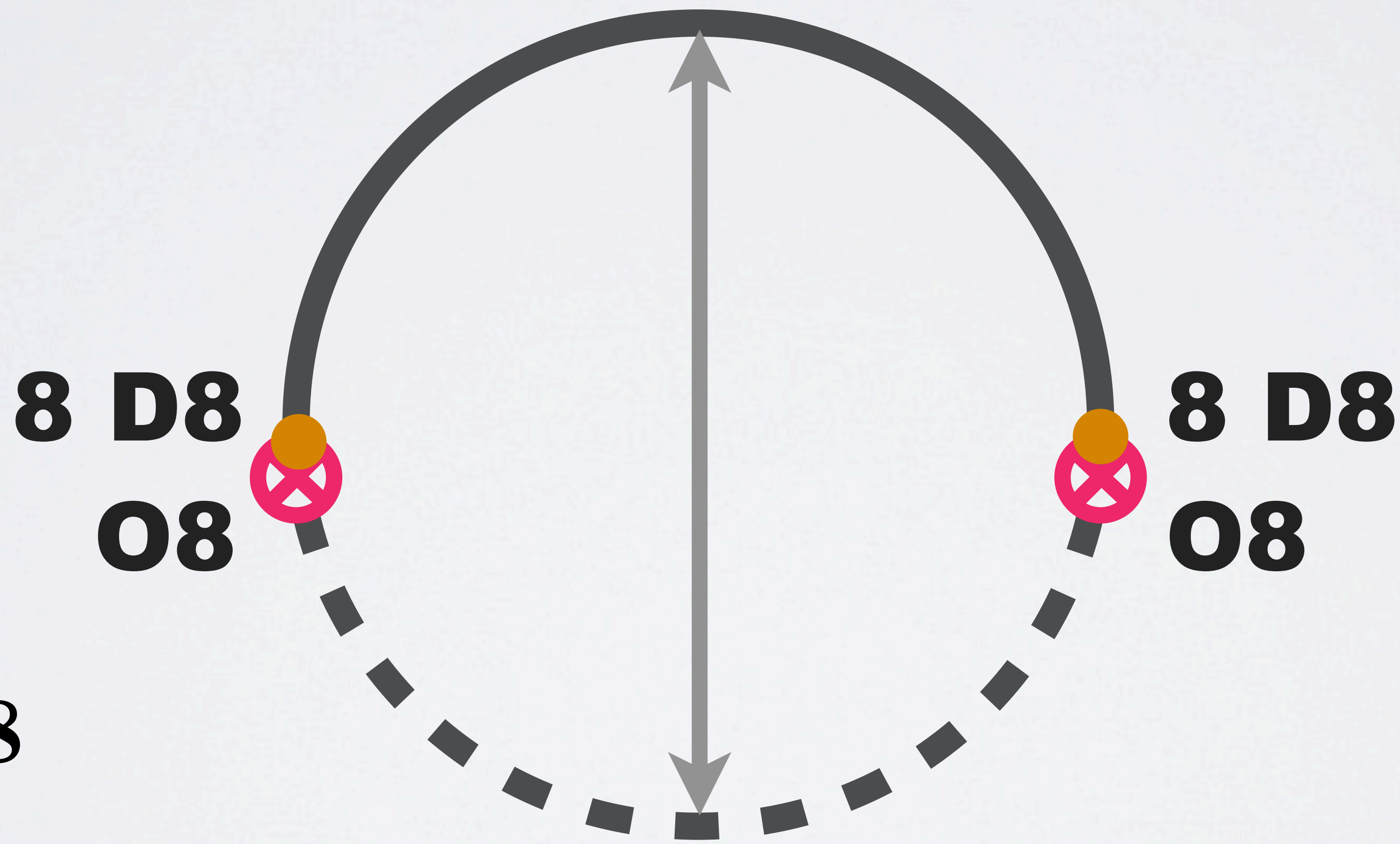

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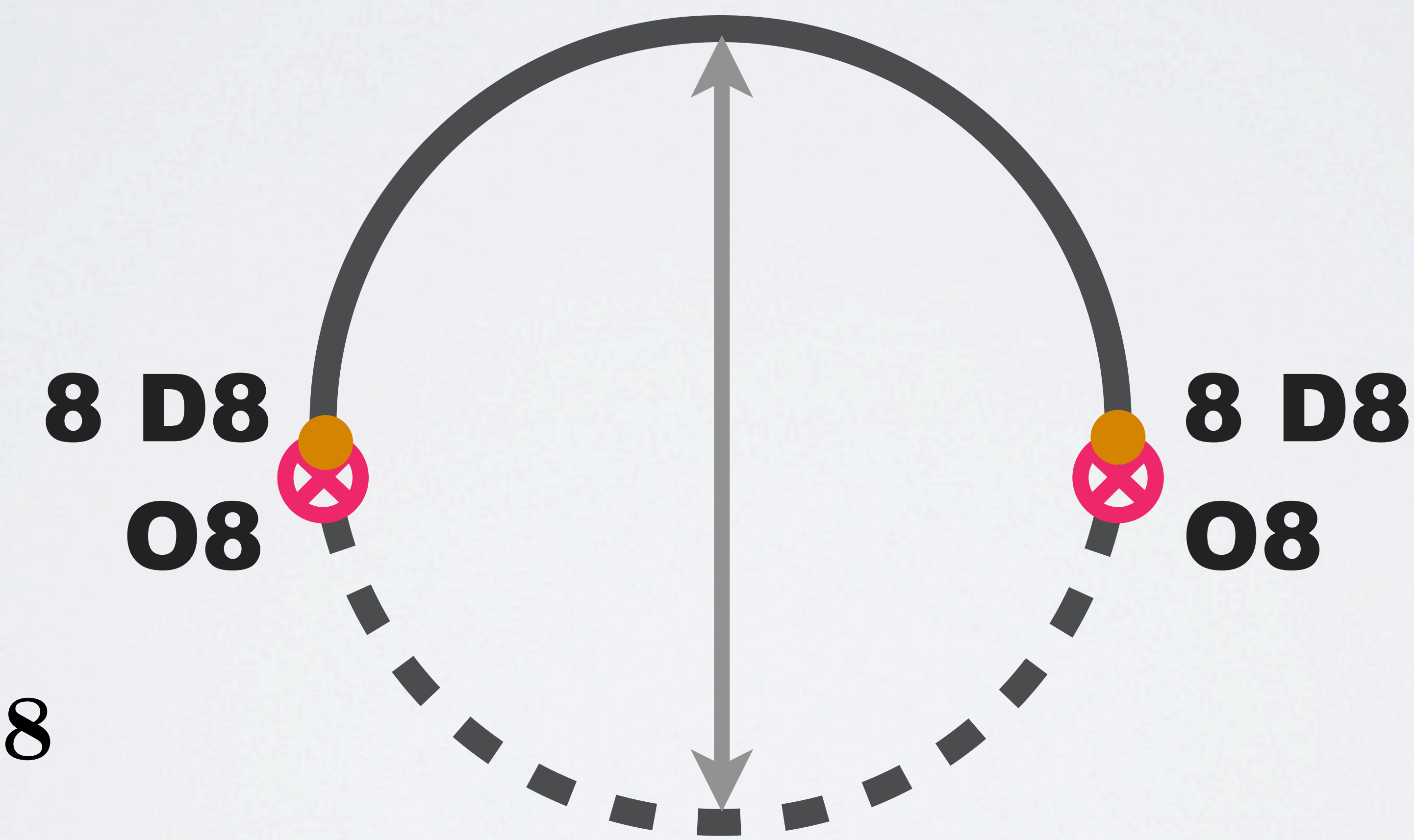
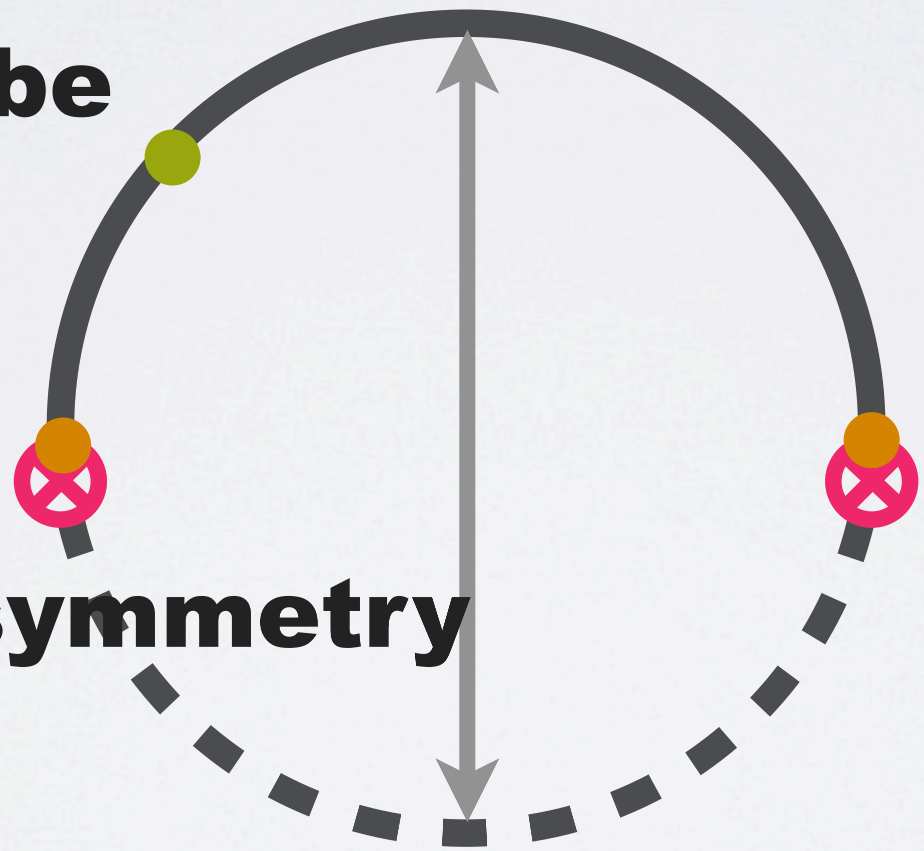


$SO(16)$

$\begin{cases} 120 \\ \textbf{D0} \;\; 128 \end{cases}$

$\longrightarrow$ 248 : E_8 !

E_n via TypeII on orbifold [Seiberg, '96]



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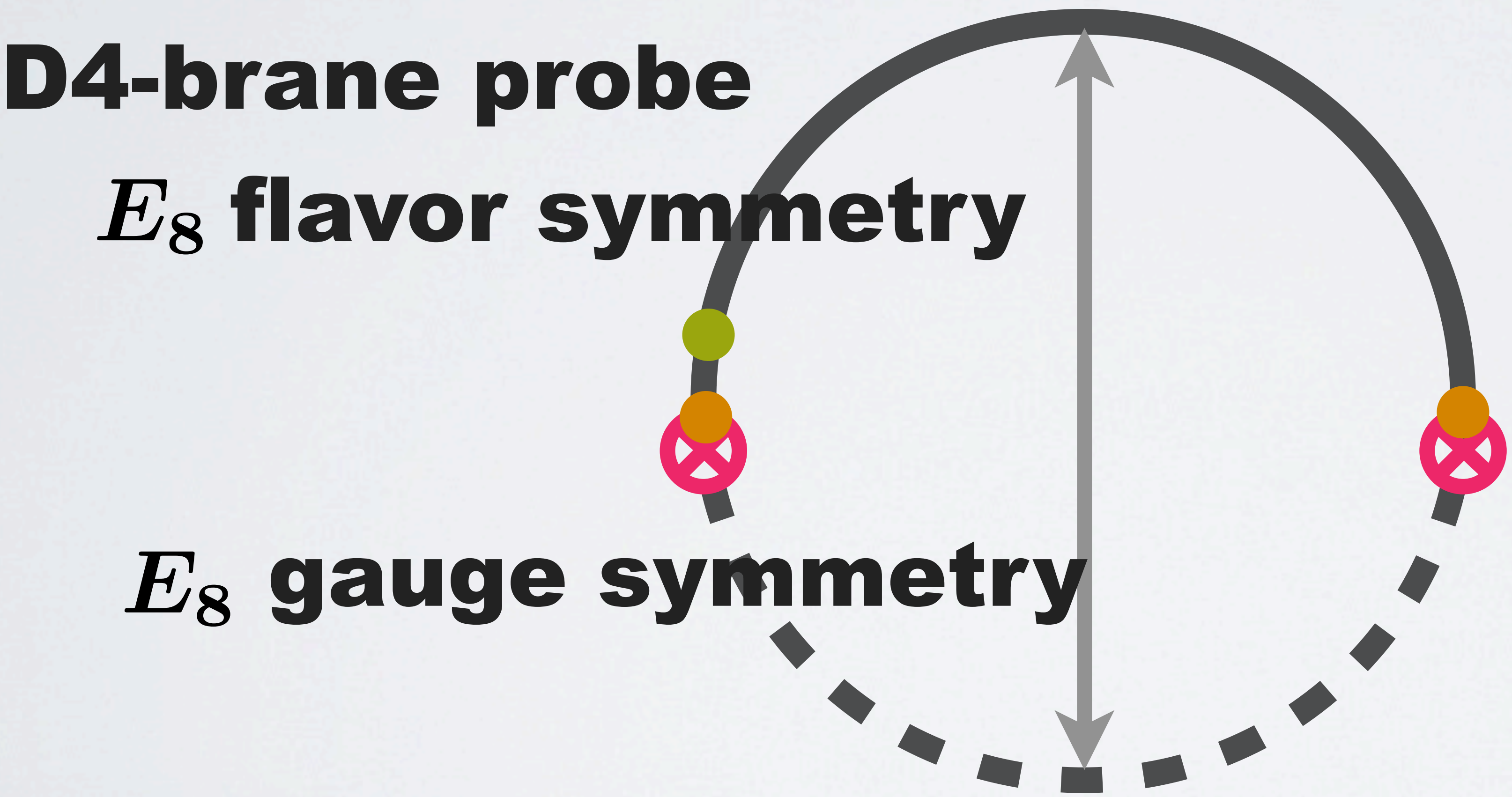

5d theory via (p,q) 5-brane web

[Aharony-Hanany, '97]

We can construct these 5d thys by using (p,q) 5-brane web in Type IIB string

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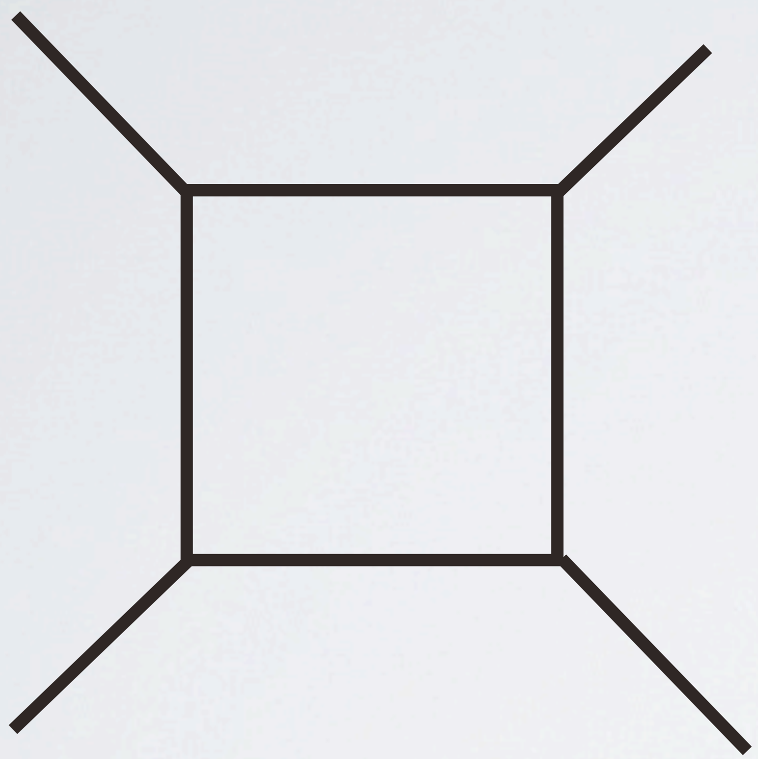
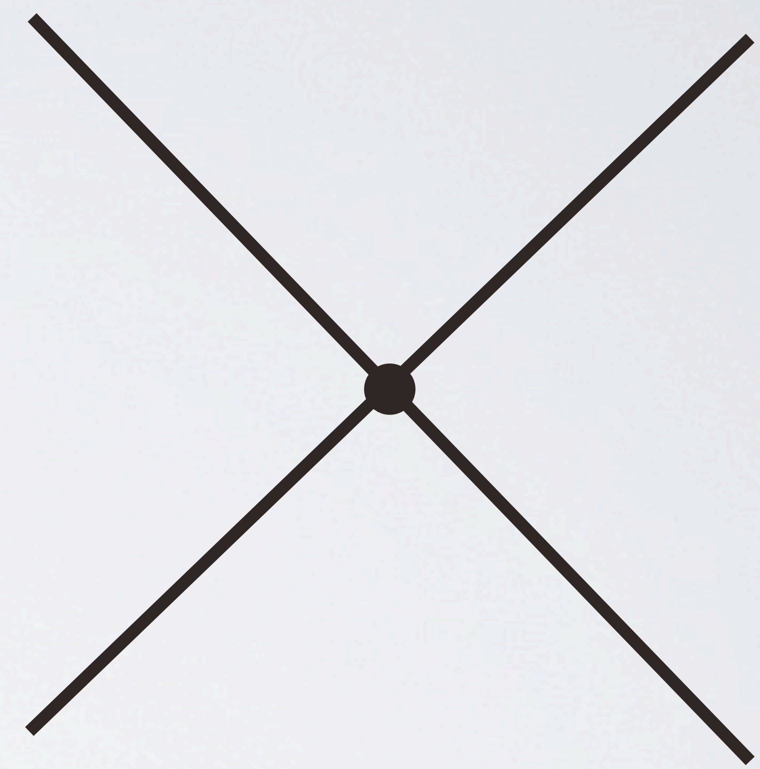
(1,0) 5-brane : D5-brane

(0,1) 5-brane : NS5-brane

pure SU(2) YM [Aharony-Hanany, ’97]

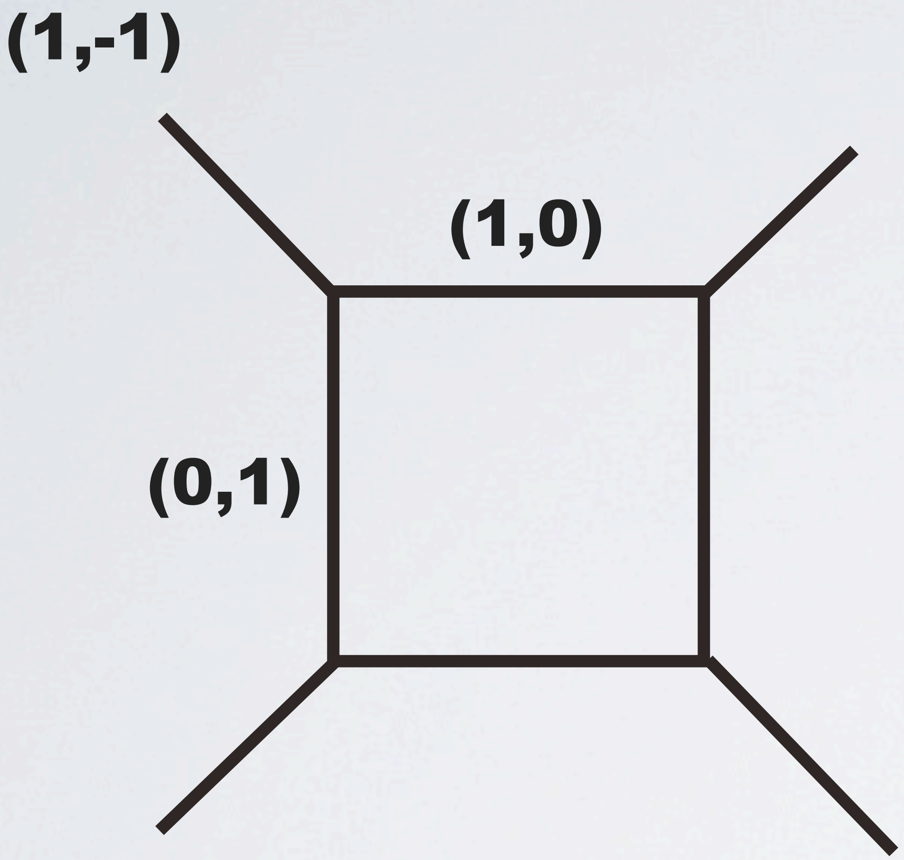


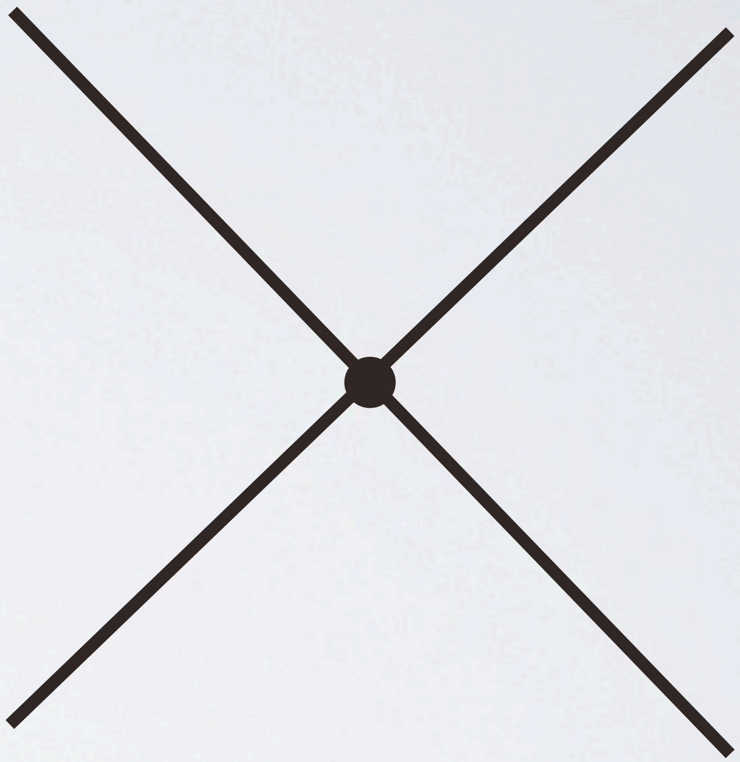


5d UV fixed pt thy

E_1 SCFT

pure SU(2) YM [Aharony-Hanany, '97]





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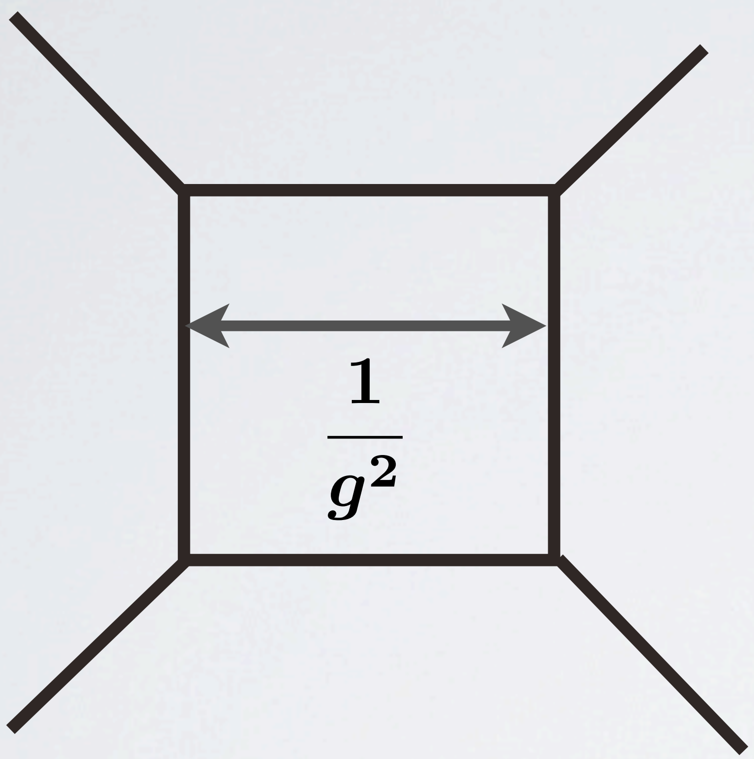
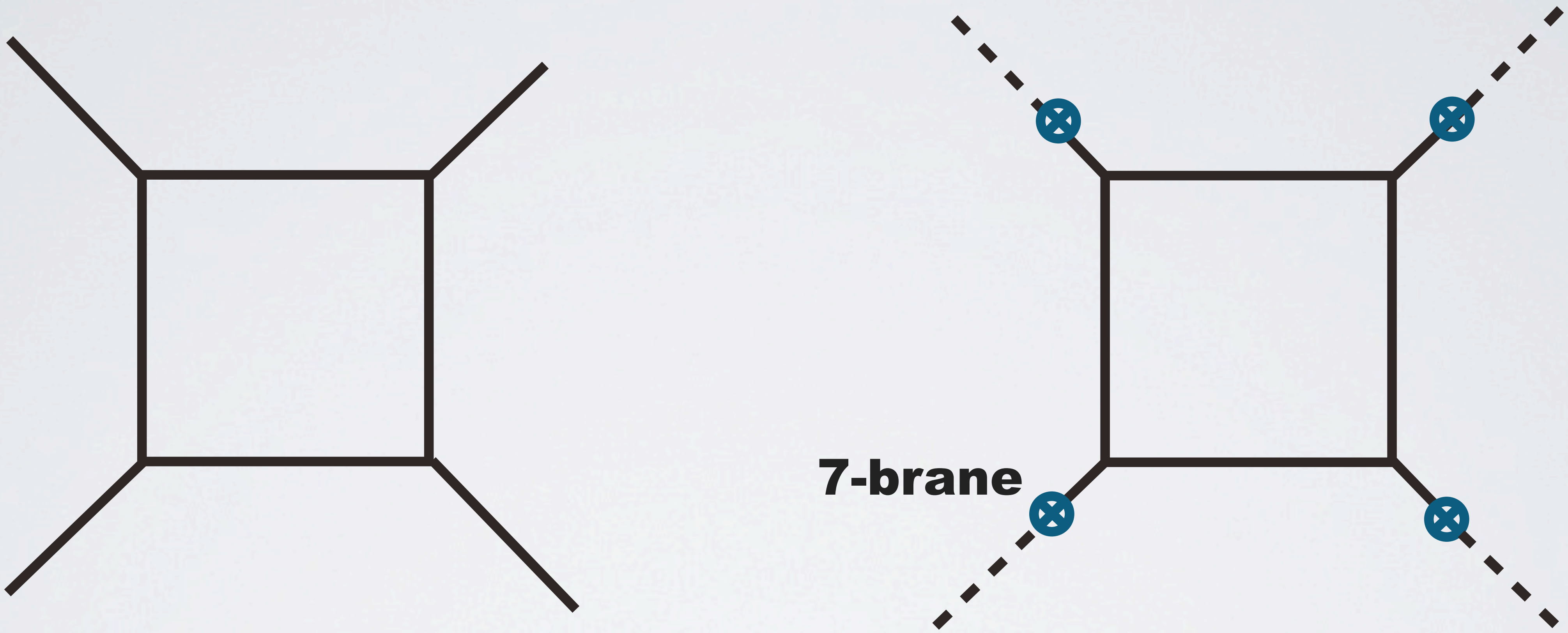


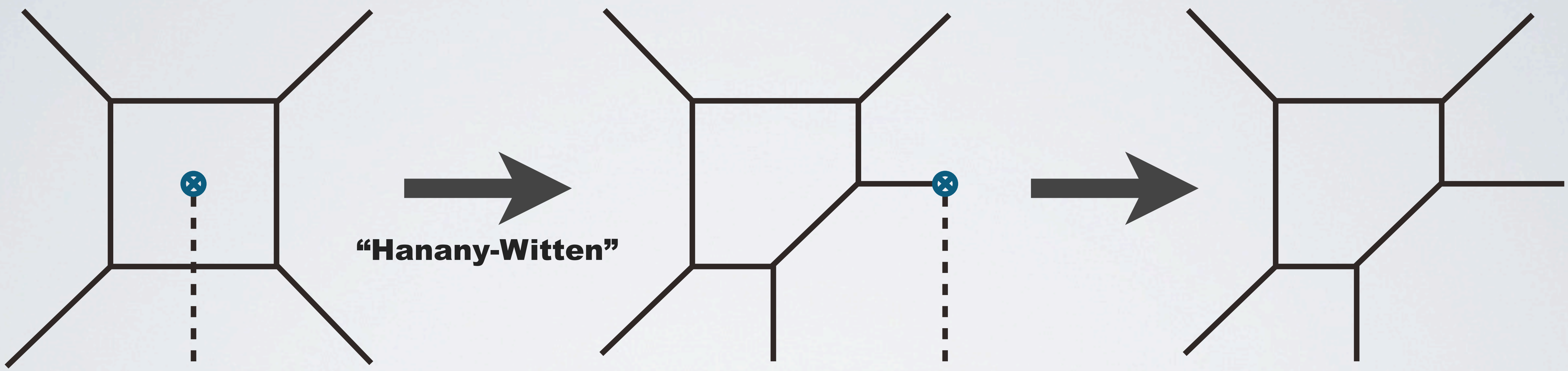
strongly-coupled

E_1 SCFT

pure SU(2) YM [DeWolfe-Hanany-Iqbal-Katz, ’99]



N_f=1 SU(2) SQCD [DeWolfe-Hanany-Iqbal-Katz, '99]



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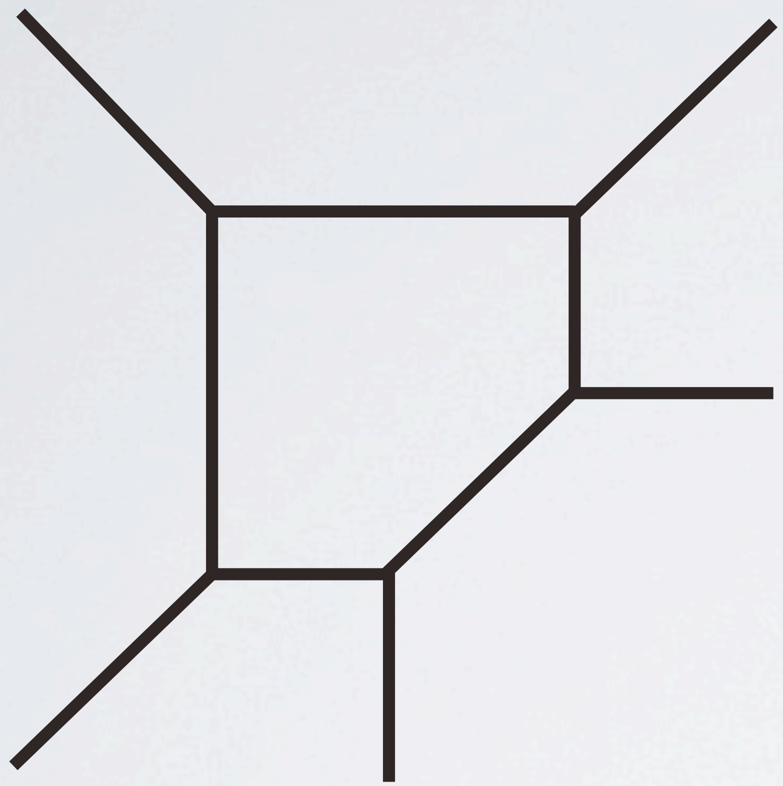

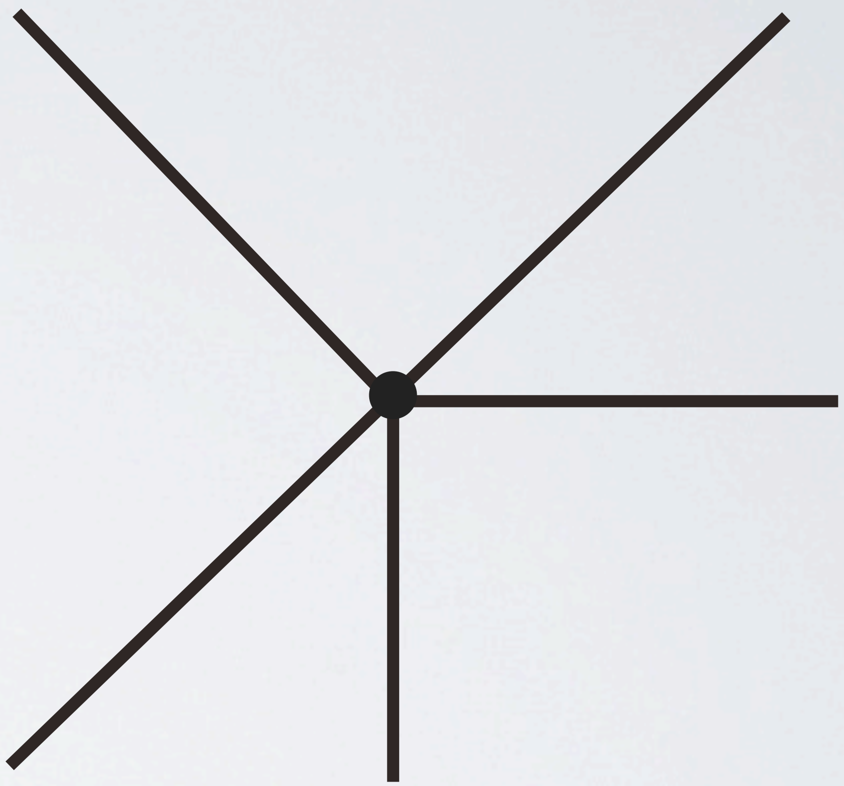

5d UV fixed pt thy

E_2 SCFT

5d theory via Calabi-Yau compactification

We can **also** construct them by using
M-thy on **toric geometry (web)**

5d theory via Calabi-Yau compactification

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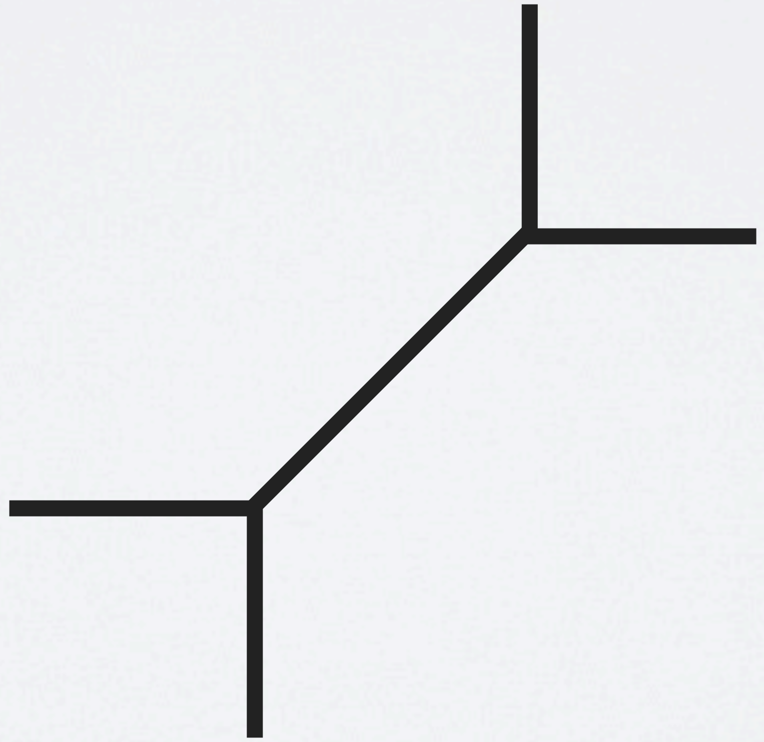
resolved conifold $A_1 B_2 - A_2 B_2 = 0$



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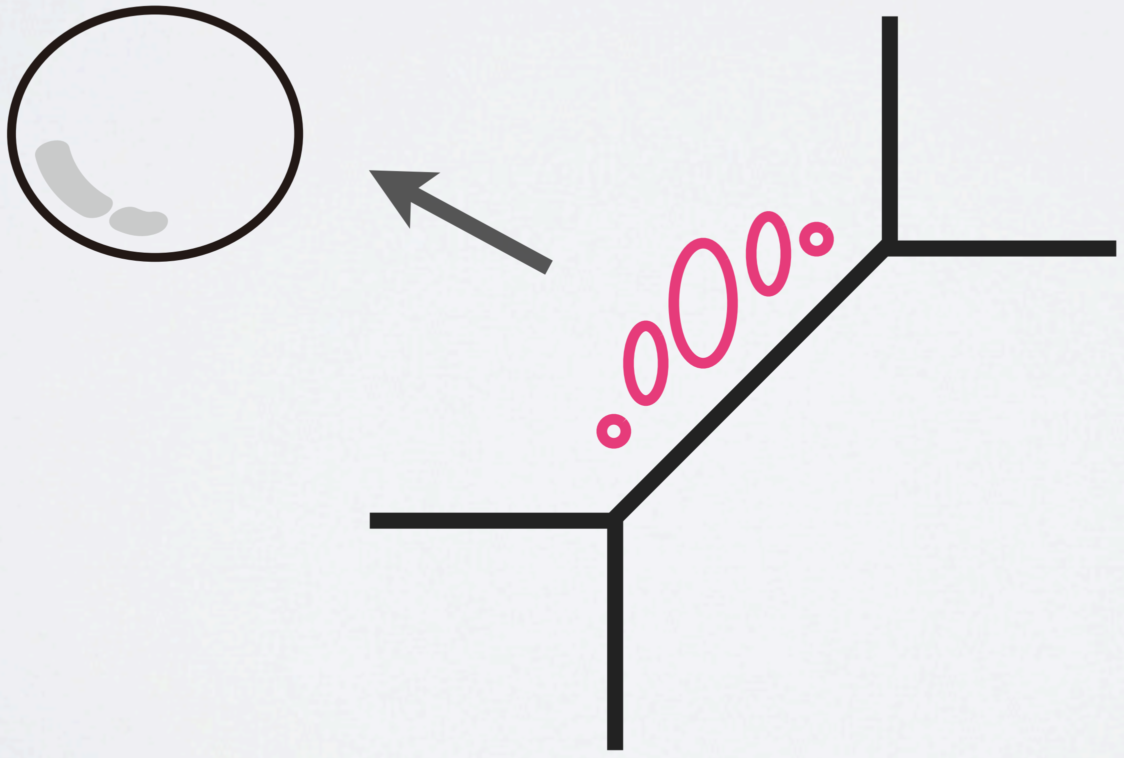

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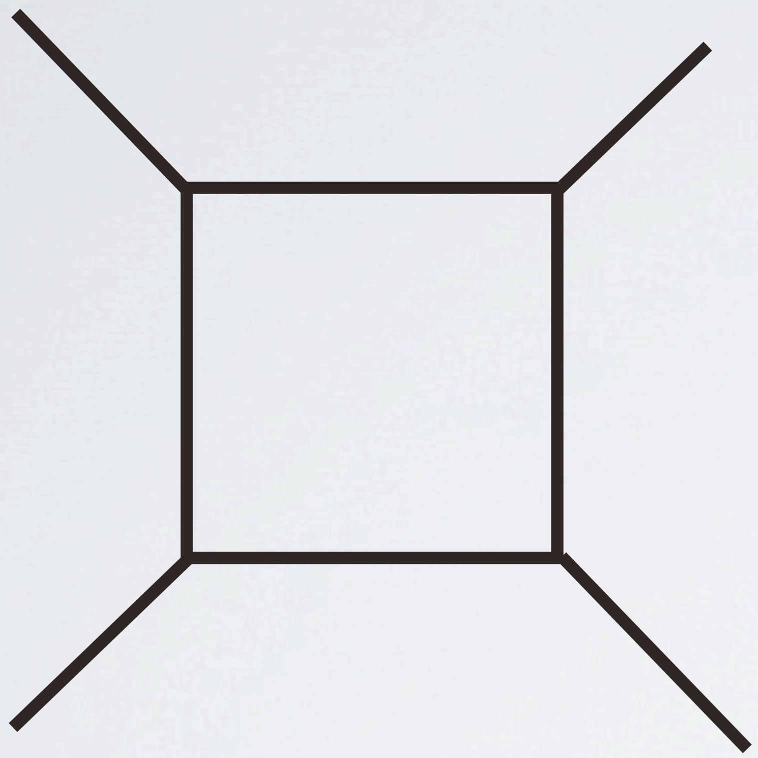
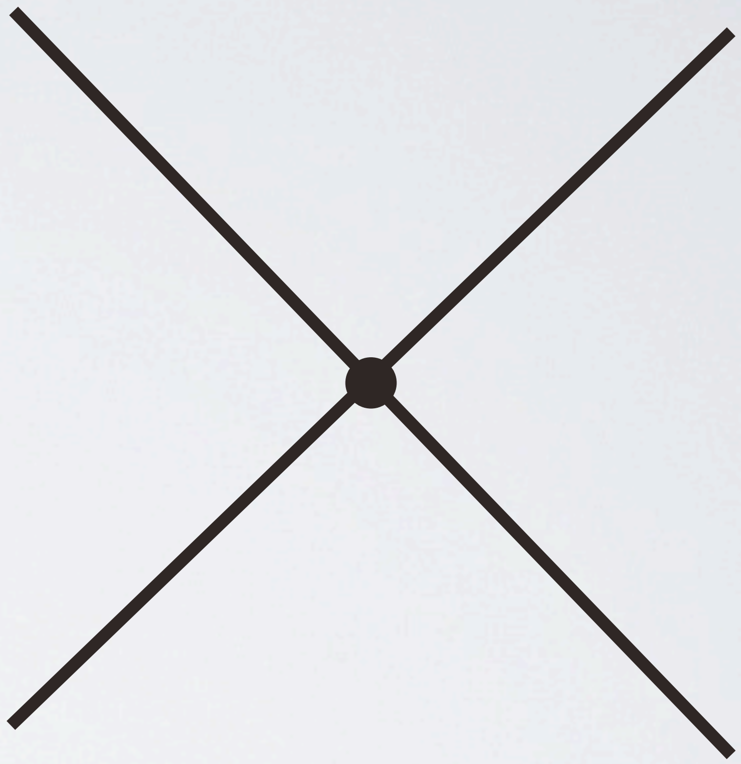
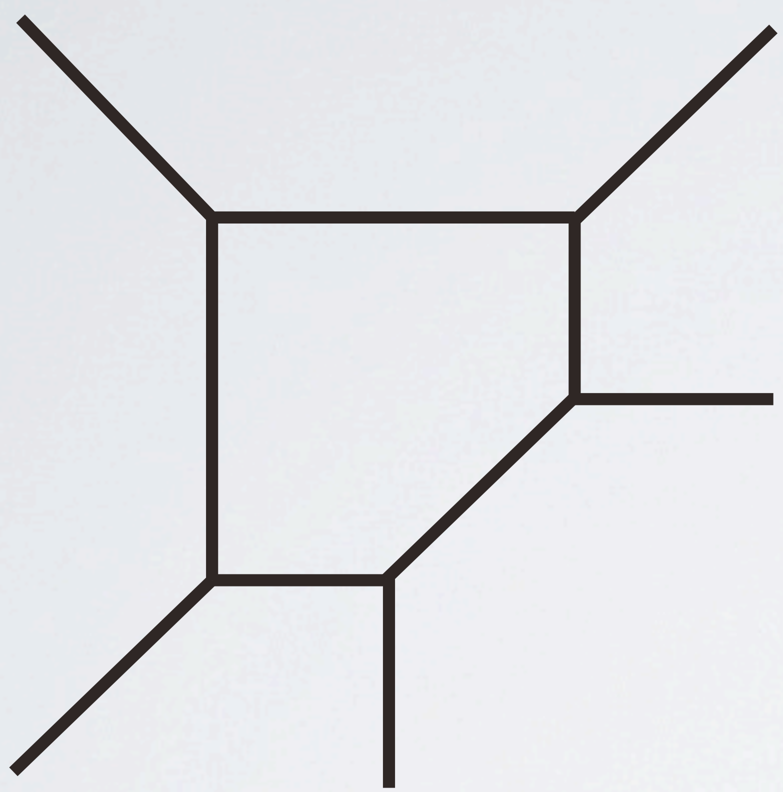
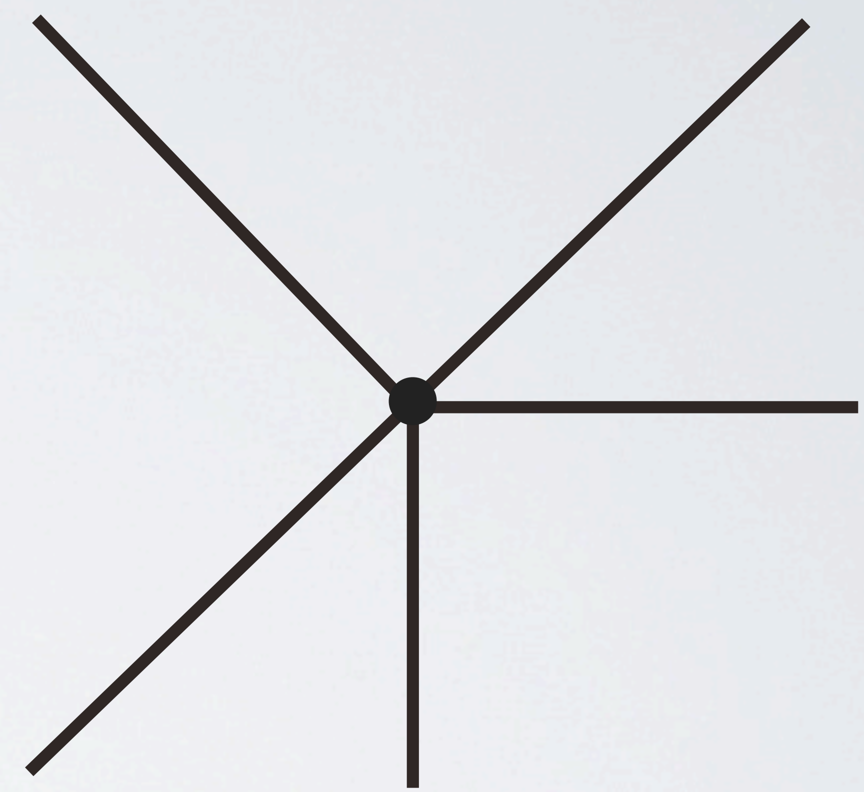




E$_1$ SCFT

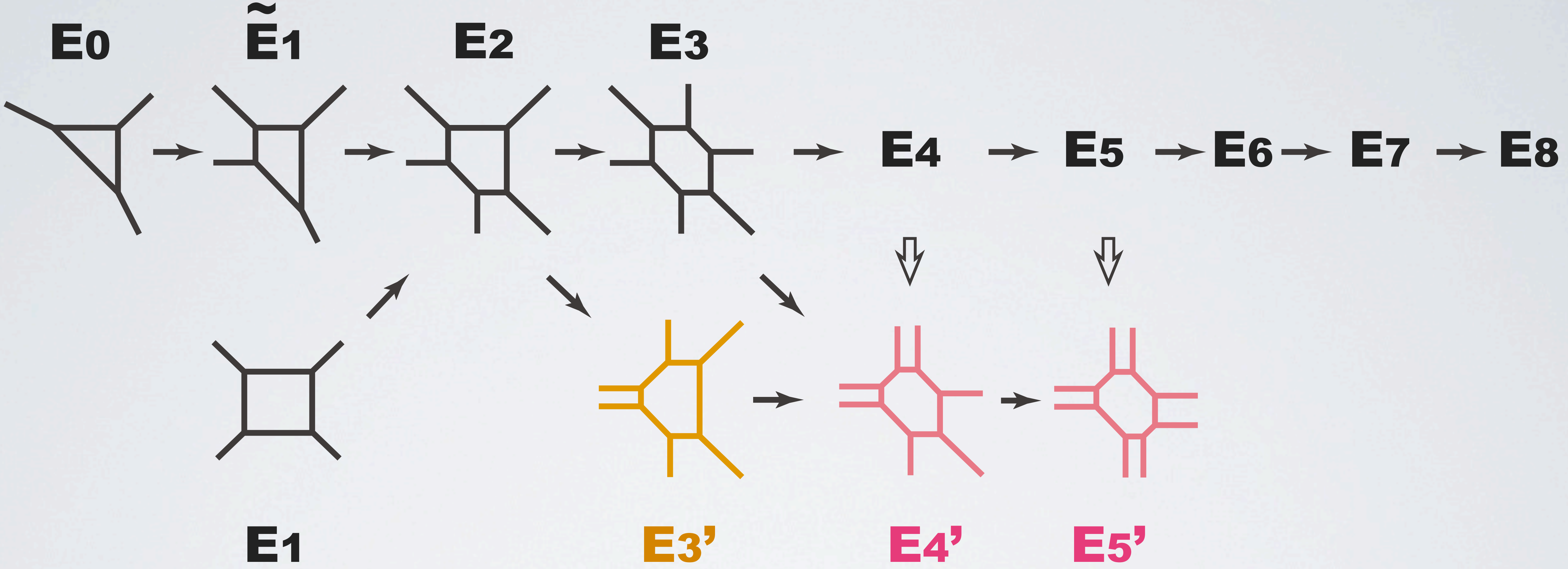
N_f =1 SU(2) SQCD





E_2 SCFT

Possible theories of this sequence



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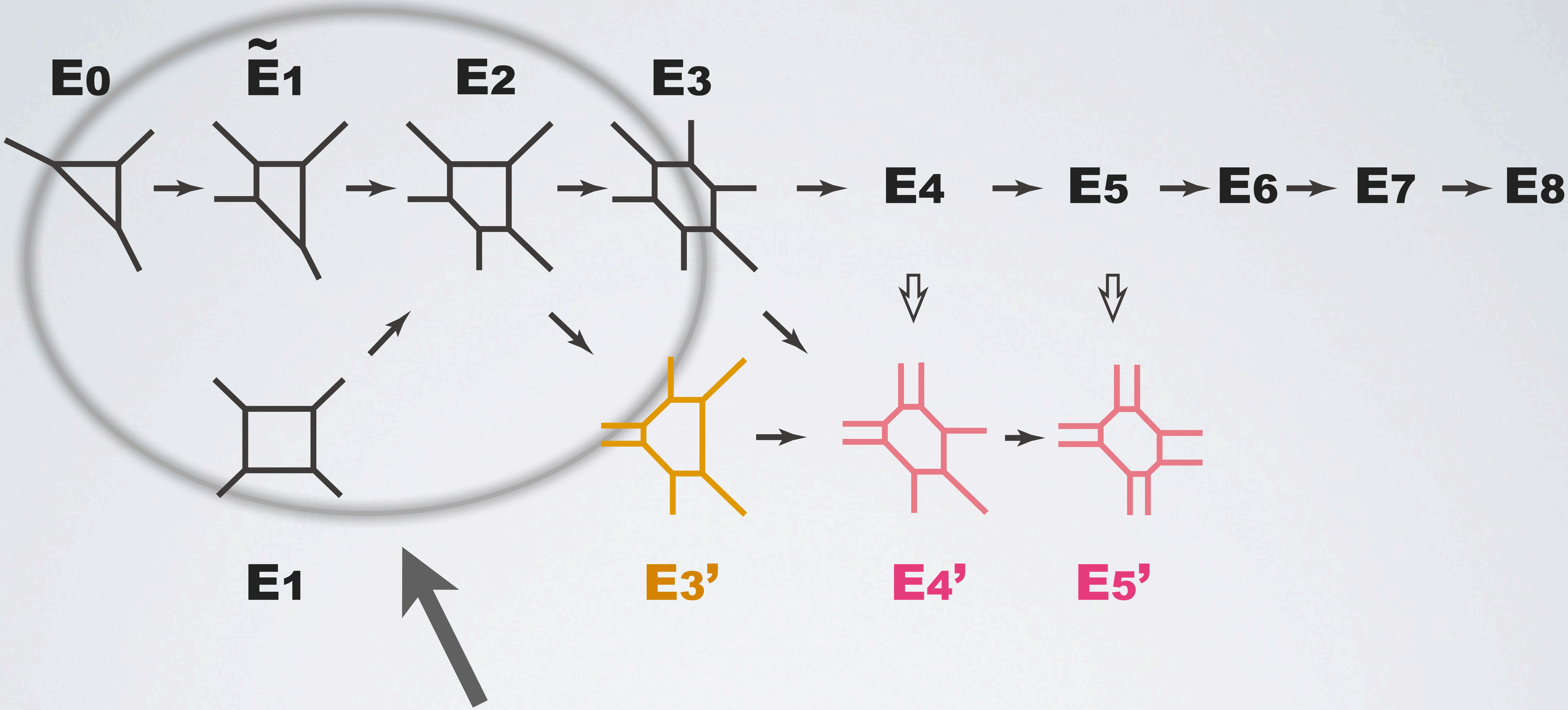
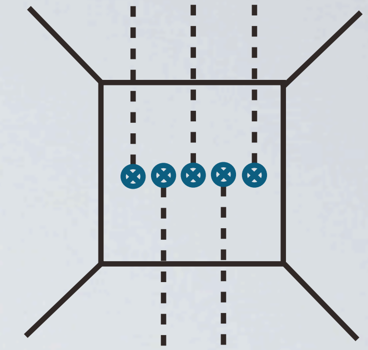
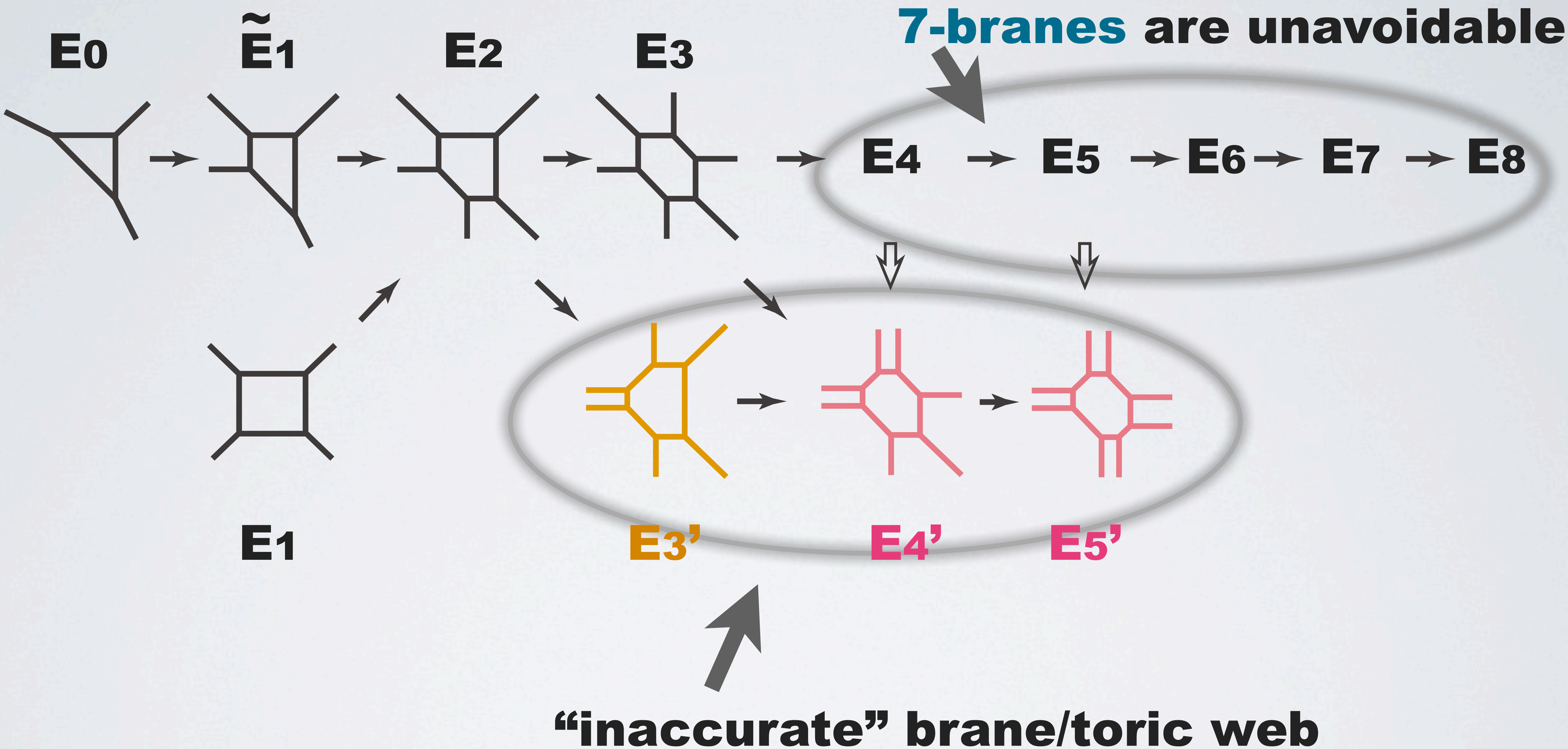


we can compute partition function
by topological vertex formalism

Possible theories of this sequence





2. Superconformal Index of 5D SCFTs

Superconformal Index

bosonic part of 5d N=1 SCA

$$SO(2,5) \times SU(2)_R$$

Superconformal Index

bosonic part of 5d N=1 SCA

$$SO(2,5) \times SU(2)_R$$

$Q = Q^{\mathbf{1}}_{\mathbf{2}}$ **a super charge**

$S = S^{\mathbf{2}}_{\mathbf{1}}$ **a superconformal charge**

Superconformal Index

bosonic part of 5d N=1 SCA

$$SO(2,5) \times SU(2)_R$$

$$Q = Q_2^1$$

$$S = S_1^2$$

← $SU(2)_R$ **index**

Superconformal Index

bosonic part of 5d N=1 SCA

$$SO(2,5) \times SU(2)_R$$

$$Q = Q^{\mathbf{1}}_{\mathbf{2}}$$

$$S = S^{\mathbf{2}}_{\mathbf{1}}$$

spinor index for 5d Lorentz

Superconformal Index

bosonic part of 5d N=1 SCA

$$SO(2,5) \times SU(2)_R$$

$$Q = Q^{\mathbf{1}}_{\mathbf{2}}$$

$$S = S^{\mathbf{2}}_{\mathbf{1}}$$

$$\Delta = \{Q, S\} = \epsilon_0 - 2j_1 - 3R \geq 0$$

1/8 BPS with dim. $\epsilon_0 = 2j_1 + 3R$

Superconformal Index

bosonic part of 5d N=1 SCA

$$SO(2,5) \times SU(2)_R$$

$$Q = Q^1_2$$

$$S = S^2_1$$

$$\Delta = \{Q, S\} = \epsilon_0 - 2j_1 - 3R \geq 0$$

1/8 BPS with dim. $\epsilon_0 = 2j_1 + 3R$

Let's count these states

Superconformal Index

$$\boldsymbol{I} = \mathrm{Tr}\,(-\mathbf{1})^{\boldsymbol{F}} = \boldsymbol{N}_{\text{bosons}} - \boldsymbol{N}_{\text{fermions}}$$

Superconformal Index

$$\boldsymbol{I} = \mathrm{Tr}\,(-1)^{\boldsymbol{F}} = \boldsymbol{N}_{\mathrm{bosons}} - \boldsymbol{N}_{\mathrm{fermions}}$$

$$= \infty - \infty \; !$$

To make the expression meaningful, we need to introduce regulators which correspond to the Cartan generators commuting with Q, S, and each other

$$\boldsymbol{\Delta}, \quad \boldsymbol{j_1} + \boldsymbol{R}, \quad \boldsymbol{j_2}, \quad \boldsymbol{H_i}, \quad \boldsymbol{J} = *\mathrm{tr}(\boldsymbol{F} \wedge \boldsymbol{F})$$

Superconformal Index

$$I(x, y, m, u) =$$

$$\mathrm{Tr}\,(-1)^F e^{-\beta\{Q,S\}} x^{2(j_1+R)} y^{2j_2} e^{-i\sum m_i H_i} u^k$$

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5d SCFTs are strongly-coupled,

so free field limit is not available.

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→ **Localization works !!**

Superconformal Index

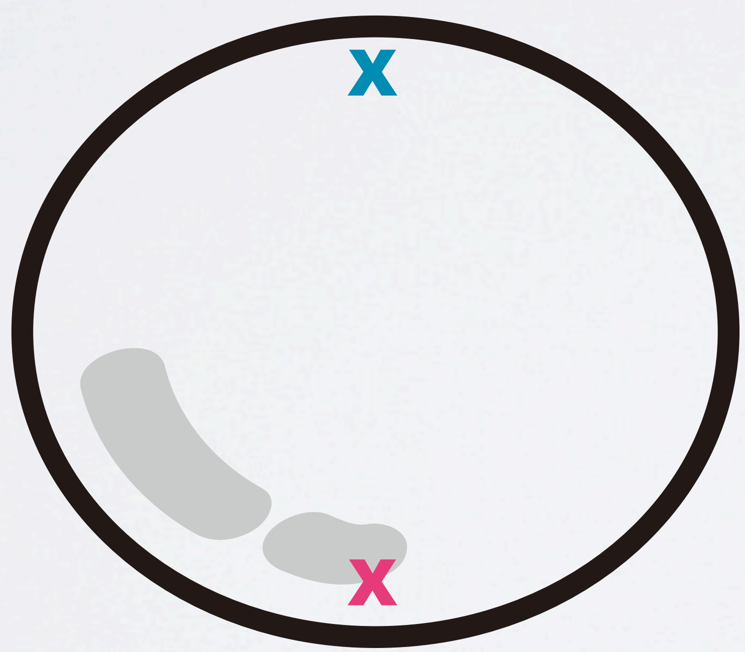
$I(x, y, m, u) =$ **a path integral expression on** $S^1 \times S^4$

This is the form where we can apply the localization method ! **[Kim-Kim-Lee, '12]**

Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

$$I(x, y, m, u) = \int [da] \left| Z^{\text{5d}}_{\text{Nek}}(t, q, m, u, a) \right|^{2}$$

$S^1 \times$



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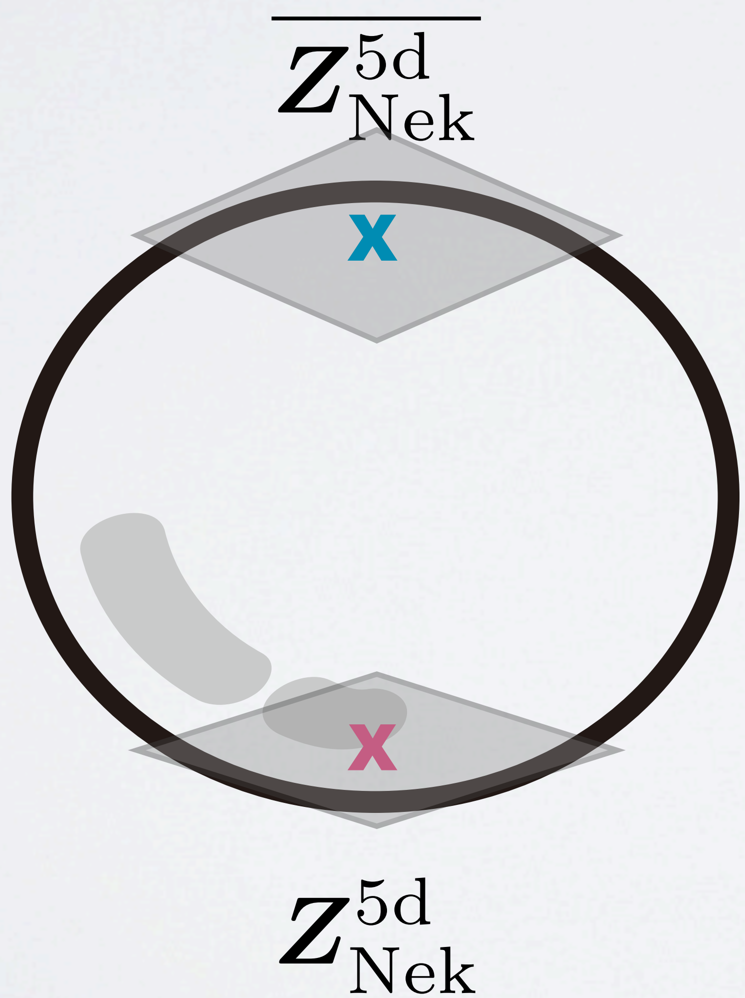

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[Hayashi-Kim-Nishinaka]

Superconformal Index [Kim-Kim-Lee, '12], [Iqbal-Vafa, '12]

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Q.

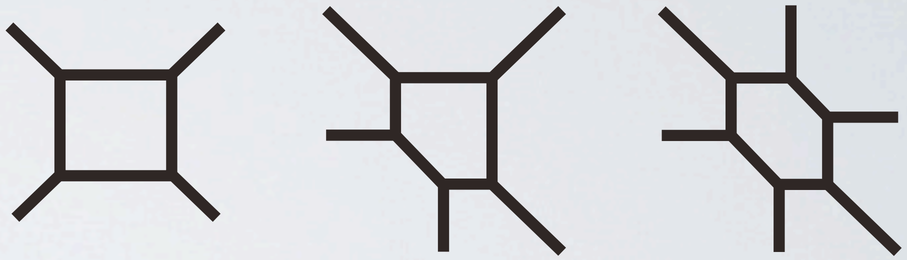
$$Sp(1) \neq SU(2) \text{ !?}$$

in some sense (physically).

3. Solution
& partition function
for 7-brane b.k.g.d.

5d gauge theory & topological string

[Nekrasov, '03] etc

topological string theory on 

$\cdots$

||

5d SU(2) gauge theory

5d gauge theory & topological string

[Nekrasov, ’03] etc.

$$Z_{\text{top. str.}}$$

$$||$$

$$Z^{\text{5d}}_{\text{Nek}}$$

5d gauge theory & topological string

[Nekrasov, '03] etc.

$$Z_{\text{top. str.}}$$ **(5-brane web)**

$$\|$$

$$Z^{\text{5d}}_{\text{Nek}}$$

$$\|$$

SU(2) partition function (E_n SCFT)

5d gauge theory & topological string

[Nekrasov, ’03] etc.

$$Z_{\text{top. str.}}$$ **(5-brane web)**

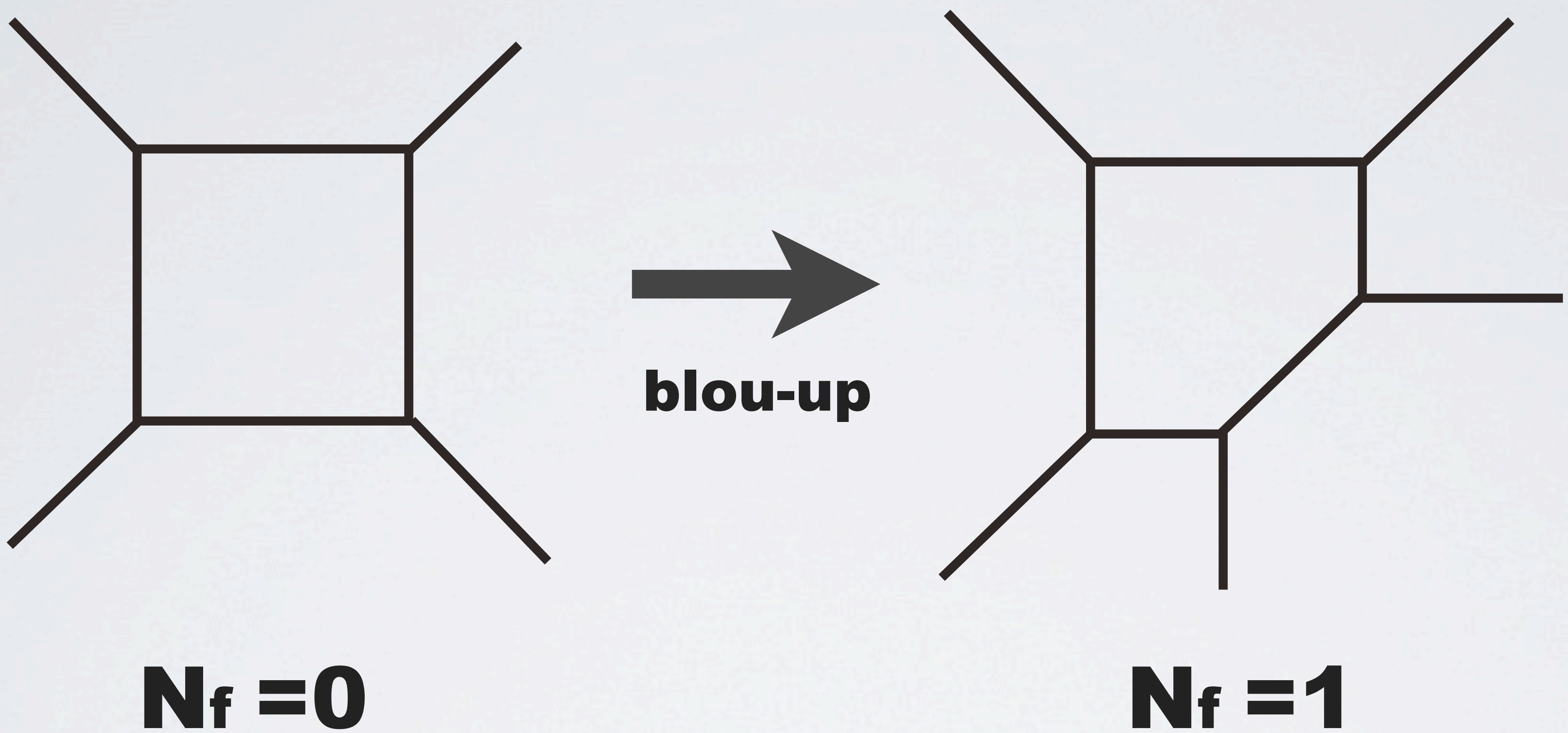
$$||$$

$$Z^{\text{5d}}_{\text{Nek}}$$

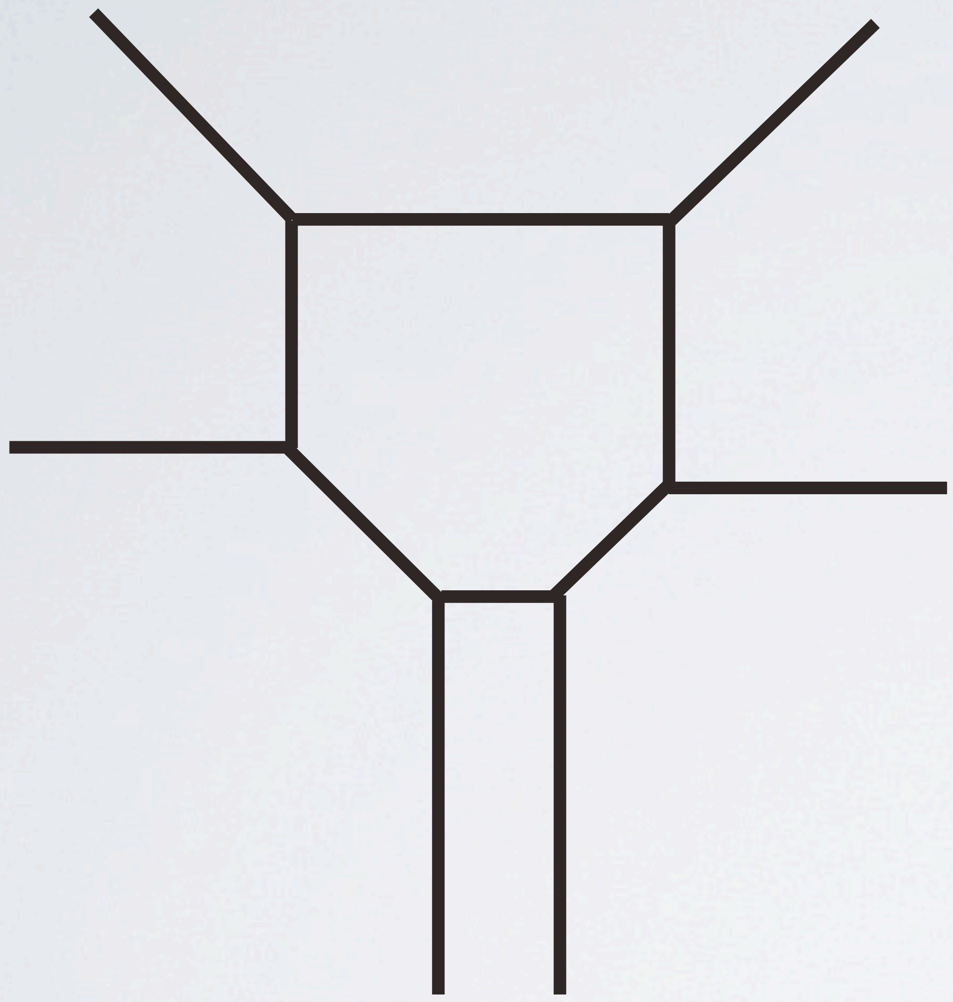
$$||$$ ← **not correct ! (preconception)**

SU(2) partition function (E_n SCFT)

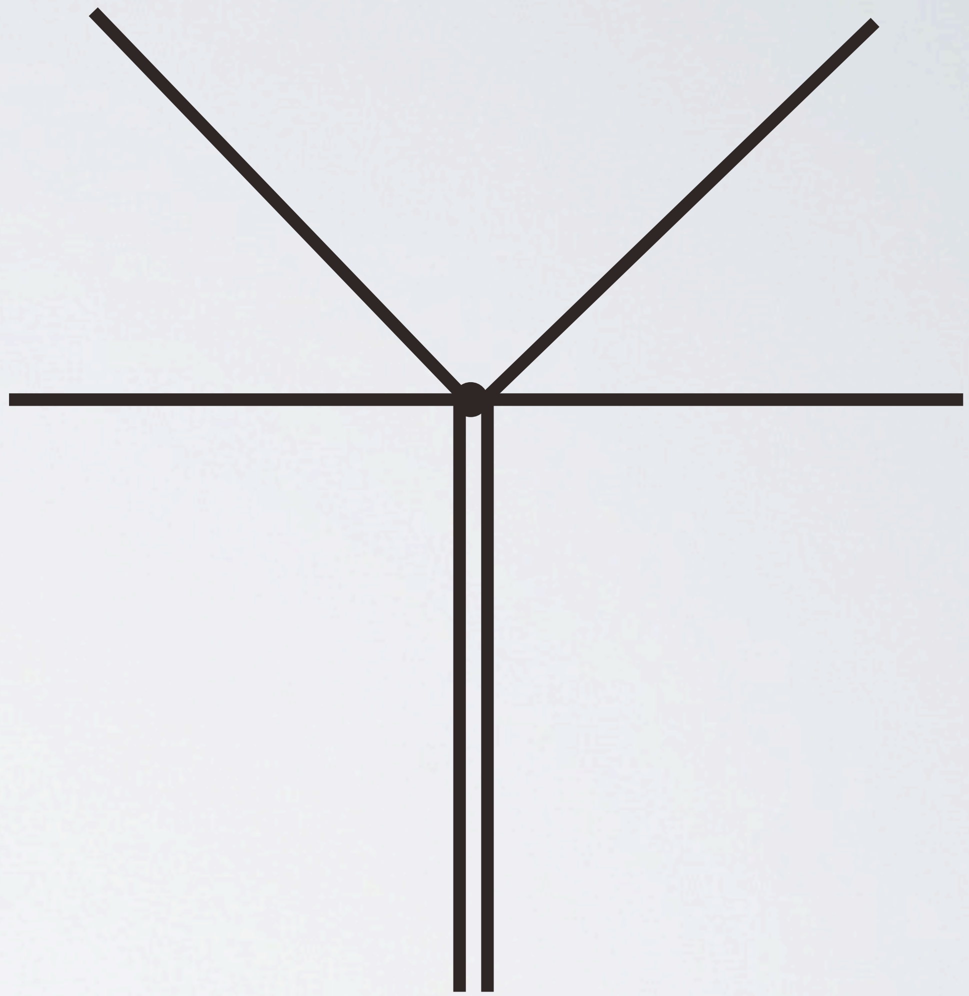
5d gauge theory & topological string



5d gauge theory & topological string

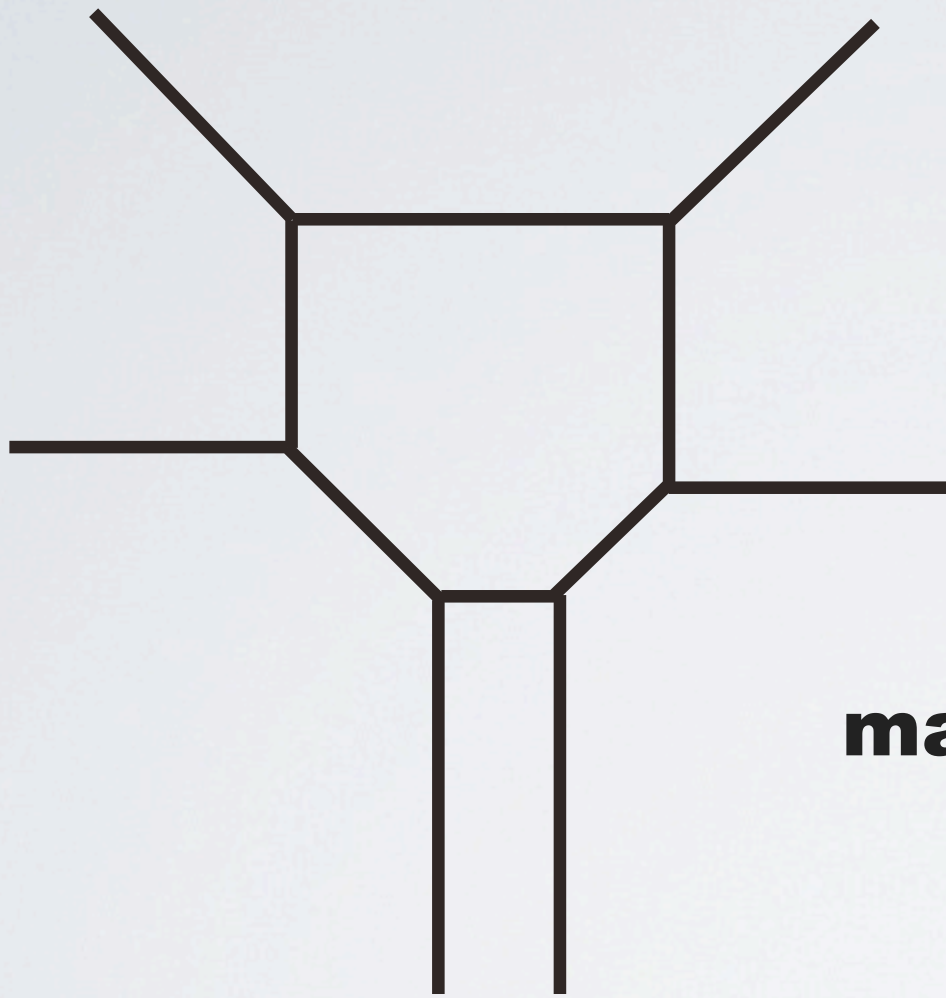


$N_f = 2$



E_3 ?

5d gauge theory & topological string



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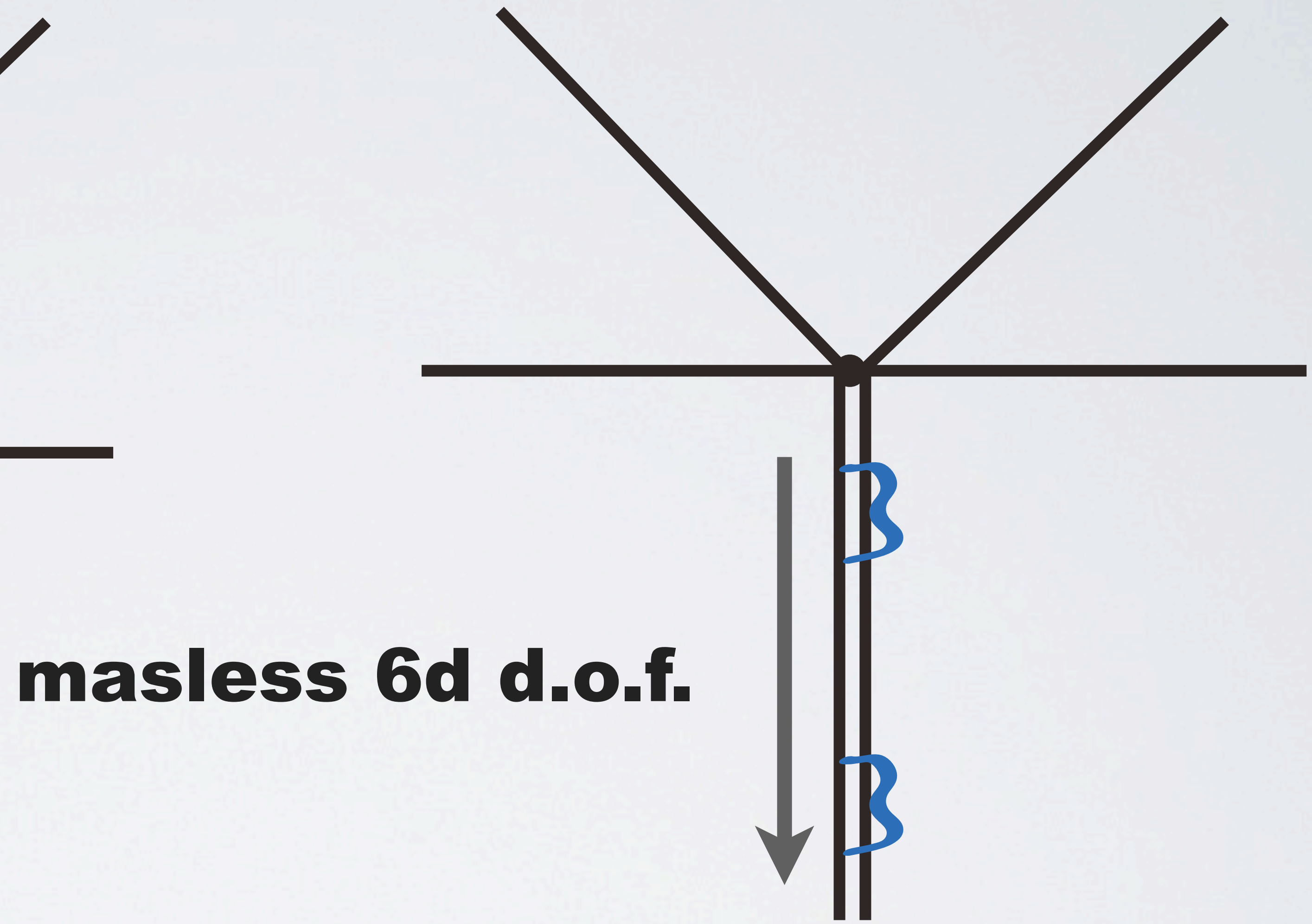
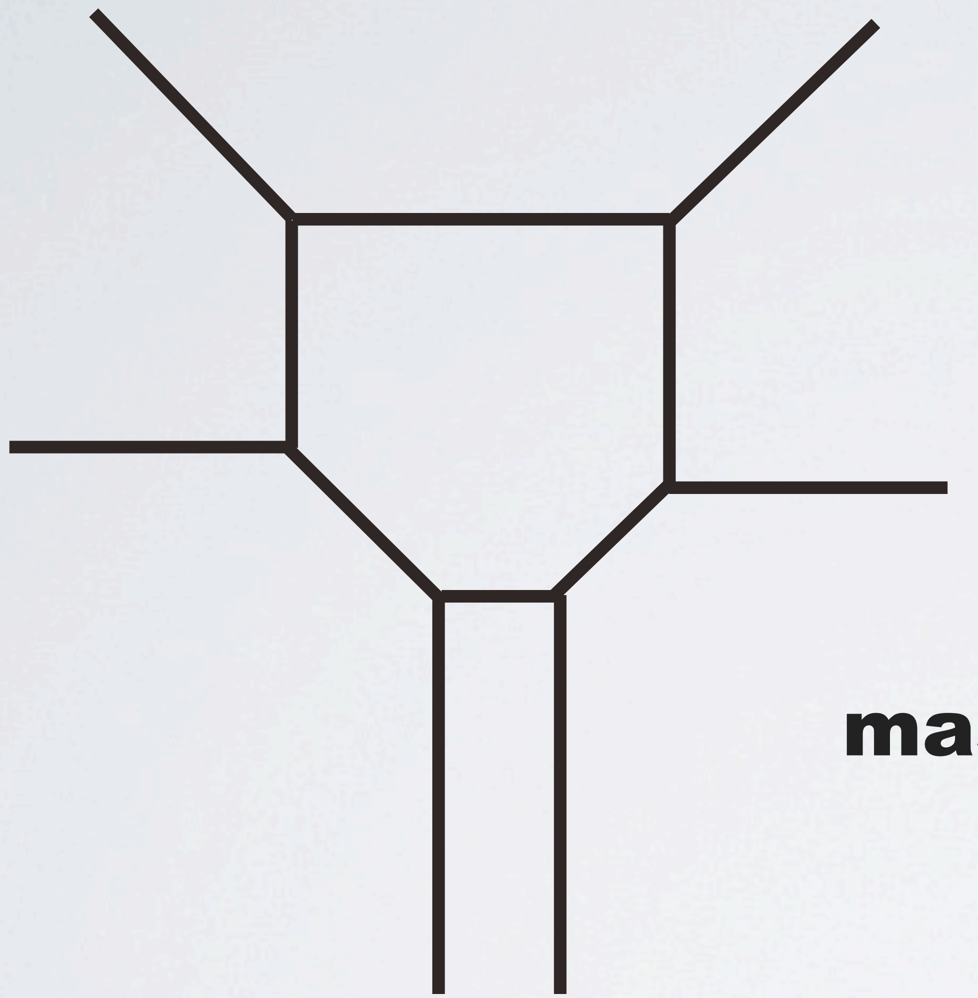
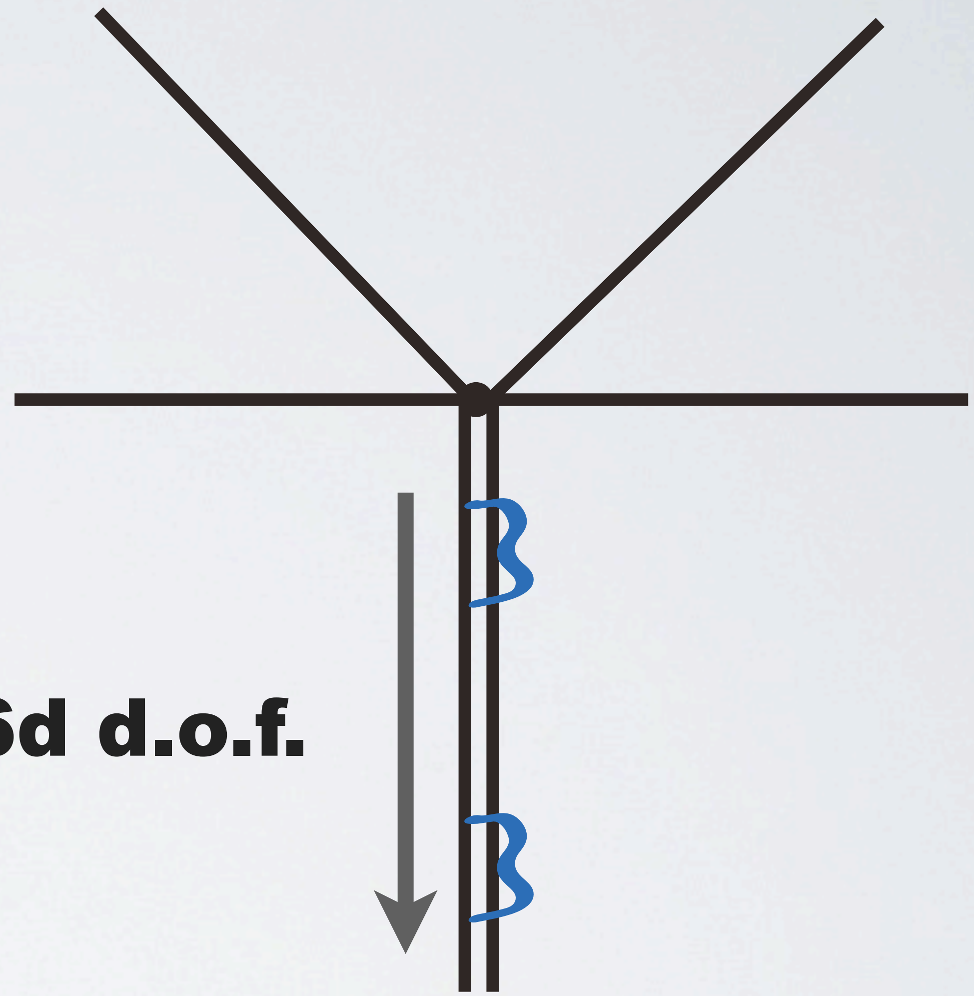


E_3 ?

5d gauge theory & topological string





masless 6d d.o.f.

$N_f = 2$

NOT 5d SCFT !

5d gauge theory & topological string

Modified statement

[Bao-Mitev-Pomoni-M.T.-Yagi] [Hayashi-Kim-Nishinaka]

$$Z_{\mathrm{top.\,str.}}$$

$$||$$

genuine $Z^{\mathrm{5d}}_{\mathrm{Nek}}$ **for SU(2)**

with 6d additional contribution

5d gauge theory & topological string

Modified statement

[Bao-Mitev-Pomoni-M.T.-Yagi] [Hayashi-Kim-Nishinaka]

conventional SU(2) (U(2)) Nekrasov p. f.

$$||$$

$$Z_{\mathrm{top.\,str.}}$$

$$||$$

genuine $Z^{\mathrm{5d}}_{\mathrm{Nek}}$ **+ extra contribution**

5d gauge theory & topological string

Claim [BMPT] [HKN]

$$Z^{\rm 5d}_{\rm Nek} = \frac{Z_{\rm top.\,str.}}{Z_{\rm extra}}$$

$$I = \frac{I[Z_{\rm top.\,str.}]}{I_{\rm extra}}$$

extra contribution can be factored out

5d gauge theory & topological string

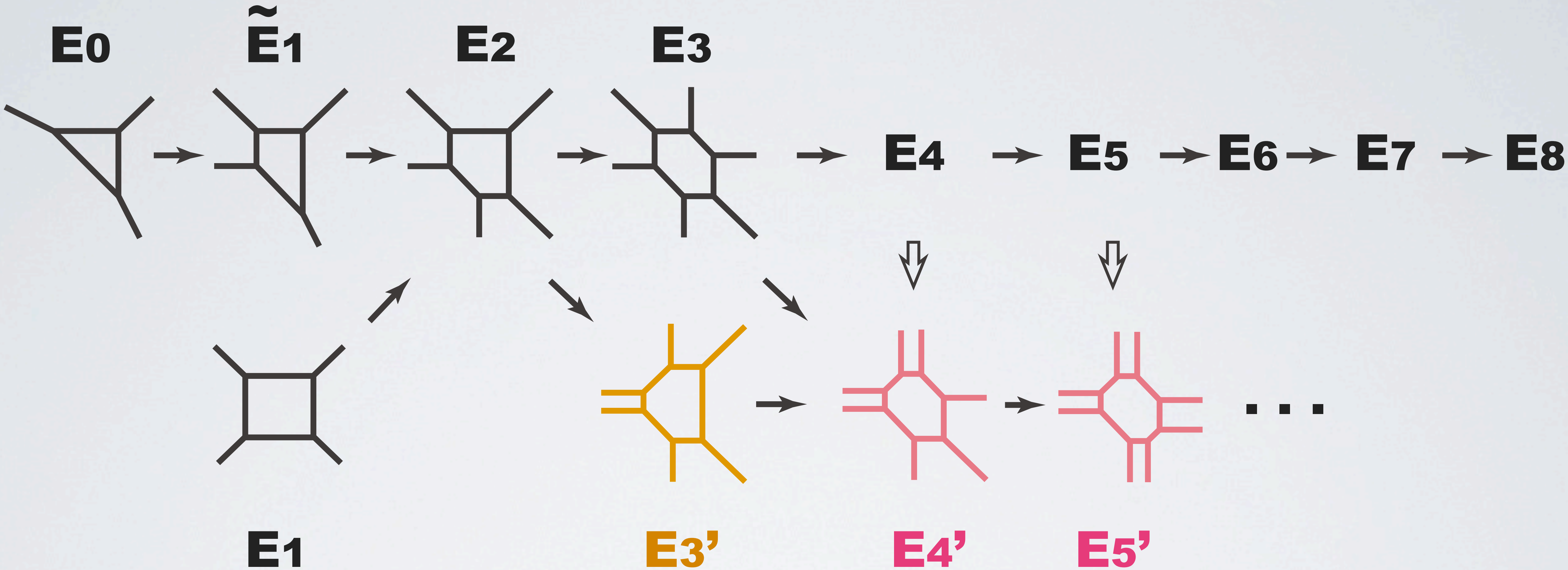
Claim [BMPT] [HKN]

$$Z^{\rm 5d}_{\rm Nek} = \frac{Z_{\rm top.\,str.}}{Z_{\rm extra}}$$

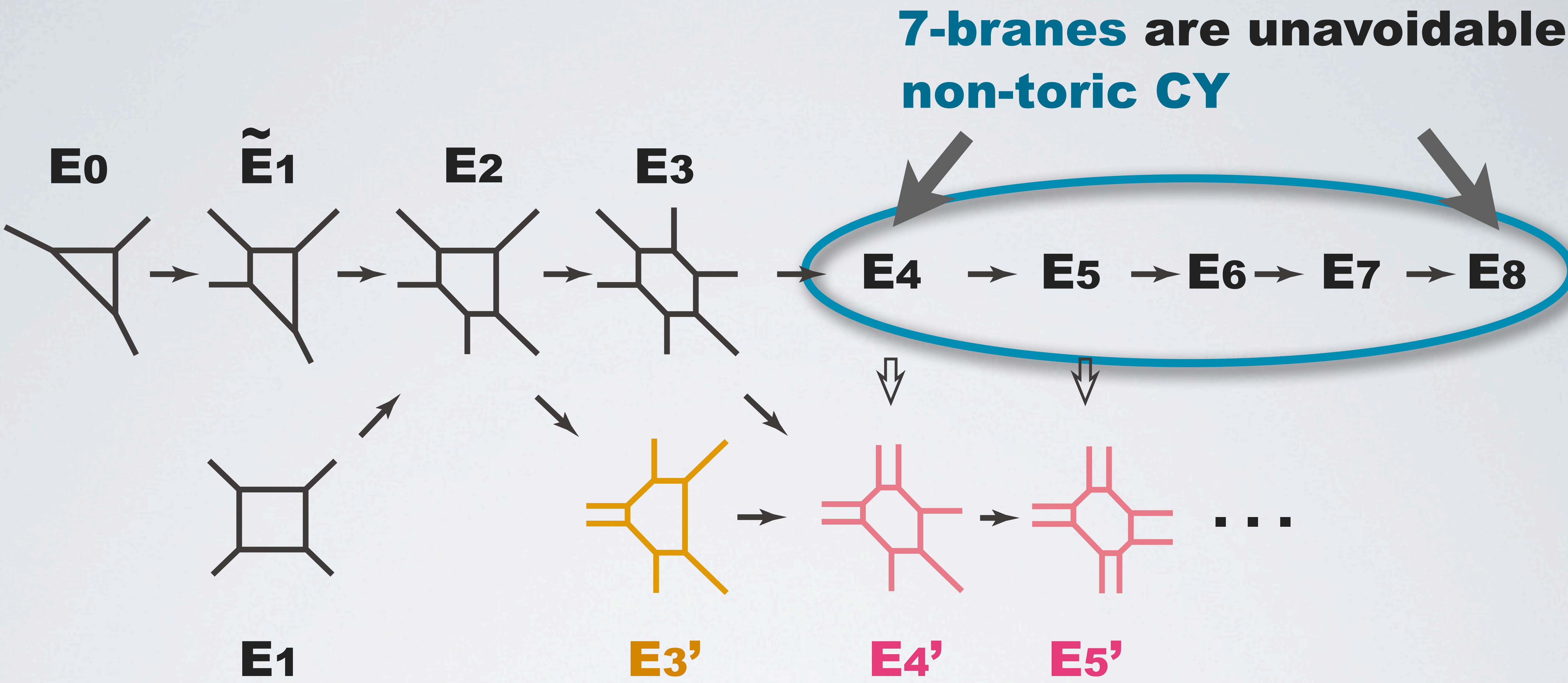
$$I = \frac{I[Z_{\rm top.\,str.}]}{I_{\rm extra}}$$

Q. How can we compute $Z_{\rm extra}$ & $I_{\rm extra}$??

Extra contribution

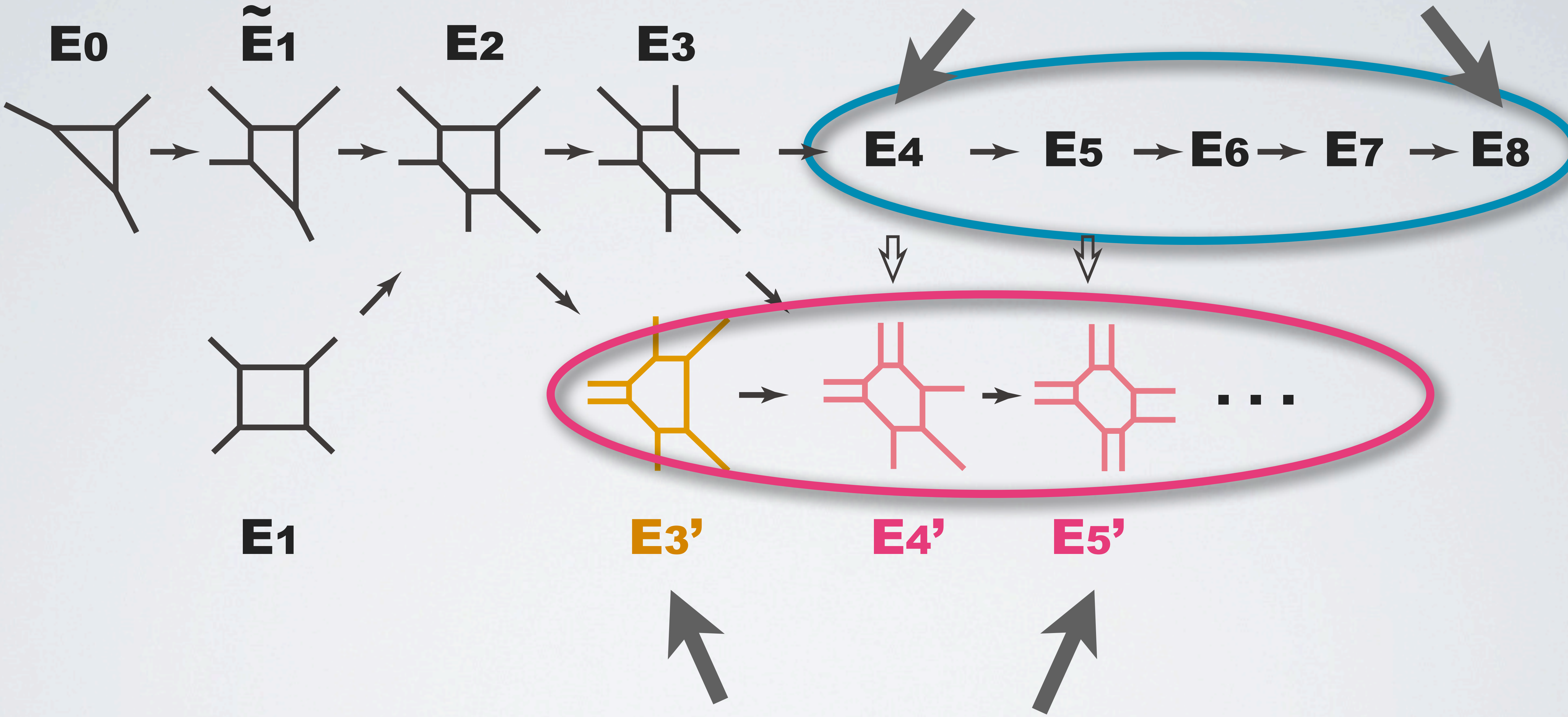


Extra contribution



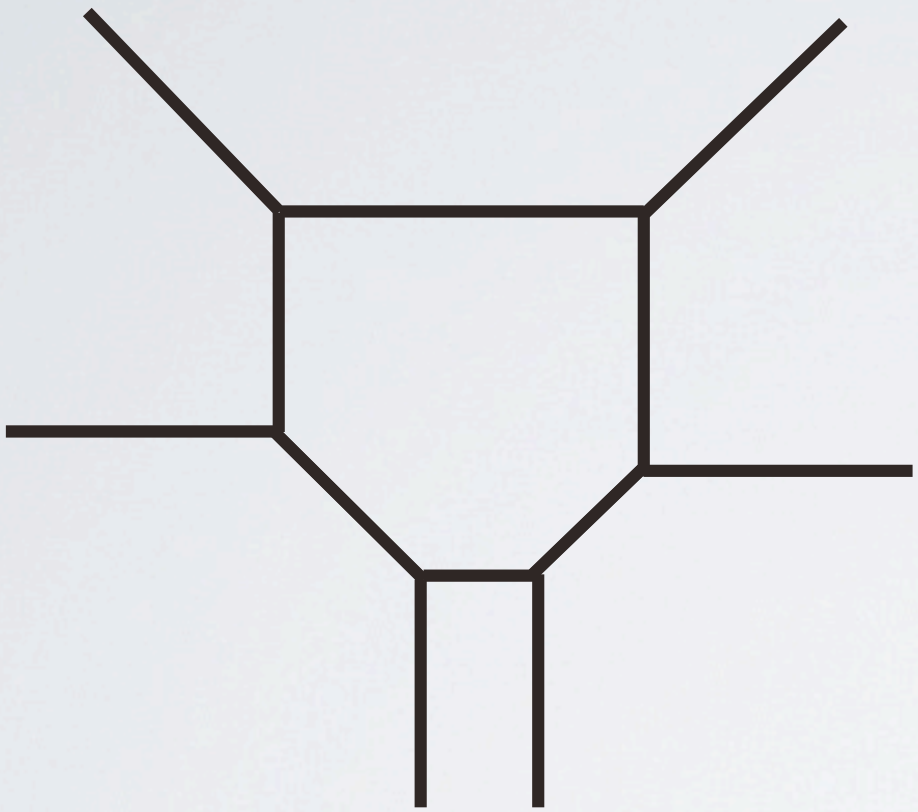
Extra contribution

7-branes **are unavoidable**
non-toric CY

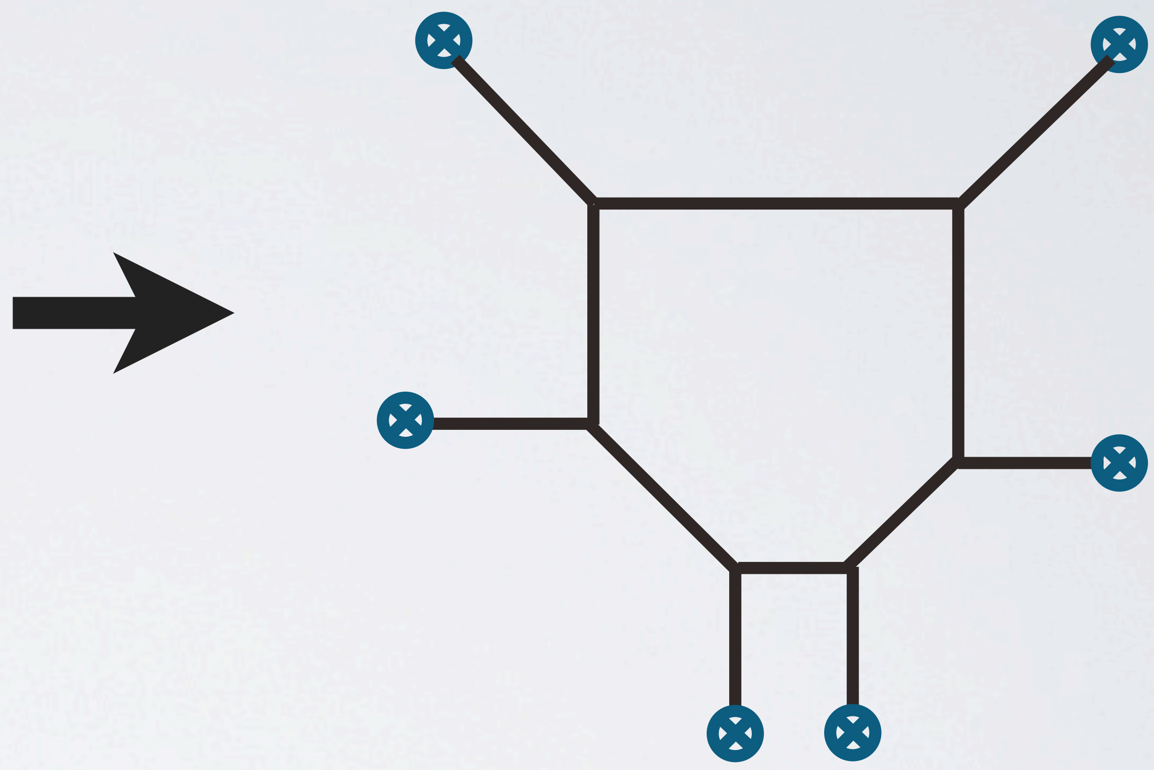


No 7-branes
toric CY
BUT they cause problem!

Extra contribution: typical example

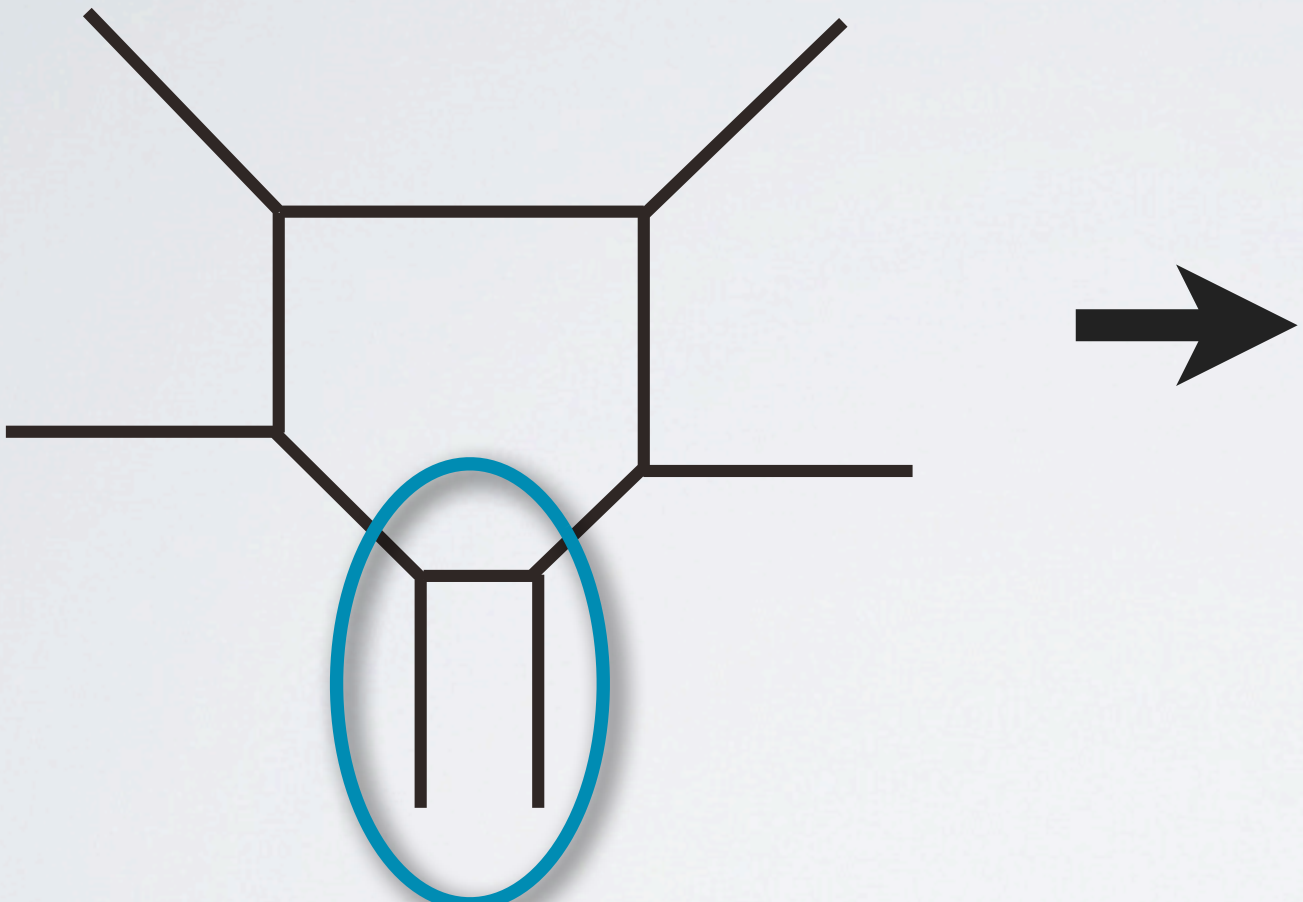


$Z_{\rm top.\,str.}$

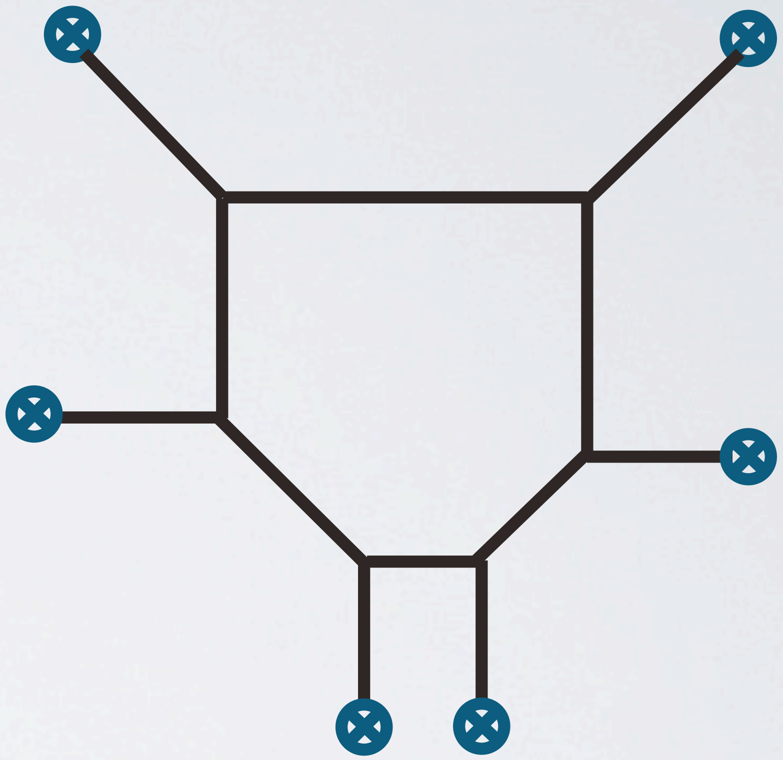


genuine $Z^{\rm 5d}_{\rm Nek}$

Extra contribution: typical example

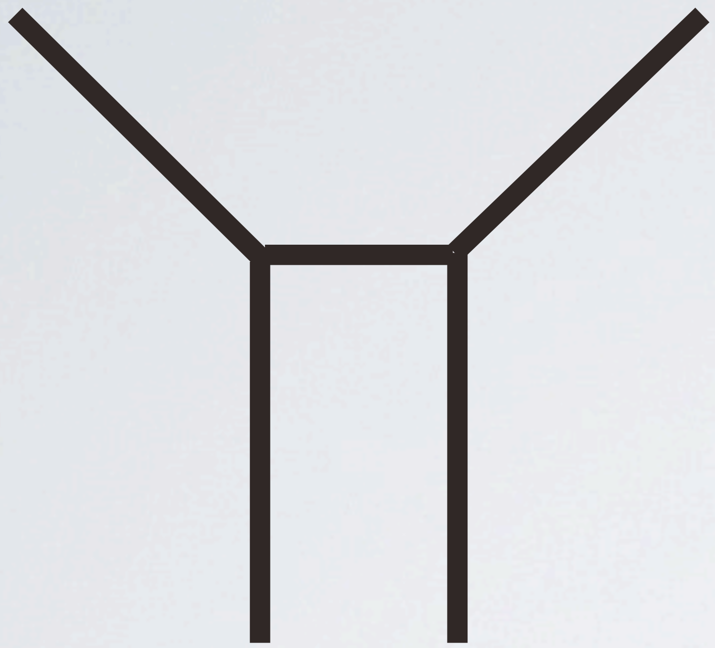


$Z_{\text{top. str.}}$



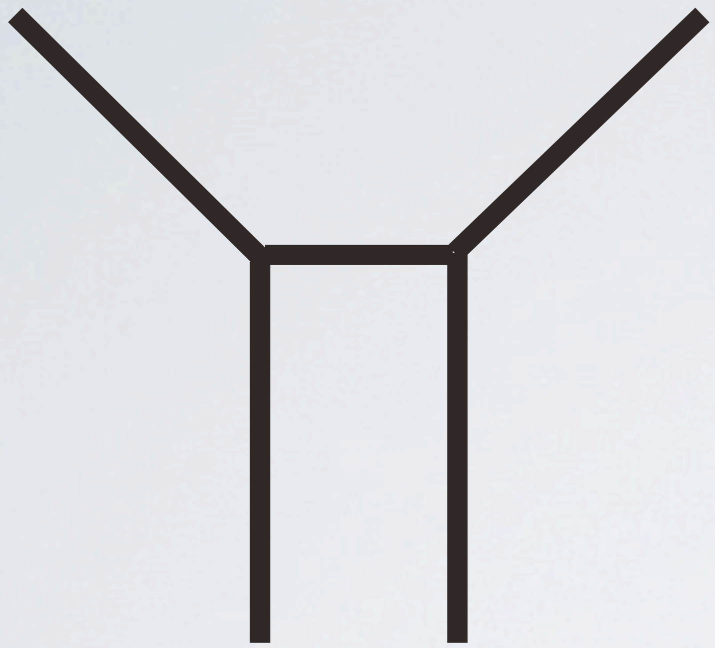
genuine $Z^{5\text{d}}_{\text{Nek}}$

Extra contribution: typical example



$$= Z_{\mathrm{extra}}$$

Extra contribution: typical example



$$= Z_{\text{extra}}$$

$$= \frac{1}{\prod_{i,j=1}^{\infty}(1 - u\, e^{-R(m_1+m_2)}\, t^i\, q^{j-1})}$$

Extra contribution

Then we can check the appearence of E_3

$$\boldsymbol{I} = \frac{\boldsymbol{I}[Z_{\text{top. str.}}]}{\boldsymbol{I}_{\text{extra}}}$$

Extra contribution

Then we can also check

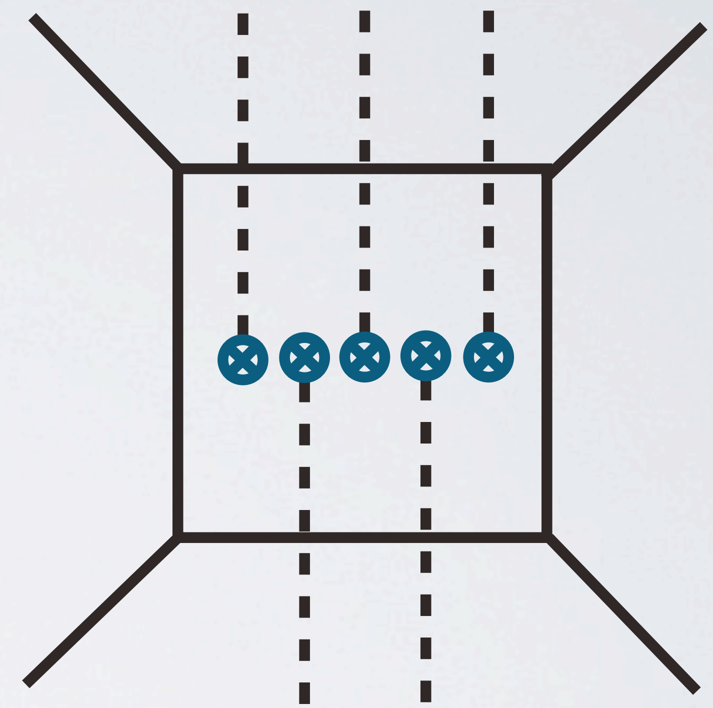
$$\boldsymbol{I} = \frac{\boldsymbol{I}[Z_{\text{top. str.}}]}{\boldsymbol{I}_{\text{extra}}}$$

for E_3, E_4, E_5, E_6 ... theories !

4. E_6 SCFT
&
Gaiotto's T_3 SCFT

E_6 SCFT

SU(2) theory with 5 flavors



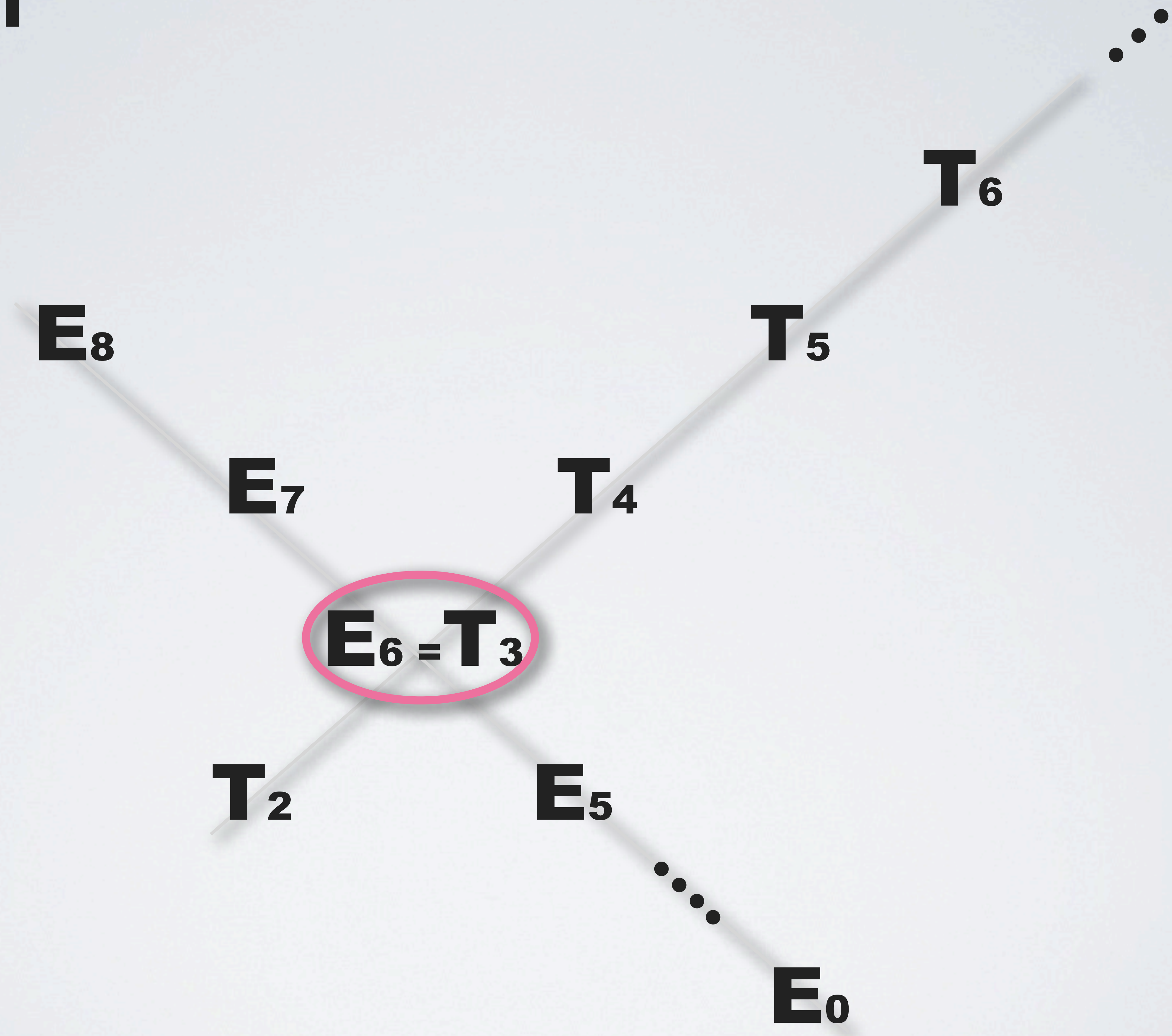
- ✔ web involves 7-branes
- ✔ CY is **not** toric (no genuine web)

E_6 SCFT [Benini-Benvenuti-Tachikawa,'09]

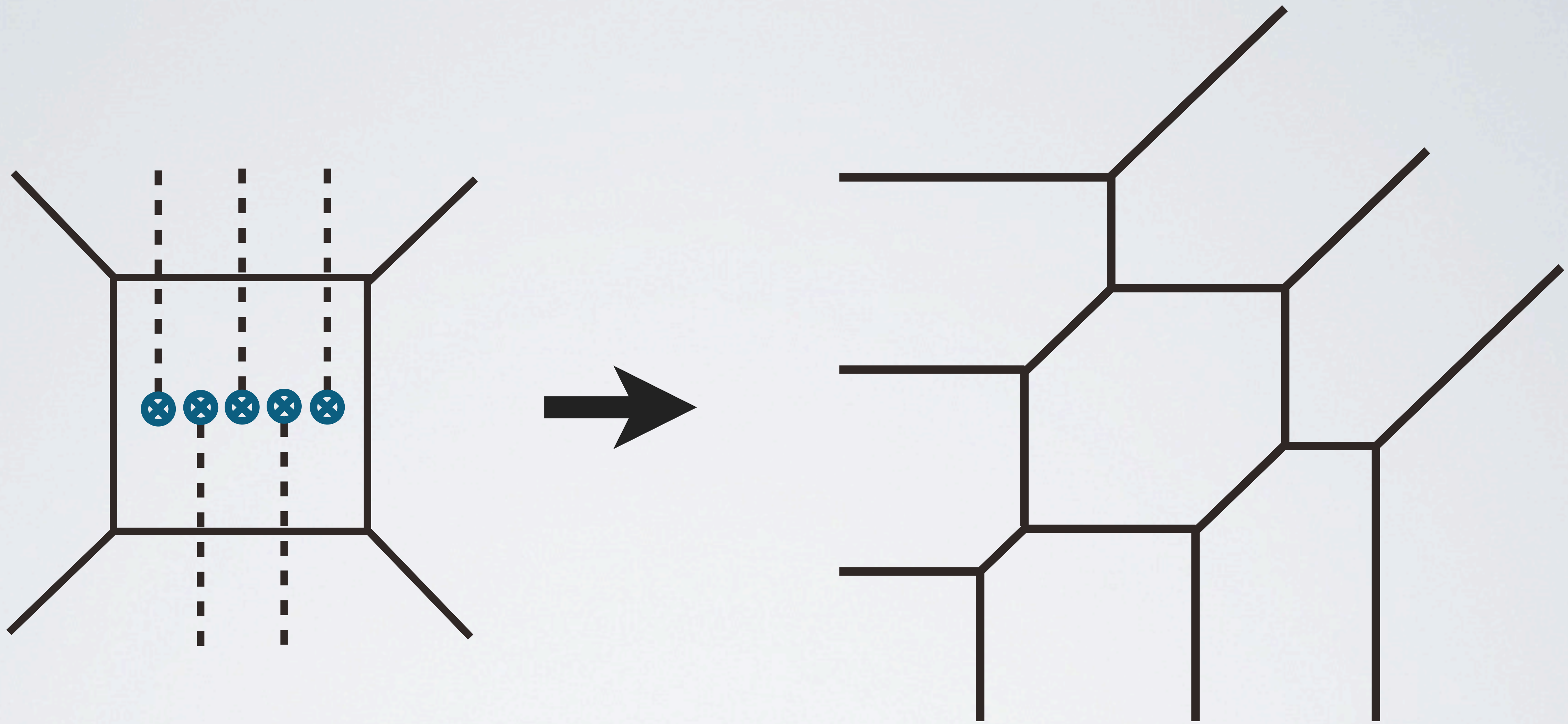
three M5-branes on sphere (Gaiotto's 4d T_3)

- ✔ 5d uplift of the Gaiotto thy
- ✔ N M5s gives T_N theory

E_6 SCFT

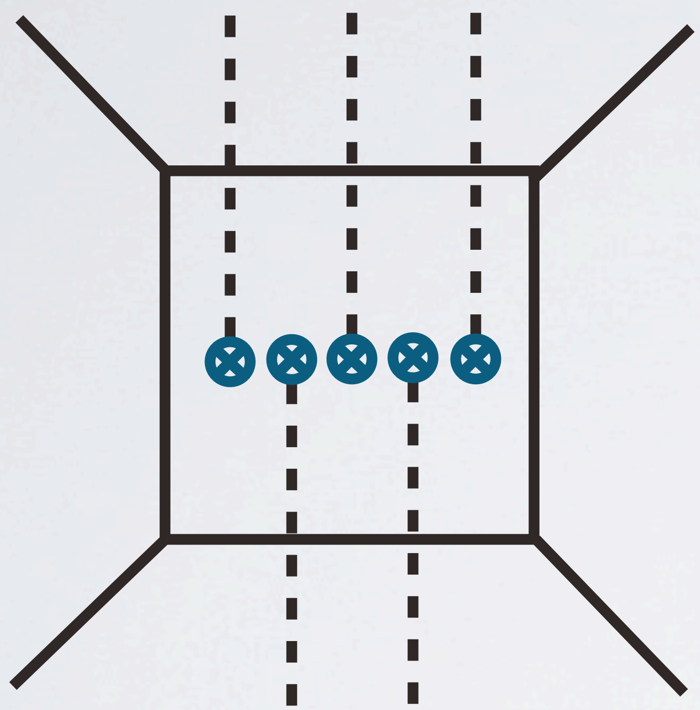


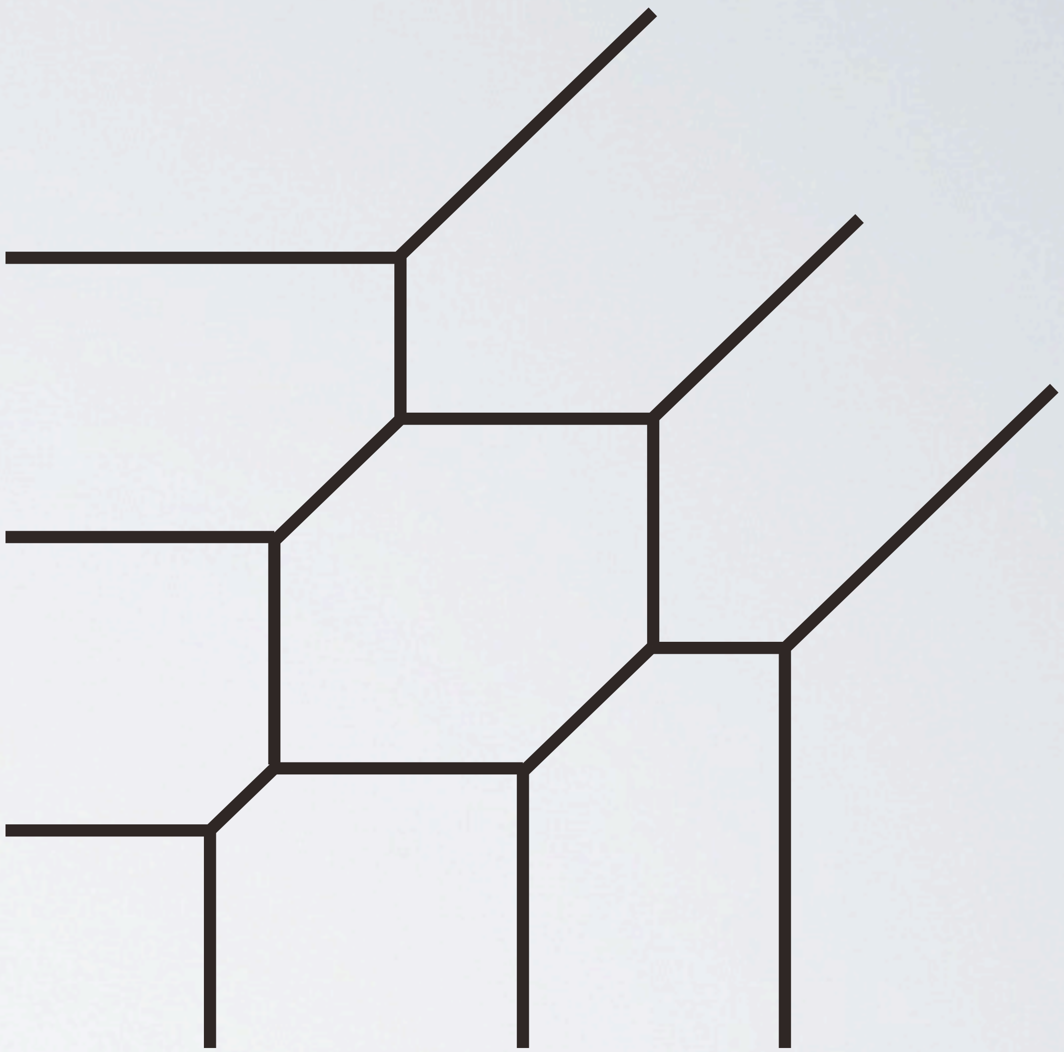
E_6 superconformal index



[Benini-Benvenuti-Tachikawa,'09]

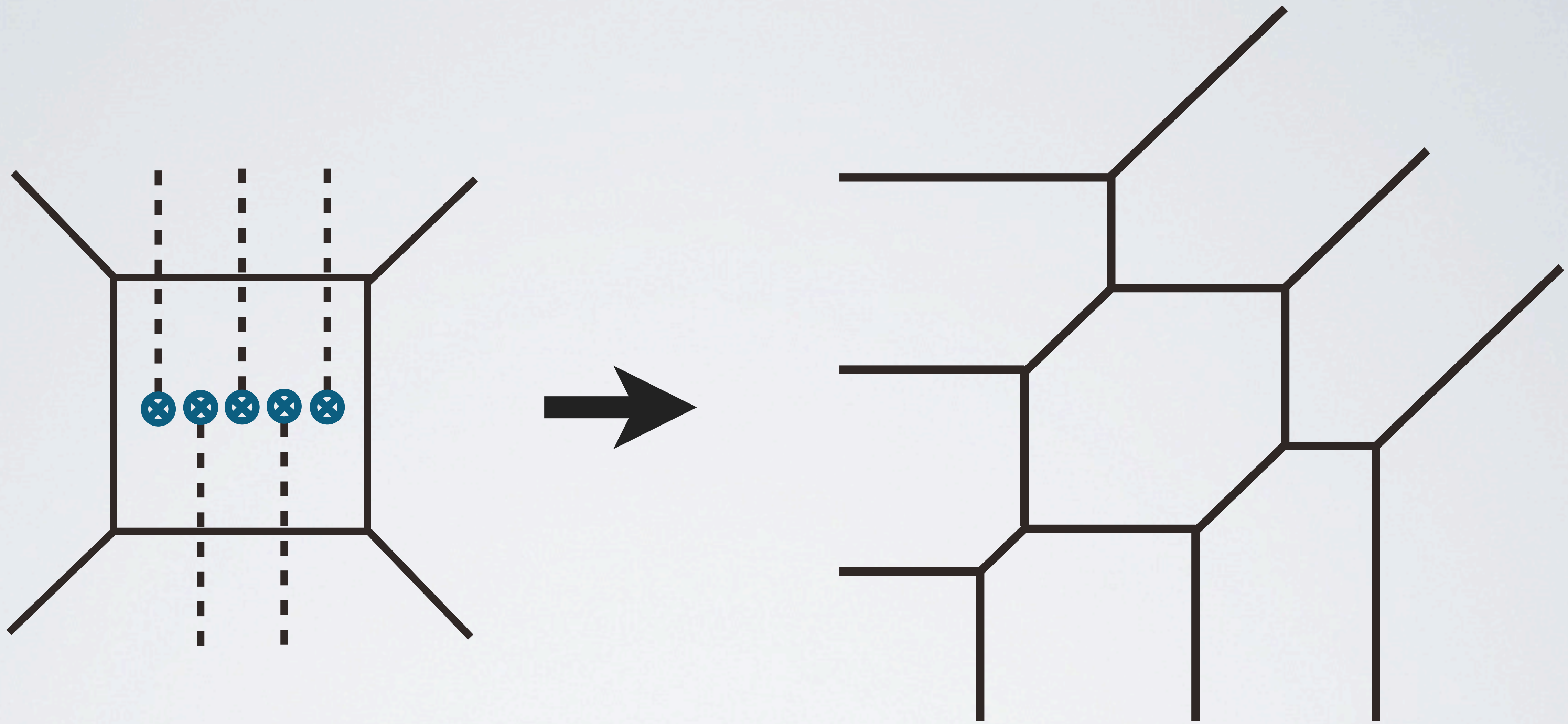
E_6 superconformal index





5-brane junction

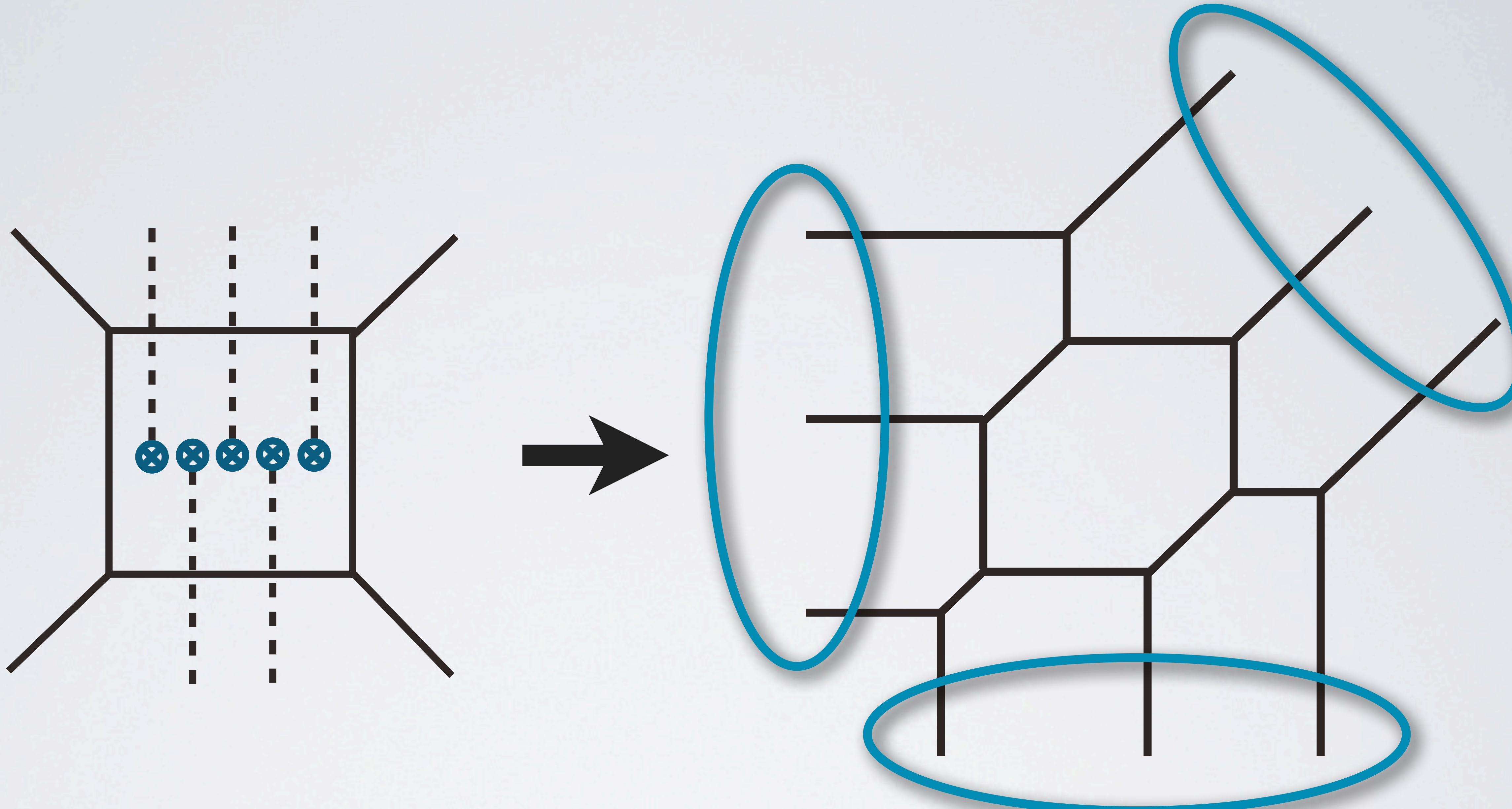
E$_6$ superconformal index



we need this web partition function

we can compute it by topological vertex

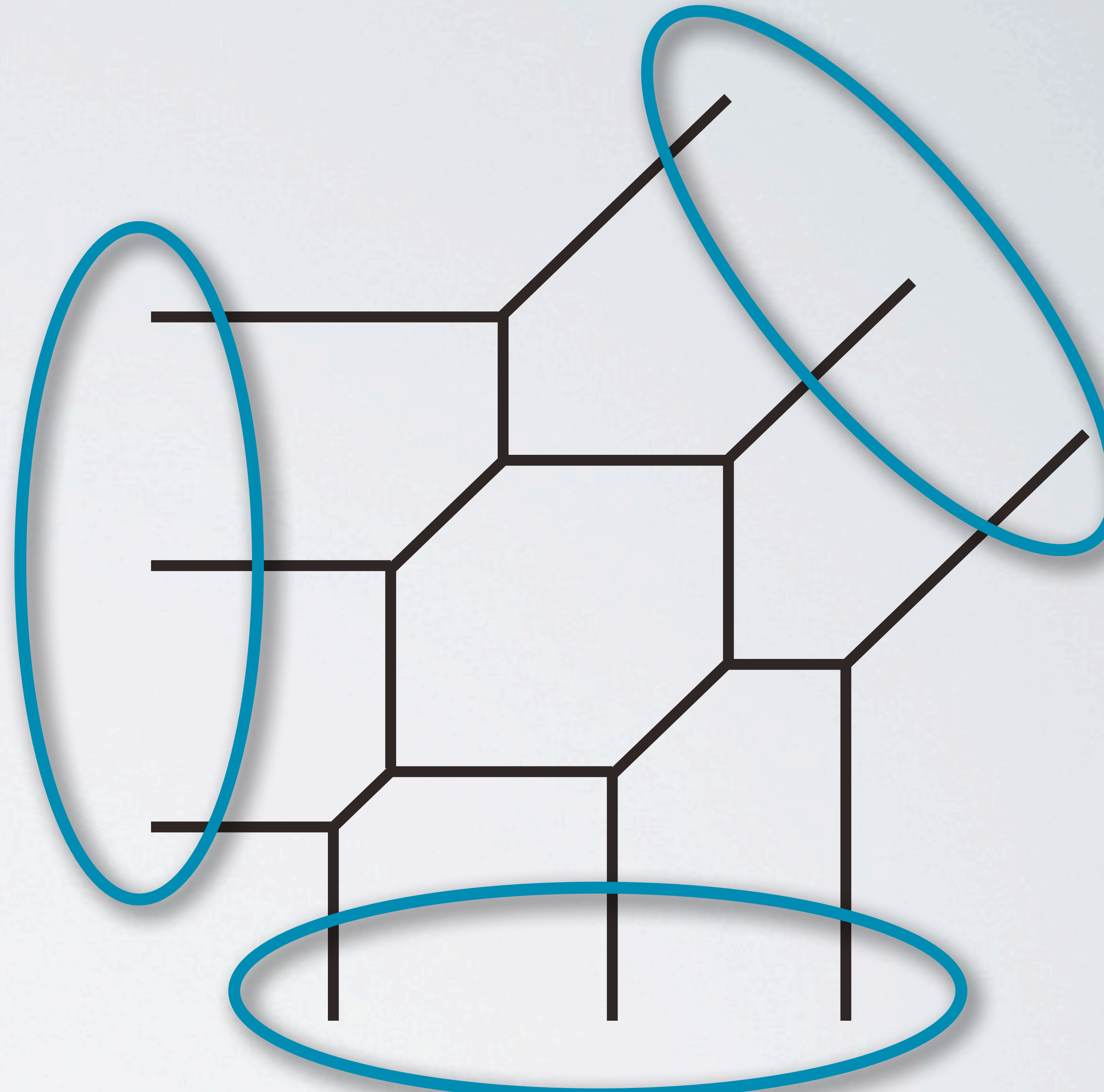
E_6 superconformal index



these three stacks
cause the discrepancy

E6 superconformal index

$$I_{E_6} = \frac{I_{T_3\,\mathrm{web}}}{I_{3\,\mathrm{stacks}}}$$



these three stacks cause the discrepancy

E_6 superconformal index

$$I_{E_6} = \frac{I_{T_3\,\mathrm{web}}}{I_{3\,\mathrm{stacks}}}$$

$$= 1 + \chi^{E_6}_{\mathbf{78}} x^2 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_1(y) x^3$$

$$+(1 + (1 + \chi^{E_6}_{\mathbf{78}})\chi_2(y) + \chi^{E_6}_{\mathbf{2430}}) x^4 + \cdots$$

agrees with [Kim-Kim-Lee]'s Sp(1) index !!

generalization

Our method should work for generic theories without purely 5-brane web description (i.e. generalized webs involving 7-branes)

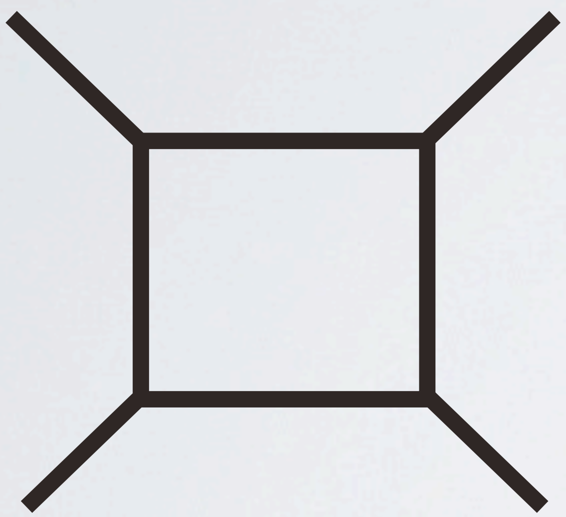
→ new theory !?

These set-up are dual to TypeII theory on complicated singularities. So far it is difficult to deal with them.

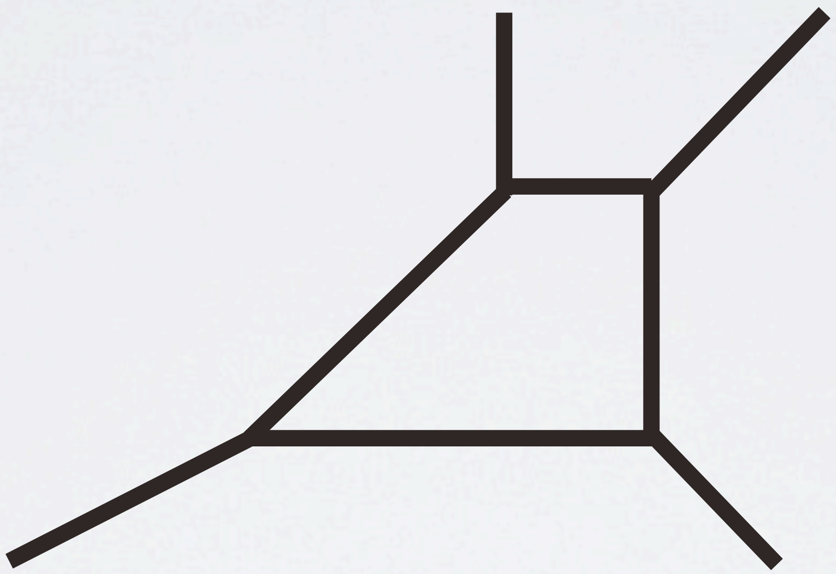
rank-one theories [M.T. to appear]

discussion so far

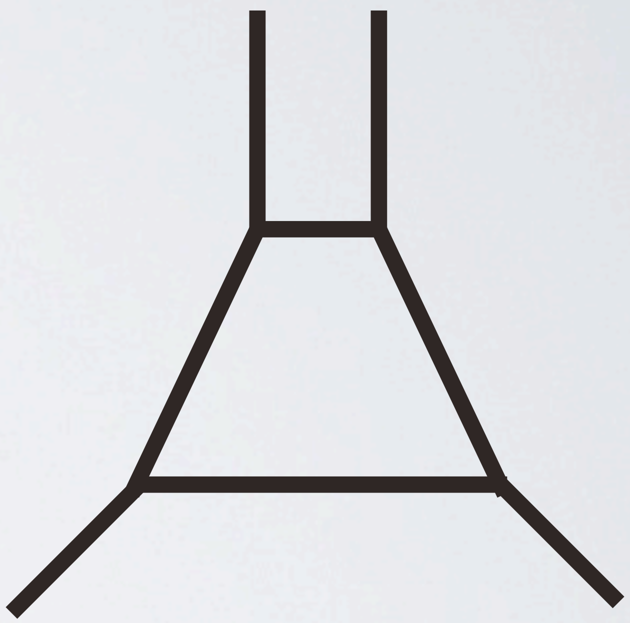
[Douglas-Katz-Vafa,’97][Aharony-Hanany,’97]
[Aharony-Hanany-Kol,’97] ...



$\mathbb{F}_0$



$\mathbb{F}_1$

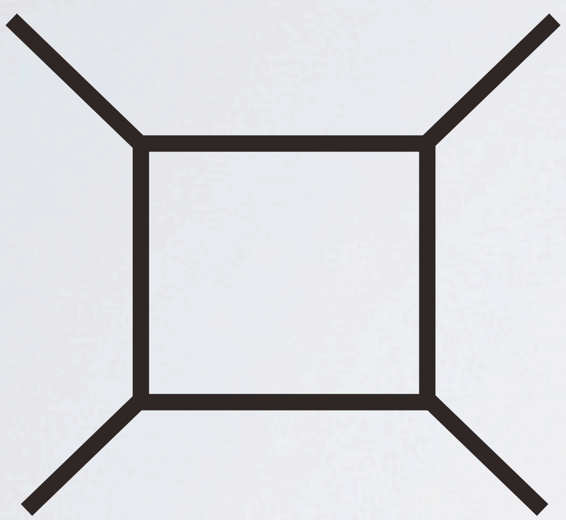


$\mathbb{F}_2$

rank-one theories [M.T. to appear]

discussion so far

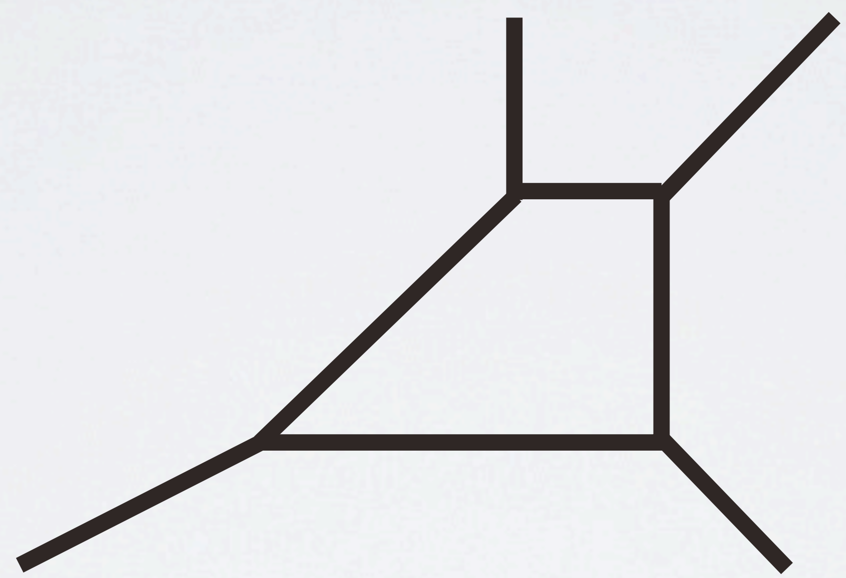
[Douglas-Katz-Vafa,'97][Aharony-Hanany,'97]
[Aharony-Hanany-Kol,'97] ...



$\mathbb{F}_0$

E_1

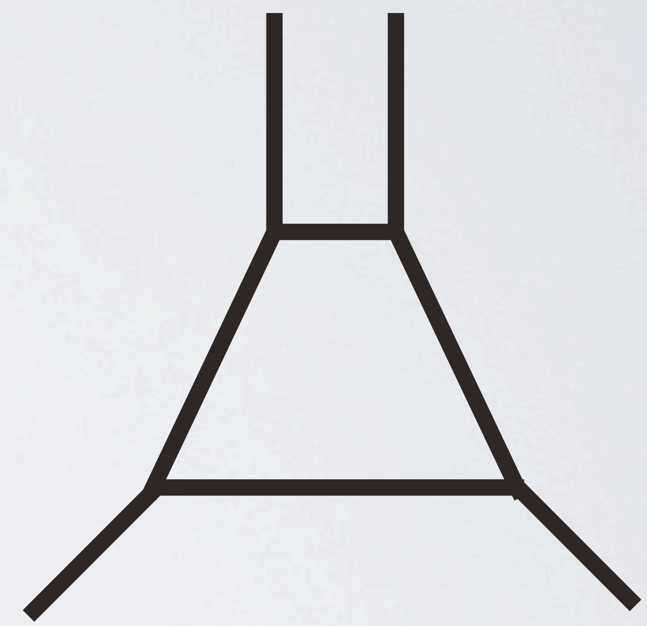
SU(2) YM θ=0



$\mathbb{F}_1$

$\tilde{E}_1$

SU(2) YM θ=π



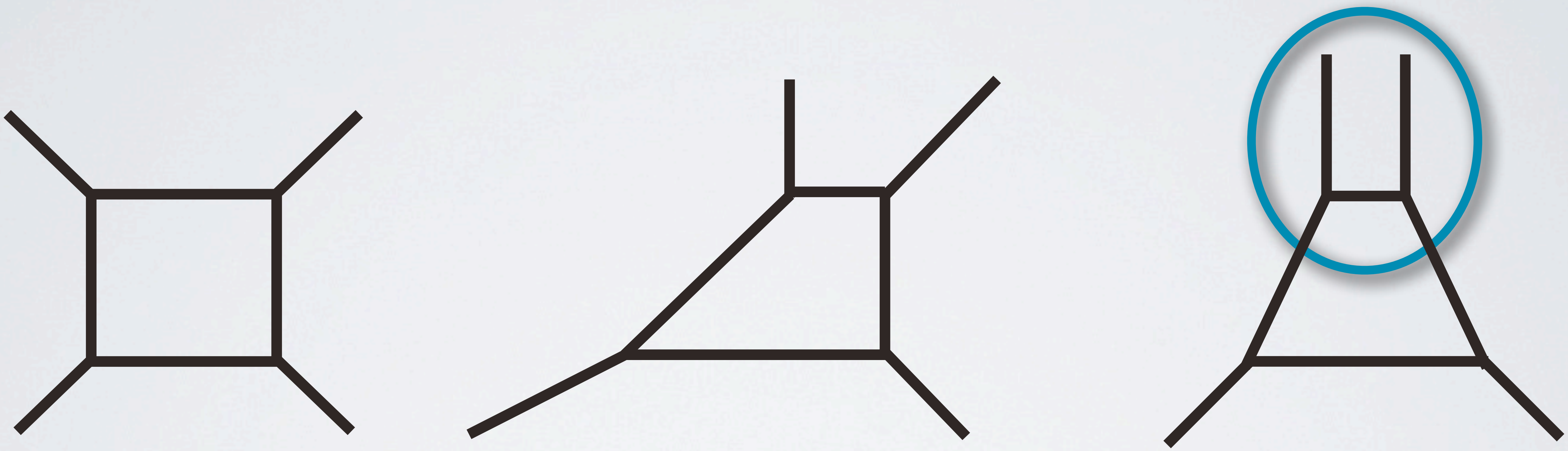
$\mathbb{F}_2$

??

rank-one theories [M.T. to appear]

discussion so far

[Douglas-Katz-Vafa,'97][Aharony-Hanany,'97]
[Aharony-Hanany-Kol,'97] ...



$\mathbb{F}_0$	$\mathbb{F}_1$	$\mathbb{F}_2$
E_1	$\tilde{E}_1$	??
SU(2) YM θ=0	SU(2) YM θ=π	

$I_{\mathbb{F}_2}(u;x,y)$

$$
\begin{aligned}
&= 1+\chi_3^{E_1}(u)x^2+\chi_2(y)\left(1+\chi_3^{E_1}(u)\right)x^3+\left\{\chi_3(y)\left(1+\chi_3^{E_1}(u)\right)+1+\chi_5^{E_1}(u)\right\}x^4\\
&+\left\{\chi_4(y)\left(1+\chi_3^{E_1}(u)\right)+\chi_2(y)\left(1+\chi_3^{E_1}(u)+\chi_5^{E_1}(u)\right)\right\}x^5\\
&+\left\{\chi_5(y)\left(1+\chi_3^{E_1}(u)\right)+2\chi_3(y)\left(1+\chi_3^{E_1}(u)+\chi_5^{E_1}(u)\right)-1+\chi_3^{E_1}(u)+\chi_7^{E_1}(u)\right\}x^6\\
&+\left\{\chi_6(y)\left(1+\chi_3^{E_1}(u)\right)+\chi_4(y)\left(2+4\chi_3^{E_1}(u)+2\chi_5^{E_1}(u)\right)\right.\\
&\left.+\chi_3(y)\left(1+3\chi_3^{E_1}(u)+2\chi_5^{E_1}(u)+\chi_7^{E_1}(u)\right)\right\}x^7+\left\{\chi_7(y)\left(1+\chi_3^{E_1}(u)\right)\right.\\
&+\chi_5(y)\left(4+5\chi_3^{E_1}(u)+3\chi_5^{E_1}(u)\right)+\chi_3(y)\left(2+7\chi_3^{E_1}(u)+3\chi_5^{E_1}(u)+2\chi_7^{E_1}(u)\right)\\
&\left.+3+2\chi_3^{E_1}(u)+2\chi_5^{E_1}(u)+\chi_9^{E_1}(u)\right\}x^8+\left\{\chi_8(y)\left(1+\chi_3^{E_1}(u)\right)\right.\\
&+\chi_6(y)\left(4+7\chi_3^{E_1}(u)+3\chi_5^{E_1}(u)\right)+\chi_4(y)\left(6+10\chi_3^{E_1}(u)+6\chi_5^{E_1}(u)+3\chi_7^{E_1}(u)\right)\\
&\left.+\chi_2(y)\left(4+7\chi_3^{E_1}(u)+4\chi_5^{E_1}(u)+2\chi_7^{E_1}(u)+\chi_9^{E_1}(u)\right)\right\}x^9+\cdots
\end{aligned}
$$

$$I_{\mathbb{F}_2}(u;x,y)$$

$$= 1 + \chi_3^{E_1}(u)x^2 + \chi_2(y)\left(1+\chi_3^{E_1}(u)\right)x^3 + \Big\{\chi_3(y)\left(1+\chi_3^{E_1}(u)\right) + 1 + \chi_5^{E_1}(u)\Big\}x^4$$

$$+\Big\{\chi_4(y)\left(1+\chi_3^{E_1}(u)\right) + \chi_2(y)\left(1+\chi_3^{E_1}(u)+\chi_5^{E_1}(u)\right)\Big\}x^5$$

$$+\Big\{\chi_5(y)\left(1+\chi_3^{E_1}(u)\right) + 2\chi_3(y)\left(1+\chi_3^{E_1}(u)+\chi_5^{E_1}(u)\right) - 1 + \chi_3^{E_1}(u) + \chi_7^{E_1}(u)\Big\}x^6$$

$$+\Big\{\chi_6(y)\left(1+\chi_3^{E_1}(u)\right) + \chi_4(y)\left(2+4\chi_3^{E_1}(u)+2\chi_5^{E_1}(u)\right)$$

$$+\chi_3(y)\left(1+3\chi_3^{E_1}(u)+2\chi_5^{E_1}(u)+\chi_7^{E_1}(u)\right)\Big\}x^7 + \Big\{\chi_7(y)\left(1+\chi_3^{E_1}(u)\right)$$

$$+\chi_5(y)\left(4+5\chi_3^{E_1}(u)+3\chi_5^{E_1}(u)\right) + \chi_3(y)\left(2+7\chi_3^{E_1}(u)+3\chi_5^{E_1}(u)+2\chi_7^{E_1}(u)\right)$$

$$+3 + 2\chi_3^{E_1}(u) + 2\chi_5^{E_1}(u) + \chi_9^{E_1}(u)\Big\}x^8 + \Big\{\chi_8(y)\left(1+\chi_3^{E_1}(u)\right)$$

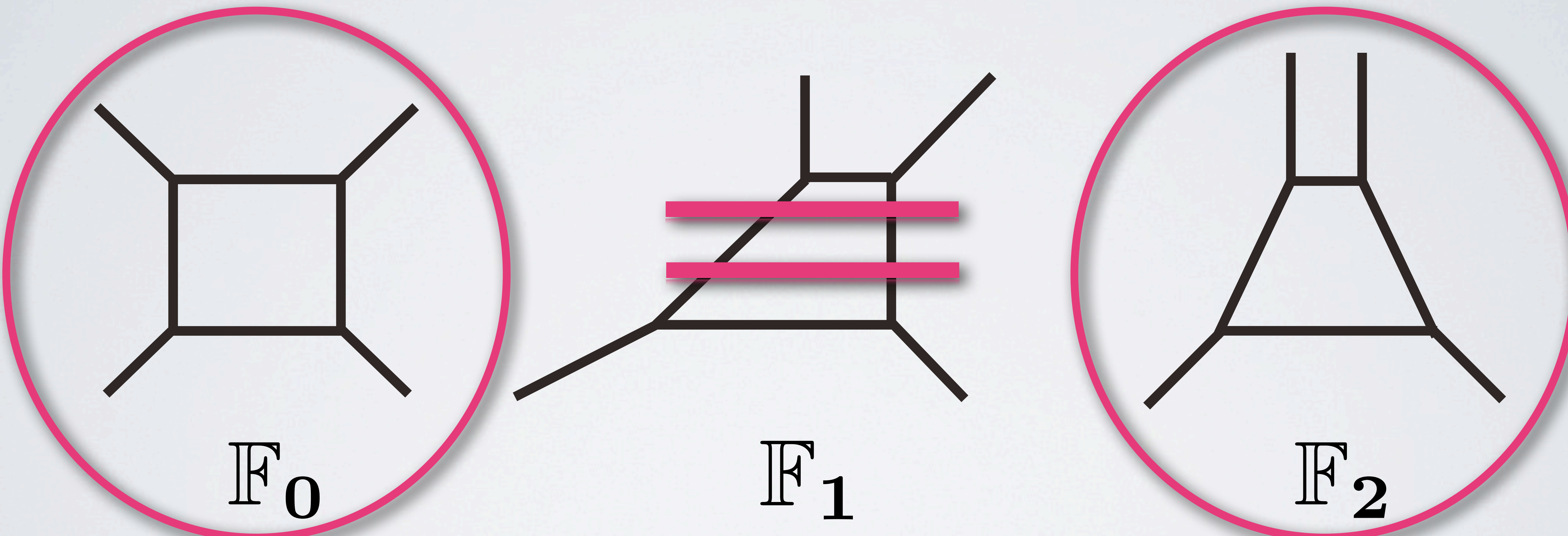
$$+\chi_6(y)\left(4+7\chi_3^{E_1}(u)+3\chi_5^{E_1}(u)\right) + \chi_4(y)\left(6+10\chi_3^{E_1}(u)+6\chi_5^{E_1}(u)+3\chi_7^{E_1}(u)\right)$$

$$+\chi_2(y)\left(4+7\chi_3^{E_1}(u)+4\chi_5^{E_1}(u)+2\chi_7^{E_1}(u)+\chi_9^{E_1}(u)\right)\Big\}x^9 + \cdots$$

$$= I_{\mathbb{F}_0}(u;x,y)$$

rank-one theories [M.T. to appear]

Computing index, we find

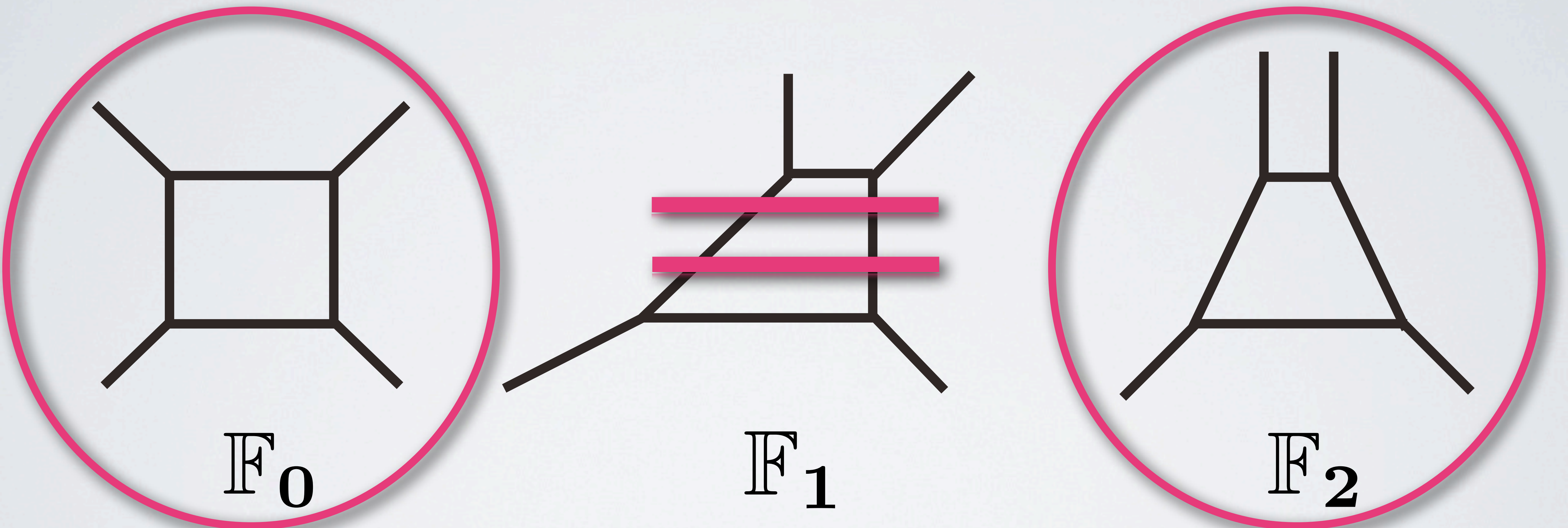


E_1	$\tilde{E}_1$	E_1
SU(2) YM θ=0	SU(2) YM θ=π	SU(2) YM θ=0

rank-one theories [M.T. to appear]

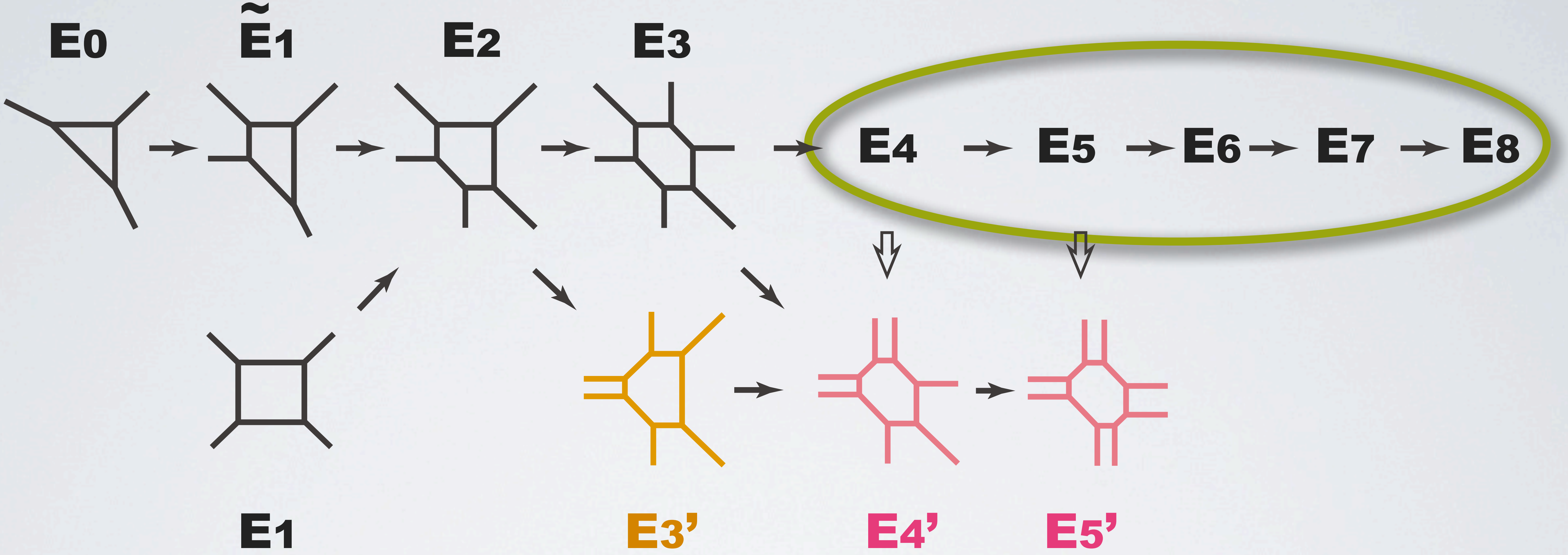
[Bergman-Gomez-Zafrir,'13]

Computing index, we find

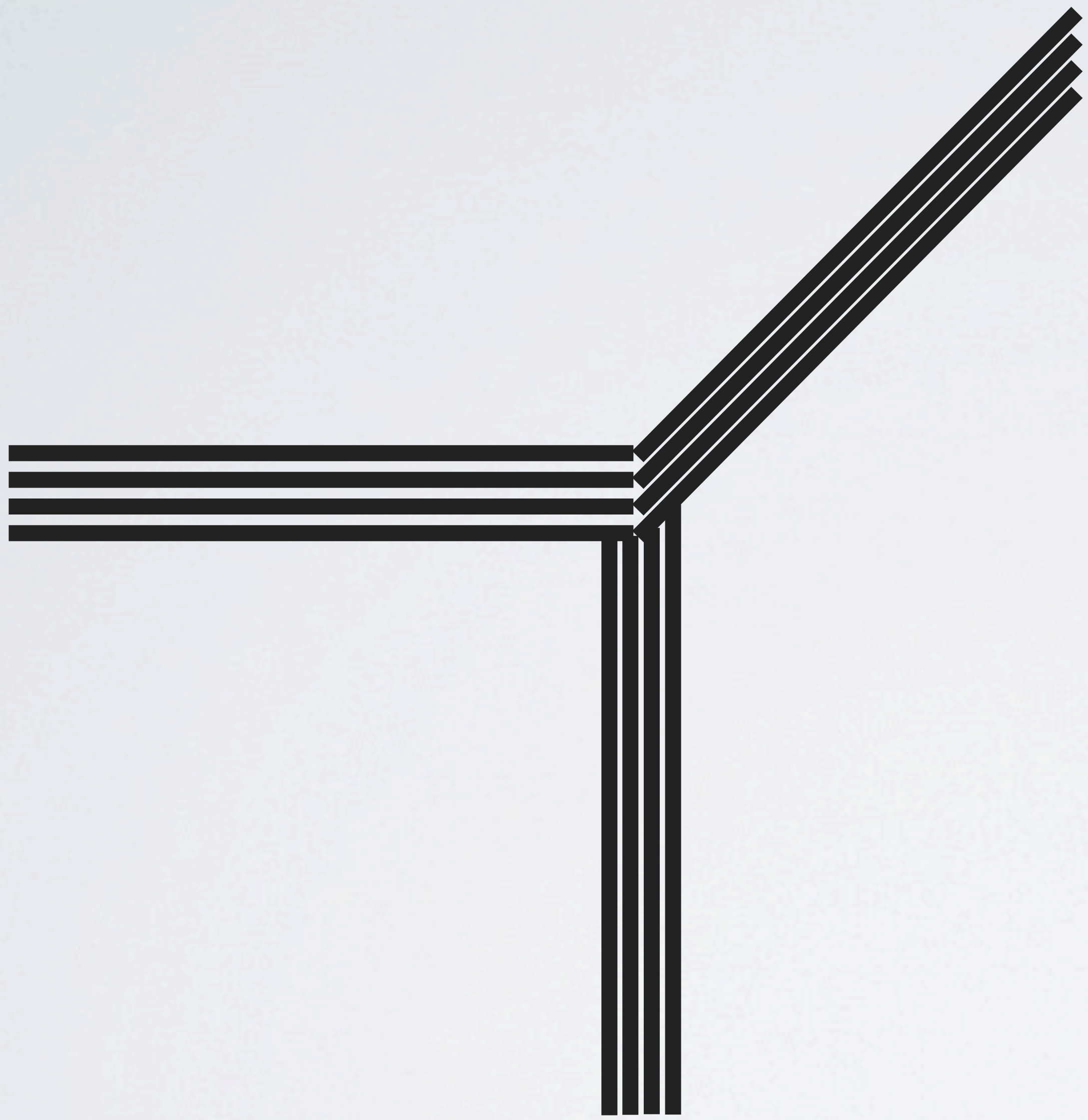


E_1	$\tilde{E}_1$	E_1
SU(2) YM θ=0	SU(2) YM θ=π	SU(2) YM θ=0

generalization: non del Pezzo cases



generalization: T_N junction



N M5-branes

T_4 junction : [Hayashi-Kim-Nishinaka]

5. 5d SCFT
&
AGT relation

4d AGT (weak version)

(Pestun's) partition function on S^4

$$||$$

correlation function of 2d Toda CFT

4d SU(2) theory v.s. 2d Liouville CFT

5d AGT [Awata-Yamada][Nieri-Pasquetti-Passerini][BMPTY]

(Pestun's) partition function on $S^1 \times S^4$

||

correlation function of 2d q-Toda CFT

5d AGT [Awata-Yamada][Nieri-Pasquetti-Passerini][BMPTY]

5d superconformal index $I(x, y, m, u)$

$$||$$

correlation function of 2d q-Toda CFT

5d AGT [Awata-Yamada][Nieri-Pasquetti-Passerini][BMPTY]

5d superconformal index $I(x, y, m, u)$

$$||$$

correlation function of 2d q-Toda CFT

This proposal may resolve the long-standing difficulty of Toda CFT

5d AGT [BMPTY]

correlation function of 2d (q-)Toda CFT

↖

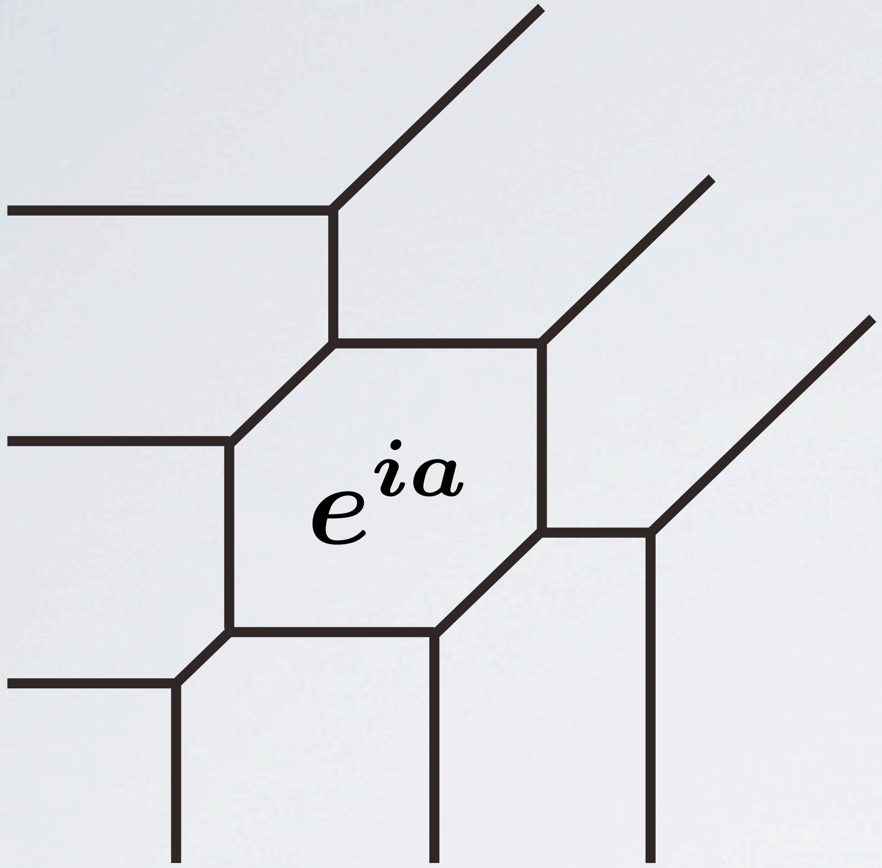
three point functions of W-primaries are not enough !

e.g. A_2 Toda

$$\langle \alpha_1 | (W_{-1})^n V_{\alpha_2} | \alpha_3 \rangle$$

one integral parameter n

5d AGT [BMPTY]

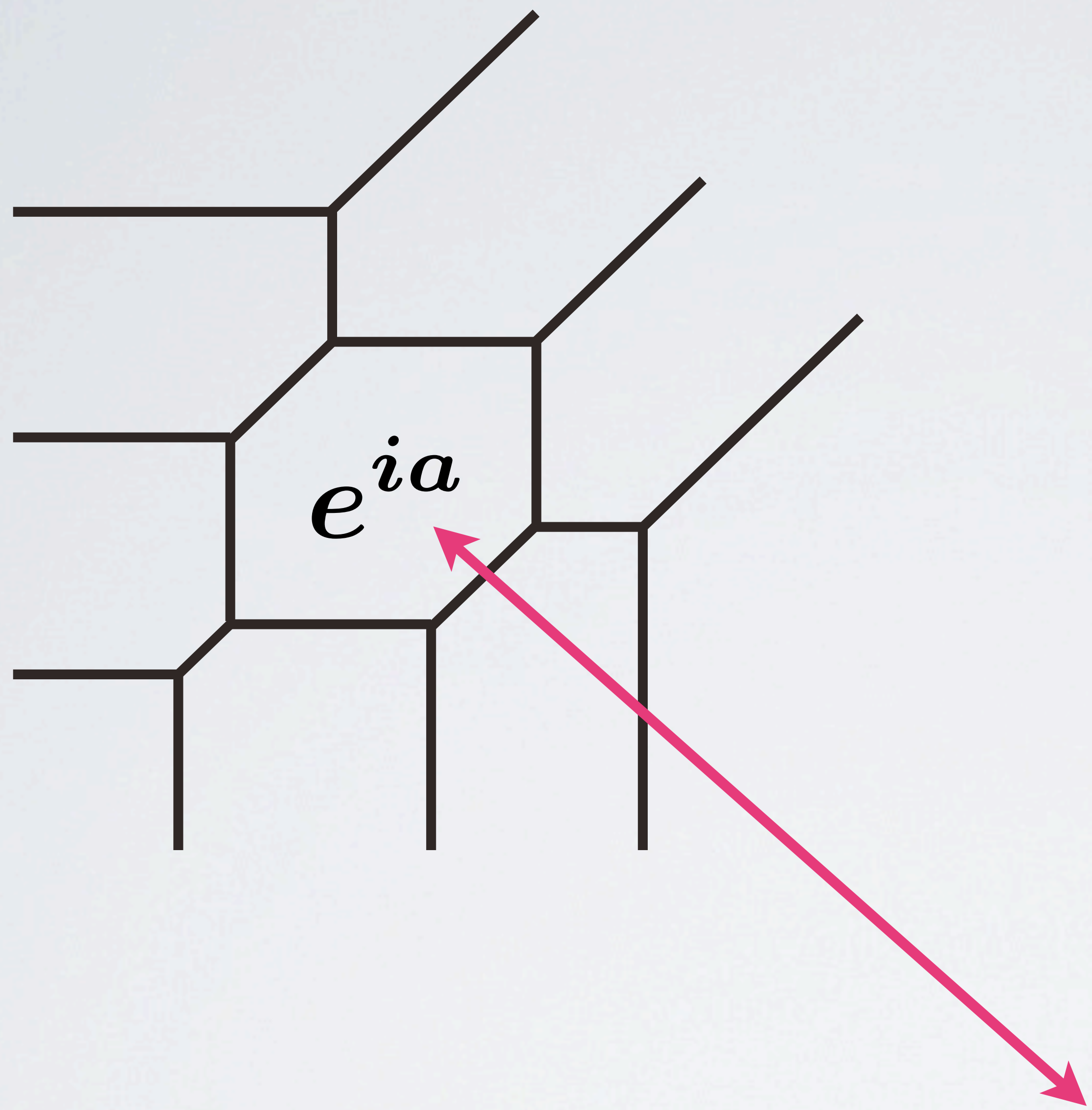


$$I = \oint d(e^{ia}) \cdots$$

$$\langle \alpha_1 | (W_{-1})^n V_{\alpha_2} | \alpha_3 \rangle$$

one integral parameter n

5d AGT [BMPTY]



$$I = \oint d(e^{ia}) \cdots$$

$$\langle \alpha_1 | (W_{-1})^n V_{\alpha_2} | \alpha_3 \rangle$$

6. Summary

we revisit 5d SCFT from the perspective of index

7-brane is now treatable !

5d index is a junction of various topics:
UV CFT, non-toric CY, AGT, ...

FIN