Magnetic Vortex Dynamics

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We review the recent theoretical and experimental achievements on dynamics of spin vortices in patterned ferromagnetic elements. We first demonstrate the theoretical background of the research topic and briefly list the analytical and experimental approaches dealing with magnetic vortices. Then we report on the most remarkable studies devoted to steady state vortex excitations, switching processes, and coupled-vortex dynamic phenomena including the design of artificial crystals where the micromagnetic energy transfer takes place via the magnetic dipolar interaction among excited vortices. Finally we summarize the present state of the research with respect to novel prospects from both the fundamental and the application viewpoints.

KEYWORDS: magnetization, Landau–Lifshitz–Gilbert equation, spin vortex, polarity, chirality, dynamic switching, spin waves, time-resolved Kerr microscopy, permalloy, spin transfer torque

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1. Introduction

One of the most remarkable manifestations of the recent progress in magnetism is the establishment of microfabrication procedures employing modern magnetic materials. Electron or ion beam lithographies combined with the conventional thin film deposition techniques yield a variety of laterally patterned nanoscale structures such as arrays of magnetic nanodots or nanowires.¹,² Among them, submicron ferromagnetic disks have drawn particular interest due to their possible applications in high density magnetic data storage,³ magnetic field sensors,⁴ logic operation devices,⁵ etc.

It has been revealed both theoretically and experimentally that for particular ranges of dimensions of cylindrical and other magnetic elements (Fig. 1) a curling in-plane spin configuration (vortex) is energetically favored, with a small spot of the out-of-plane magnetization appearing at the core of the vortex.⁶–⁸ Such a system, which is sometimes referred to as a magnetic soliton⁹ and whose potentialities have already been discussed in a few recent review papers,¹⁰,¹¹ is thus characterized by two binary properties ("topological charges"), a chirality (counter-clockwise or clockwise direction of the in-plane rotating magnetization) and a polarity (the up or down direction of the vortex core's magnetization), each of which suggests an independent bit of information in future high-density nonvolatile recording media. For this purpose various properties have been investigated such as the appearance and stability of vortices when subjected to quasistatic or short-pulse magnetic fields and variations of those properties when the dots are densely arranged into arrays. The properties are identified with experimentally measured and theoretically calculated quantities called nucleation and annihilation fields, effective magnetic susceptibilities, etc.

Most recently, the time-resolved response to applied magnetic field pulses or spin-polarized electrical currents with sub-nanosecond resolution has been extensively studied, providing results on the time-dependence of the location, size, shape, and polarity deviations of the vortex cores, eigenfrequencies and damping of time-harmonic trajectories of the cores, the switching processes, and the spin waves involved. In this paper we will review the recent achievements in this research area with a particular interest in submicron cylindrical ferromagnetic disks with negligible magnetic anisotropy, for which permalloy (Py) has been chosen as the most typical material. We will demonstrate the theoretical background of the research topic according to the description by Hubert and Schäfer¹² (§2) and briefly describe the achievements in analytical approaches (§3) and experimental techniques (§4). Then we will review the research of various authors devoted to steady state excitations (§5), dynamic switching of vortex states (§6), and excitations of magnetostatically coupled vortices (§7). Finally we will summarize the present state of the research with respect to future prospects and possible applications (§8). We will accompany our description by our simulations using the Object-Oriented Micromagnetic Framework (OOMMF),¹² and in some cases by demonstrative examples provided by their original authors.

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Fig. 1. (Color online) Examples of vortices appearing in a cylindrical (a), rectangular (b), elliptic (c), multilayered (d), and ring-shaped (e) elements. Each vortex's center contains an out-of-plane polarized core except for the ring. Classical multidomain structures appear in larger elements where the anisotropy energy is predominant (f).
2. Theoretical Background

The unique spin distributions favored in ferromagnetic materials are governed by the exchange interaction between nearest neighbor spins $s_i, s_j$ described by the Heisenberg Hamiltonian

$$\mathcal{H} = - \sum_{i<j} J_{ij} s_i \cdot s_j \quad (1)$$

or by more general formulas if particular anisotropies are taken into account. For the sake of solving many-spin problems, the discrete spin distribution is replaced by magnetization $\mathbf{M}(\mathbf{r}, t)$, a continuous function of space and time, or by unit magnetization $\mathbf{m} = \mathbf{M}/M_s$, where $M_s$ is a saturation constant. Accordingly, the total energy of a ferromagnet is determined as the sum

$$E_{\text{tot}} = E_{\text{exch}} + E_d + E_{\text{ext}} + E_{\text{an}} + \cdots, \quad (2)$$

which demonstrates the competition among exchange, demagnetizing, external-field, anisotropy, and other forms of energy (such as magneto-elastic interaction or magnetostriction), whose particular integrals per volume of a ferromagnet are

$$E_{\text{exch}} = \int A(\nabla \mathbf{m})^2 \, dV, \quad (3)$$

$$= - \int \mu_0 M_s \frac{1}{2} \mathbf{m} \cdot \mathbf{H}_{\text{exch}} \, dV \quad (4)$$

$$E_d = - \int \mu_0 M_s \frac{1}{2} \mathbf{m} \cdot \mathbf{H}_{d} \, dV, \quad (5)$$

$$E_{\text{ext}} = - \int \mu_0 M_s \mathbf{m} \cdot \mathbf{H}_{\text{ext}} \, dV, \quad (6)$$

e tc., where $A$ is the exchange stiffness constant, $\mu_0$ the permeability of vacuum, and $\mathbf{H}_{\text{exch}}, \mathbf{H}_{d}, \mathbf{H}_{\text{ext}}$ are the “effective exchange field”, demagnetizing (stray) field, and external magnetic field, respectively, altogether forming the total effective field

$$H_{\text{eff}} = H_{\text{exch}} + H_{d} + H_{\text{ext}} + \cdots. \quad (7)$$

While the exchange interaction forces the nearest spins to align into a uniform distribution, the demagnetizing field makes the opposite effect on the long-range scale. It can be evaluated via a potential $\Phi_d$ as

$$H_d = - \nabla \Phi_d, \quad \nabla^2 \Phi_d = - M_s \rho_d, \quad (8)$$

whose sources are volume and surface “magnetic charges”

$$\rho_d = - \nabla \cdot \mathbf{m}, \quad \sigma_d = \mathbf{m} \cdot \mathbf{n}, \quad (9)$$

where $\mathbf{n}$ is a unit vector normal to the surface of the magnetized element. Hence, the magnetization tends to align parallel to the surface in order to minimize the surface charges, leading to the occurrence of vortex distributions as depicted in Figs. 1(a)–1(c). Moreover, the singularity at the center of a vortex is replaced by an out-of-plane magnetized core in order to reduce the exchange energy. On the other hand, in large samples, where the anisotropy energy predominates the surface effects of the disk edges, the magnetization forms conventional domain patterns with magnetization aligned along easy axes [Fig. 1(f)].

When we slowly apply an external magnetic field, the competition among all the energies breaks the symmetry of the vortex, shifting its core so that the area of magnetization parallel to the field enlarges, until the vortex annihilates (at the “annihilation field”), resulting in the saturated (uniform) state. Then, when we reduce the external field, the uniform magnetization changes into a curved “C-state”, until the vortex nucleates again (at the “nucleation field”). Reducing the field further to negative values causes the symmetrically analogous process, as depicted in Fig. 2.

For the vortex dynamics the main area of interest is the range of states before the vortex annihilates, which is represented in Fig. 2 by the slightly curved line whose tangent $\gamma = \delta M/\delta H$ is called the effective magnetic susceptibility defined both statically and dynamically as a function of frequency $\omega$. The dynamic response to fast changes of external field is considerably different from that described by the hysteresis loop, and is in general governed by the Landau–Lifshitz–Gilbert (LLG) equation

$$\frac{\partial \mathbf{m}}{\partial t} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}, \quad (10)$$

where $\gamma$ denotes the gyromagnetic ratio, $\alpha$ the Gilbert damping parameter, and $t$ the time.

Instead of applying external field, the vortex distribution can be excited by an electrical current propagating through the ferromagnetic disk.\textsuperscript{14–16} It has been revealed that this process, referred to as spin-transfer torque (STT), can be (in the adiabatic approximation) evaluated as an additional term on the right-hand side of the LLG equation

$$T_{\text{STT}}^{(1)} = - (v_i \cdot \nabla) \mathbf{m}, \quad (11)$$

$$T_{\text{STT}}^{(2)} = - g \frac{h}{2e} \mathbf{m}_2 \times (\mathbf{m}_2 \times \mathbf{m}_1), \quad (12)$$

where the first equation corresponds to the in-plane current with the velocity $v_i = j_i P g \mu_B/2e M_s$, whereas the second corresponds to the perpendicular current propagating through a multilayer depicted in Fig. 1(d) from the bottom to the top ferromagnetic layer (with magnetization distributions $\mathbf{m}_1, \mathbf{m}_2$, respectively), both of which are separated by a thin nonmagnetic interlayer (F/N/F). Here $j_i, P, g, \mu_B, e, h$, and $I_c$ denote the current density, spin polarization, $g$ value of an electron, Bohr magneton, electronic charge, Planck constant, and total electrical current, respectively.
3. Analytical Approaches

Although most authors have adopted OOMMF for its generality, simplicity, and accuracy, the development of analytical approaches is very useful for analyzing various fundamental aspects of dynamic processes. Among many attempts to reduce the number of parameters involved in vortex dynamics, perhaps the most used is treating the vortex as a quasiparticle whose motion (or motion of its center \(a = [a_x, a_y]\)) is described by an equation derived from the LLG equation by Thiele\(^{17}\) for magnetic bubbles and adopted by Huber\(^{18}\) to vortex systems,

\[
G \times \frac{da}{dt} = \frac{1}{R^2} \frac{dE_{\text{stat}}}{da} - \mathbf{D} \cdot \frac{da}{dt},
\]

where \(G = -2\pi \mu_0 M_s \mathbf{z}/\gamma\) is the gyrovector with \(p = \pm 1\) denoting the vortex’s polarity (the positive value stands for the up direction, parallel to the unit vector \(\mathbf{z}\)) and \(L\) denoting the disk’s thickness, and where \(\mathbf{D} = -2\pi \mu_0 M_s(\mathbf{\hat{x}} \times \mathbf{\hat{y}})/\gamma\) is the dissipation tensor of the second order. Thiele’s equation of motion thus found the use as one of the most convenient approaches of dealing with vortex dynamics and has further been generalized to include an additional term of “mass times acceleration”\(^{19,20}\) or to take into account STT.\(^{21}\)

To perform simulation with Thiele’s equation, one needs to evaluate \(E_{\text{stat}}(\mathbf{a})\) as a function of the vortex center’s position. For this purpose, two approximations have been utilized, the “rigid vortex” model,\(^{22–25}\) assuming the static position. For this purpose, two approximations have been undertaken to reduce the number of parameters involved in vortex magnetometry,\(^8,22\) and others. Lorentz transmission electron microscopy,\(^49,50\) magnetic force microscopy (MFM),\(^7\) spin-polarized scanning tunneling microscopy,\(^6\) magnetoresistance and Hall effect measurements,\(^33–48\) Lorentz transmission electron microscopy,\(^49,50\) magneto-optical Kerr effect (MOKE) measurements,\(^51–59\) photoelectron emission microscopy,\(^60–62\) scanning electron microscopy with spin-polarization analysis (SEMPA),\(^63,64\) and others.

On the other hand, different techniques have to be employed for time-resolved dynamic measurement such as time-resolved Kerr microscopy (TRKM) in the scanning,\(^65–72\) or wide-field mode,\(^73–75\) photoemission electron microscopy combined with pulsed x-ray lasers,\(^70\) Brillouin light scattering (BLS),\(^77\) time-resolved MFM,\(^78\) ferromagnetic resonance (FMR) technique,\(^79,80\) vector network analyzers,\(^81\) superconducting quantum interference device magnetometry,\(^82\) and others.

The most typical measurement technique, TRKM, often referred to as a “pump–probe” technique, combines a Kerr microscope of high space–time resolution achieved by ultrashort-pulse laser light source and high-quality microscopic imaging (the “probe”), and a system for operating ultrashort pulse excitations achieved practically via various transmission line configurations as depicted in Fig. 3 (the “pump”). The source for the excitation current can be generated either by a pulse generator (triggered by the laser control device) or by a photoconductive switch (when laser pulses are split between probe and pump pulses). The wavelength of light is often halved by a second harmonic generation device to increase the spatial resolution of measurement. The time dependence of magnetization evolution after excitation is determined by changing the delay time between the pump and the probe. To obtain an appropriate signal-to-noise ratio, the pump–probe measure-

\[
\begin{align*}
\dot{\theta}(r, \chi) &= \sum_{n} \sum_{m=\infty} f_{nm}(r) \cos(m\chi + \omega_m t + \delta_m), \\
\mu(r, \chi) &= \sum_{n} \sum_{m=\infty} g_{nm}(r) \sin(m\chi + \omega_m t + \delta_m),
\end{align*}
\]

where \([n,m]\) is a full set of numbers labeling magnon eigenstates and \(\delta_m\) are arbitrary phases. This approach has been successfully applied to both antiferromagnets\(^{30,31}\) and ferromagnets,\(^32–42\) and has revealed eigenfrequencies and eigenfunctions of spin wave modes propagating in cylindrical disks and \(S\)-matrices of magnon–vortex scattering.

4. Experimental Techniques

Experimental measurements of quasistatic properties of magnetic elements giving clear evidence of vortex structures, including the core’s shapes and quasi-static switching processes, have been carried out by magnetic force microscopy (MFM),\(^7\) spin-polarized scanning tunneling microscopy,\(^6\) magnetoresistance and Hall effect measurements,\(^33–48\) Lorentz transmission electron microscopy,\(^49,50\) magneto-optical Kerr effect (MOKE) measurements,\(^51–59\) photoelectron emission microscopy,\(^60–62\) scanning electron microscopy with spin-polarization analysis (SEMPA),\(^63,64\) and others.

Fig. 3. (Color online) Various types of ultrafast excitations: in-plane (a) or out-of-plane (b,c) magnetic field pulses are generated by electrical current pulses propagating through transmission lines with appropriate geometries; out-of-plane (d) and in-plane (e) currents induce excitations based on spin transfer torque.
ment must be repeated many times with exactly the same initial condition, referred to as a stroboscopic method. An example of TRKM measurement, showing radial modes propagating from the edges of a Co cylindrical dot excited by out-of-plane field, is displayed in Fig. 4.

5. Steady State Motion Phenomena

Various authors have studied dynamic excitations of vortices in cylindrical disks to observe a rich spectrum of modes. Besides the existence of the radial modes excited by out-of-plane field (Fig. 4), it has been also revealed that low-energy modes (those near the ground state) excited by in-plane field can be classified into two elementary types.

The first type, referred to as the gyrotropic mode, is an oscillatory motion of the vortex core around its position in equilibrium, whose numerical simulation is displayed in Fig. 5 for a Py disk with the diameter of 100 nm and thickness of 20 nm. This type of motion has been predicted as the solution of Thiele’s equation [eq. (13)], as the analytical solution of the LLG equation in the angular variables [eqs. (17) and (18)], and has been extensively studied by micromagnetic simulations and various experiments. It has been revealed that the core’s initial motion is parallel or antiparallel to the applied magnetic field pulse, depending on the vortex’s “handedness” (the polarity relative to the chirality). However, the clockwise or counter-clockwise sense of the core’s spiral motion only depends on the vortex’s polarity and is independent of the chirality. Owing to this rule, the vortex polarity can be magneto-optically measured via this dynamic motion, even from both the fundamental and the application viewpoints. Therefore, the current-induced motion of vortices has also been investigated to reveal phenomena analogous to those managed by the field excitation.

6. Dynamic Switching

During the last few months, immensely intensive work has been carried out to study the process of dynamic switching of vortex polarities and chiralities, which is particularly important for the data storage application. Traditionally, to switch the vortex core’s polarity, an extremely large quasi-static out-of-plane magnetic field was required. Moreover, to control chirality, the disk had to be fabricated
with a geometric asymmetry, e.g., with a "D-shape" or other shapes. Unlike that, dynamic processes have revealed considerable advantages.

Several authors have recently demonstrated that a short pulse of in-plane magnetic field of a certain amplitude and duration excites the vortex so that a pair of a new vortex and an antivortex is created, the new vortex possessing the opposite polarity, and that the antivortex annihilates together with the old vortex, as depicted in Fig. 7 for a cylindrical disk with the diameter of 200 nm and thickness of 20 nm. This process is fully controllable by applying an appropriate file pulse whose amplitude is considerably smaller than those which are necessary for quasistatic switching. The process has also been successfully observed by experimental measurements, and its variations and further details are presently researched.

For applications in spintronics, to control the switching process via an electrical current is of particular interest. In this respect various authors have recently carried out theoretical and experimental studies to reveal that the similar dynamic switching processes are possible by applying STT excitations in the both configurations as described by eqs. (11) and (12).

Similarly to the quasistatic case, to change the vortex chirality requires introducing some geometric asymmetry into the process. For this purpose, Choi et al. have applied a perpendicular current pulse to an F/N/F nanopillar, where the asymmetry is due to the magnetostatic interaction between the vortices in the two ferromagnetic layers. However, the full and reproducible control of the both binary parameters of a vortex is still a demanding task.

7. Magnetostatically Coupled Vortices

Many studies of quasistatic processes have been performed on pairs, chains, and two-dimensional arrays of magnetostatically coupled vortices, but little attention was paid to their dynamic properties, although putting magnetic disks near each other is of high importance for improving the density of data storage and studying the propagation of micromagnetic excitations through such arrays. Among dynamic studies, the pioneering experiments have been performed by means of BLS.

Recently the dynamics of magnetostatically coupled vortices was studied in pairs of disks placed near each other laterally, vertically (as an F/N/F nanopillar), and as a pair of two vortices located inside a single elliptic dot. It has been revealed, e.g., that the eigenfrequency of the synchronized steady-state motion of two vortices in the lateral arrangement is split into four distinct levels whose values depend on the lateral uniformity of the vortices’ excitation and on the combination of their polarities (but are independent on chiralities).

Large arrays of coupled vortices have also been investigated to reveal a close analogy with crystal vibrations (or phonon modes) in two-dimensional atomic lattices. The dispersion relations and the corresponding densities of states of propagating waves of vortex excitations were found to vary with different ordering of vortex polarities regularly arranged within nanodisk arrays. Moreover, for arrays of disks with two different alternating diameters a forbidden band gap has been observed, pointing at the analogy with the band gap between the acoustic and optical phonon modes in atomic lattices. Collective excitation modes have also been studied in small arrays of nanodisks and as analytical calculations using the Bloch theorem which enables dealing with infinite arrays.
8. Conclusions and Perspectives

We have reviewed the most fundamental achievements on the dynamic properties of magnetic vortices with a particular interest in soft cylindrical ferromagnetic disks. We have demonstrated the basic theoretical background widely used in analytical and numerical simulations, and briefly listed the utilized experimental approaches (including a description of TRKM as the most typical method of measurement). Then we have demonstrated the most significant results achieved by various authors. First, we have shown that the elementary excitations near the ground vortex state (steady state motions) are the important starting point to understand the whole principle of spin vortex dynamics. Then we have reported the recent results of ultrafast magnetic-field and STT based switching of the vortex binary properties (polarity and chirality) which are of high importance for the possible future application in nonvolatile magnetic recording media. Finally we have briefly presented a few potentialities of vortices densely arranged into arrays or multilayers, which were found important, e.g., for designing novel artificial metamaterials which possess propagation modes based on magnetostatic interactions between nearest neighbor elements.

The contemporary research continues in the tendency of pushing the limits of the up-to-date theoretical and measurement capabilities and exploring new directions of studying fundamental physical phenomena and utilizing them in new or higher-level applications. As regards the theoretical capabilities, reducing the element sizes to the true nanoscale requires the generalization of models to allow for the effect of surfaces and interfaces on atomic scale,11 quantum and nonlinear effects (such as nonlinear optical excitations of spins usable for entirely optical switching112,113), etc., for which the first-principle calculation will probably be employed. As regards experiment, the tendency of reducing sizes will require not only the improvement of the spatial resolution14 (for which novel techniques are interesting such as magnetic exchange force microscopy with atomic resolution15), but also further increase of the time resolution of dynamic measurements.11 Moreover, since the stroboscopic measurement of the ultrafast dynamics requires repetition with always equal initial conditions, this method cannot be used to study possible stochastic processes, which determines a challenge to develop new experimental conceptions. Another challenge from both the theoretical and the experimental viewpoints is the modulation of the dynamic properties of vortices by virtual fab10 or artificial defects designed by tricky methods of deposition116 or etching117, leading to considerable increase of vortices’ sensitivity to fields with particular strengths or frequencies. We can thus conclude that the results of numerous dynamics studies have pointed out various advantages and new potentialities in data storage, nanoscale probing of magnetic thin-film structures, and other types of sensing or controlling.

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YoshiChika Otani was born in Tokyo, Japan, in 1960. He obtained his B. Sc. (1984), M. Sc. (1986), and Ph. D. (1989) degrees from Keio University. He was a research fellow (1989–1991) at the Physics Department of the Trinity College, University of Dublin, a researcher (1991–1992) at the Laboratoire Louis Néel, CNRS. Then he was appointed to a research instructor (1992–1995) at the Department of Physics, Keio University, an associate professor at the Department of Materials Science, Graduate School of Engineering, Tohoku University, and a head (since 2002) of the Quantum Nano-Scale Magnetics Laboratory at FRS-RIKEN. Since 2002 he has also been a professor at ISSP, University of Tokyo. He has been primarily working on experimental studies on spin electronics such as magnetic and transport properties of nanostructured magnetic/nonmagnetic (superconductive) hybrid systems including vortex dynamics confined in magnetic nanodisks.

Junya Shibata was born in Osaka Prefecture, Japan, in 1974. He obtained his B. Sc. (1996) degree from Ritsumeikan University, M. Sc. (1998), and D. Sc. (2001) degrees from Tohoku University. He was a postdoctoral researcher (2001–2002) at Osaka University and (2002–2007) at the Institute of Physical and Chemical Research (RIKEN). Since 2007 he has been an associate professor at Kanagawa Institute of Technology. He has worked on the theory of condensed matter physics in nanoscale magnetism. His research is now focused on current-induced magnetization dynamics and current generation induced by magnetization dynamics.