Fast and Faithful Scale-Aware Image Filters

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Workshop in Homage to Prof. T. L. Kunii, September 2021

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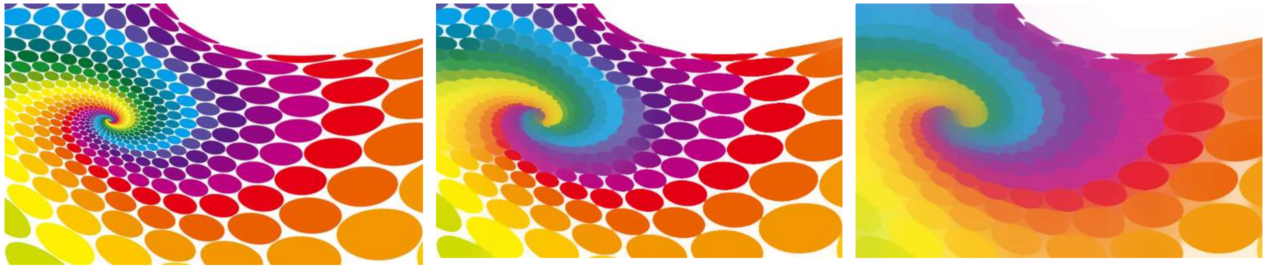
What's this talk about ?

✓ Fast & accurate framework for scale-aware image filters.

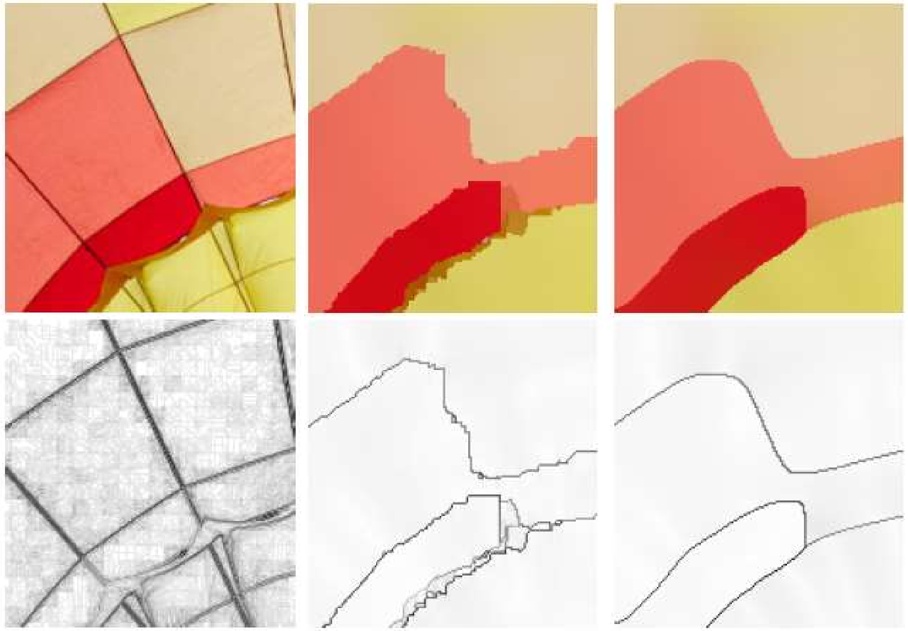
- Idea: Domain Transform & Splitting → joint (guided) Gaussian averaging.



Contributions & Benefits:

- Fast: linear complexity O(n) & O(1/σ).
- Faithful: Gaussian scale-space.
- Accurate: stable & 525σ truncation.

→ Prevent ringing artifact/phantom edges.



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In memory of Prof. Toshiyasu Kunii

- ✓ When he had been the president of University of Aizu,
  - I spent my undergraduate and master programs there.
- ✓ Also, when I had been the student of Prof. A. Belyaev in the Computer Science and Engineering Laboratory of Univ. Aizu, where Prof. Kunii was the head of laboratory.
  - Geometry Processing: e.g., Medial Axis.
  - Shape Modeling Int. and MPI-Inf. etc.
  - Research Philosophy and Passion.

Prof. T. Kunii, IEEE 2021.

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In memory of Prof. Toshiyasu Kunii

- ✓ My paper in this research dedicated to Prof. Kunii's memory was inspired from their educations of the professors of University of Aizu.

Fast and Faithful Scale-Aware Image Filters

Image Processing (**Scale-Space**) by ***Prof. S. Mori*** + Geometry (**Domain Transform**) by ***Prof. A. Belyaev*** + Numerical Analysis (**Domain Splitting**) by ***Prof. Y. Ikebe***

Prof. T. Kunii, IEEE 2021.

Hand picked/invited by Prof. Kunii for the founding University of Aizu.

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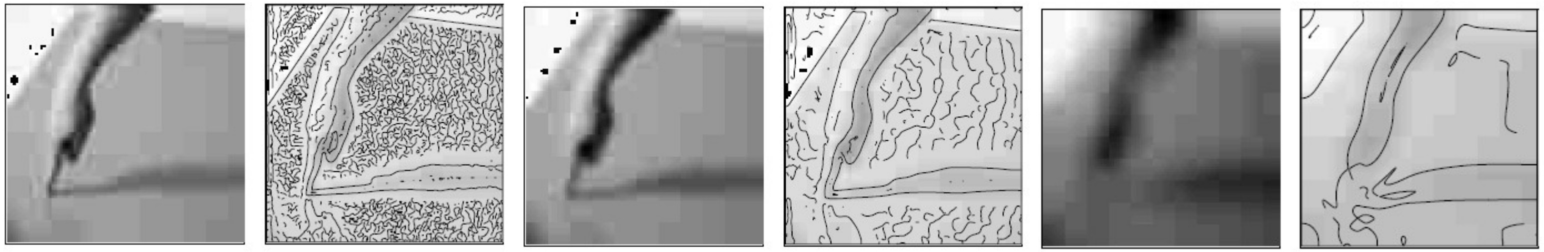
Scale-Space & Edge-Awareness

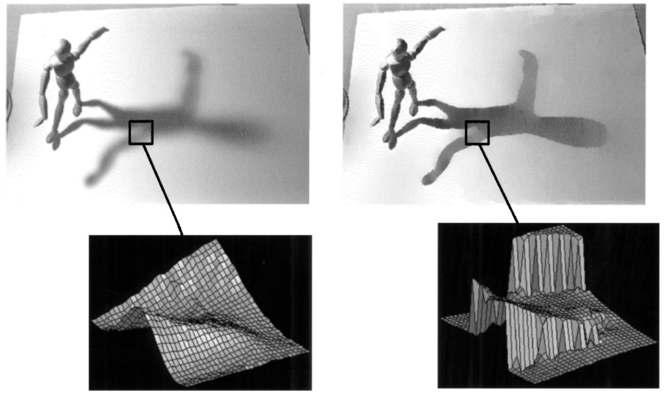
✓ Remove structures smaller than scale σ: multi-scales.

– Linear (Gaussian) space-space: *T. Iijima 1959, J. Stansfield 1980, A. Witkin, IJCAI 1983.*

+ salient edges



T. Lindeberg, Handb. Comput. Vis. Appl. 1999.



J. Elder, IJCV 1999.

✓ Edge preserving filters: bilateral, non-local means, nonlinear diffusions, etc.

– Difficult to control parameters: especially, its number of iterations.

Iterative Non-Local Means

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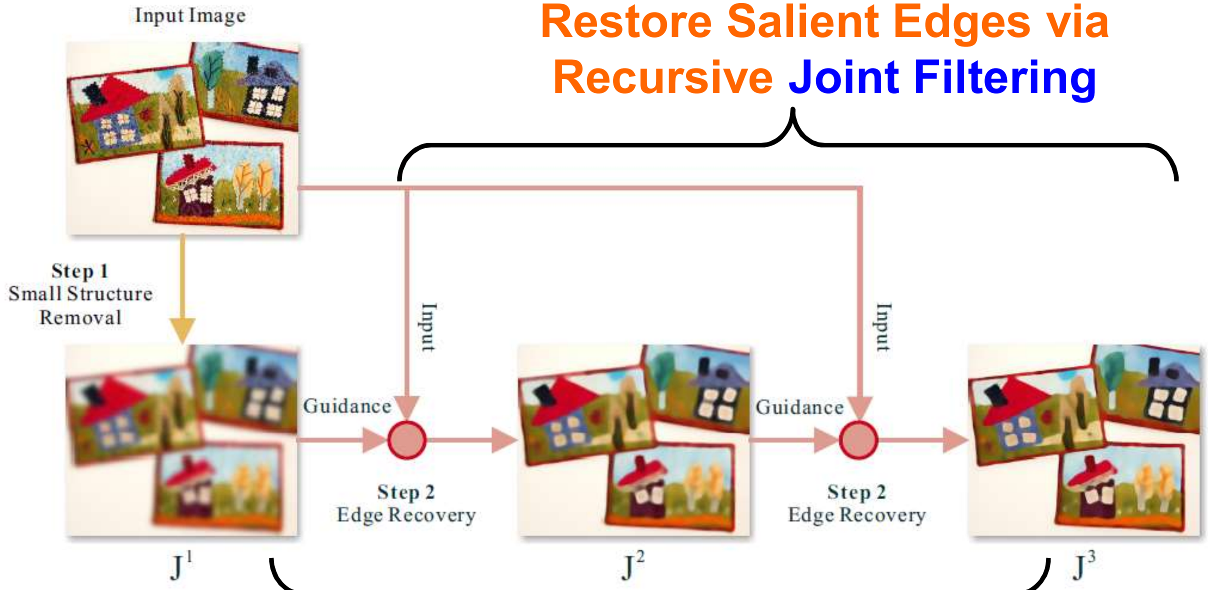
Scale-Aware Filtering

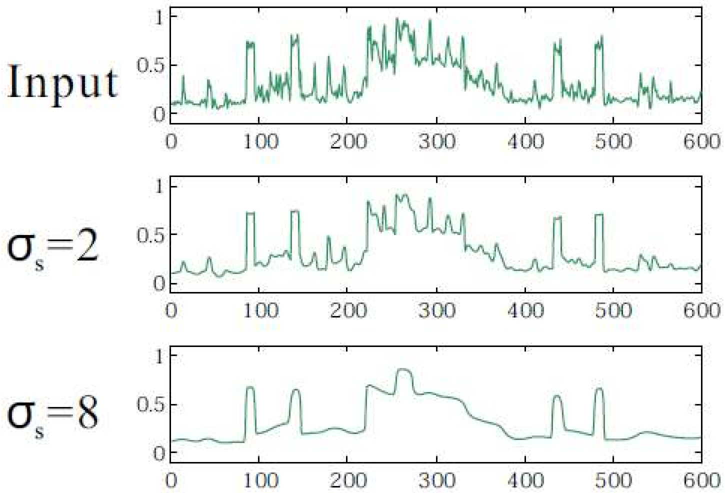
✓ RG: Rolling Guidance: *Q. Zhang et al. ECCV'14.*

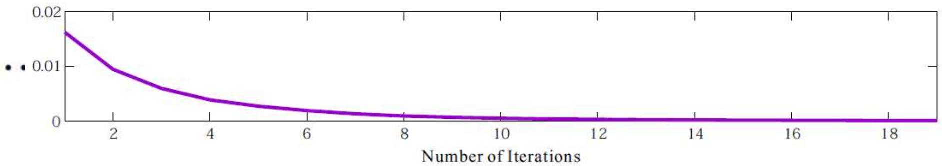
Restore Salient Edges via Recursive **Joint Filtering**



Initial Gaussian Smoothing

Guide Images





✓ Required a fast joint filter: Naive $O(n^2)$.

- Conventional (e.g., box, recursive) → Not enough accuracy → artifacts !

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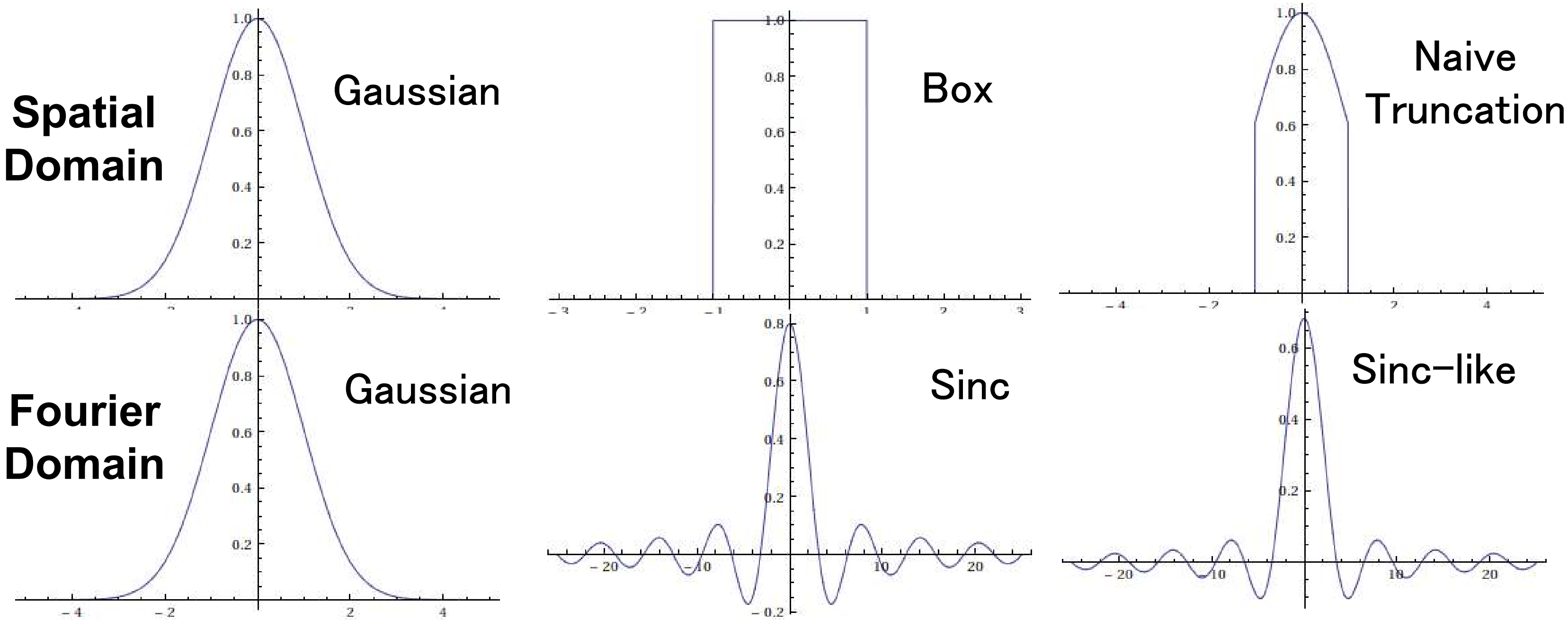
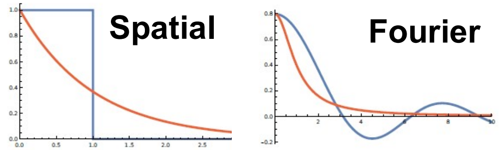
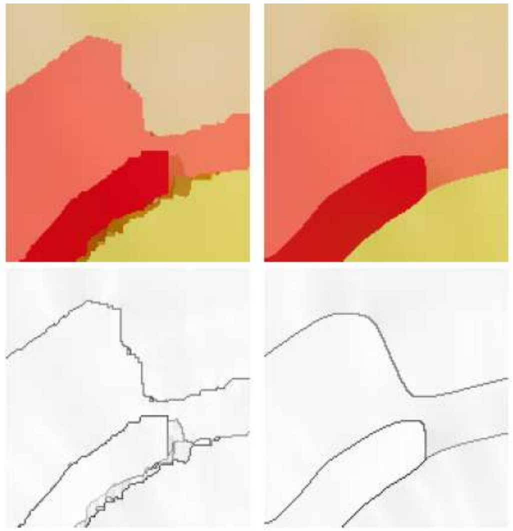
Why artifacts & how to avoid them ?

✓ Inaccurate (and/or non-Gaussian) linear convolutions produce phantom edges (Ringing artifacts) because of the ripples in Fourier domain.





Box & L^1 Gaussian



Box-based Our

✓ L^1 and L^2 Gaussian convolutions do not increase extrema.

T. Lindeberg, in Gaussian Scale-Space Theory 1997.

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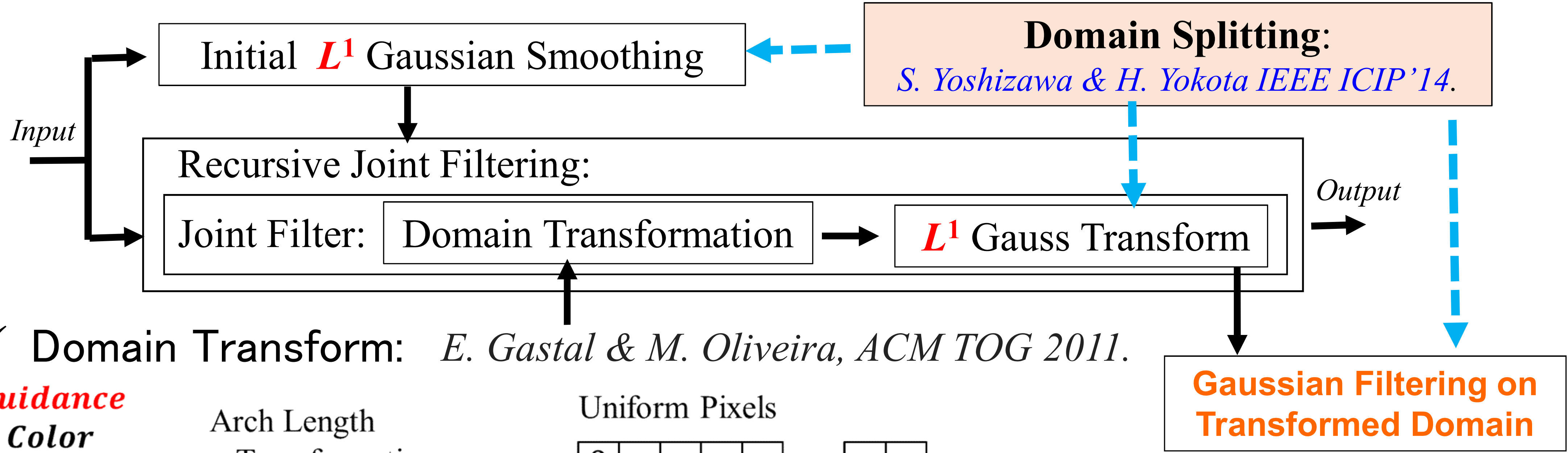
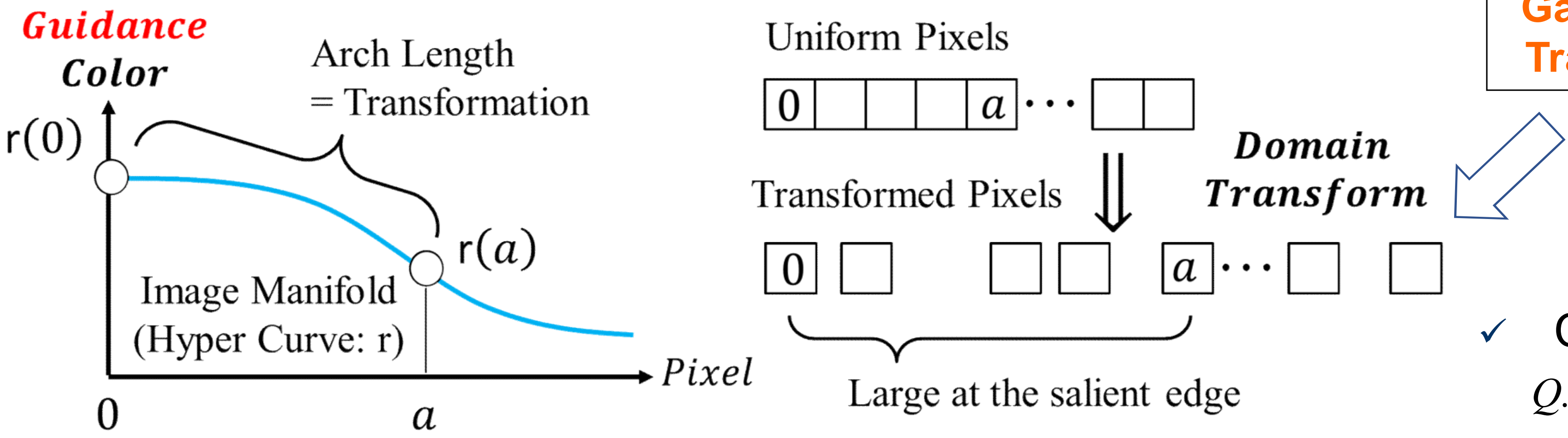
Our Framework



✓ Domain Transform: *E. Gastal & M. Oliveira, ACM TOG 2011.*



✓ Guidance Domain : *Q. Zhang et al. ECCV'14.*

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Domain Splitting

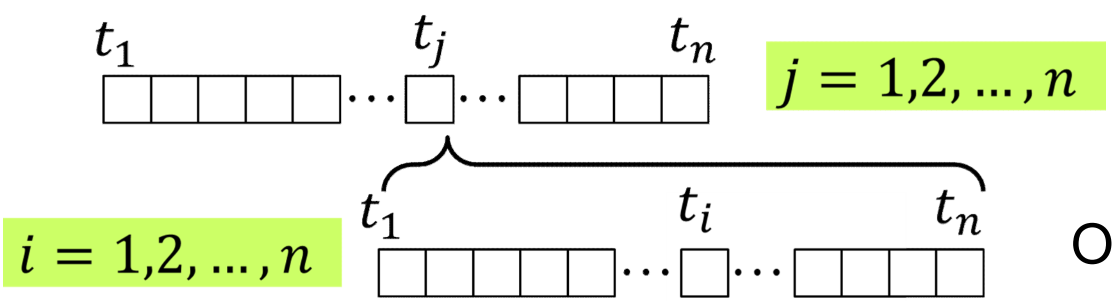
✓ Approximation technique for L^1 Gaussian convolution: *S. Yoshizawa et al., IEEE ICIP'14.*

– Decomposing the integral domain by parts via the representative points $\{\alpha_k\}$ (*poles*) to resolve dependency of ***i*** and ***j*** indexes of the Gauss transform:

$$f(t_j) = \sum_{i=1}^{n} G_\sigma(t_j - t_i)h(t_i) \qquad t_1 \le t_2 \le \ldots \le t_n,$$

Pixel Coordinates

$h(\cdot)$: Input $\quad G_\sigma(\cdot)$: L^1 Gaussian $\quad f(\cdot)$: Output $\quad n$: Pixel Number



$$G_\sigma(t_j - t_i) = \begin{cases} \dfrac{G_\sigma(t_i - \alpha_k)}{G_\sigma(t_j - \alpha_k)} & \text{if } t_j \in \Omega_k^1 \\ \dfrac{G_\sigma(t_j - \alpha_k)}{G_\sigma(t_i - \alpha_k)} & \text{if } t_j \in \Omega_k^2 \\ G_\sigma(t_j - \alpha_k)G_\sigma(t_i - \alpha_k) & \text{if } t_j \in \Omega_k^3 \end{cases}$$

$O(n^2) \rightarrow O(n)$

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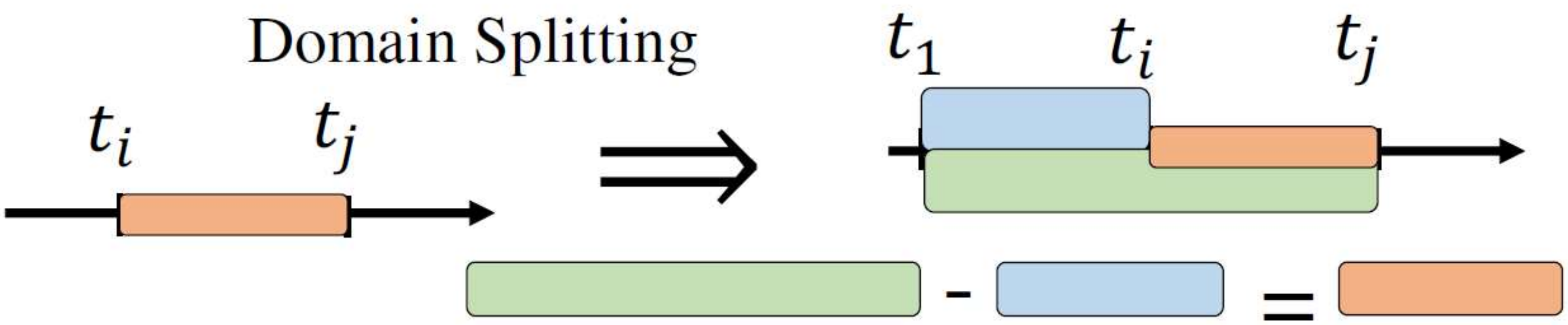
Basic Decomposition Concept

✓ Splitting L[1] norm by introducing an anchor point t_1 : $t_1 \leq t_2 \leq \ldots \leq t_n,$

Pixel Coordinates

$$\left|t_j - t_i\right| = \begin{cases} |t_i - t_1| - \left|t_j - t_1\right| & \text{if } \; t_1 \leq t_j \leq t_i : t_j \in D_1 \\ -|t_i - t_1| + \left|t_j - t_1\right| & \text{if } \; t_1 \leq t_i \leq t_j : t_j \in D_2 \end{cases}$$



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Basic Decomposition Concept

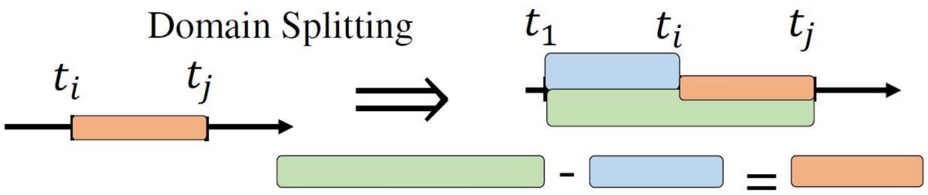
✓ Splitting L[1] norm by introducing an anchor point t_1 : $t_1 \le t_2 \le \ldots \le t_n,$

Pixel Coordinates

$$G_\sigma(x) = \exp\left(-\frac{|x|}{\sigma}\right)$$

$$\exp(x+y) = \exp(x)\exp(y)$$

$$\exp(x-y) = \frac{\exp(x)}{\exp(y)}$$



$$G_\sigma(t_j - t_i) = \begin{cases} \dfrac{G_\sigma(t_i - t_1)}{G_\sigma(t_j - t_1)} & \text{if } t_j \in D_1 \\ \dfrac{G_\sigma(t_j - t_1)}{G_\sigma(t_i - t_1)} & \text{if } t_j \in D_2 \end{cases}$$

$$|t_j - t_i| = \begin{cases} |t_i - t_1| - |t_j - t_1| & \text{if } t_1 \le t_j \le t_i : t_j \in D_1 \\ -|t_i - t_1| + |t_j - t_1| & \text{if } t_1 \le t_i \le t_j : t_j \in D_2 \end{cases}$$

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One Anchor to Multi-poles

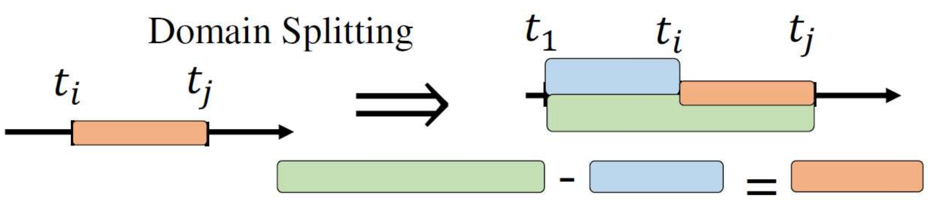
✓ To solve the numerical instability, $\{\alpha_k\}$ is introduced.

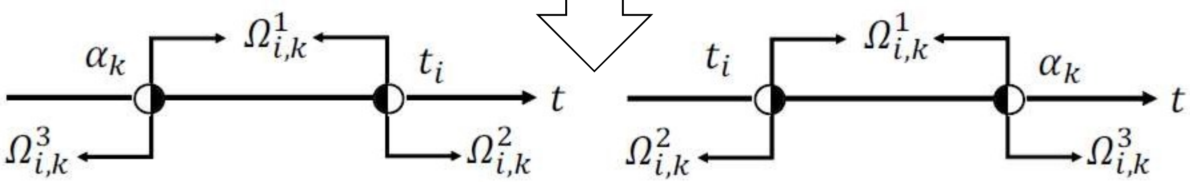
$$t_1 \leq t_2 \leq \ldots \leq t_n,$$

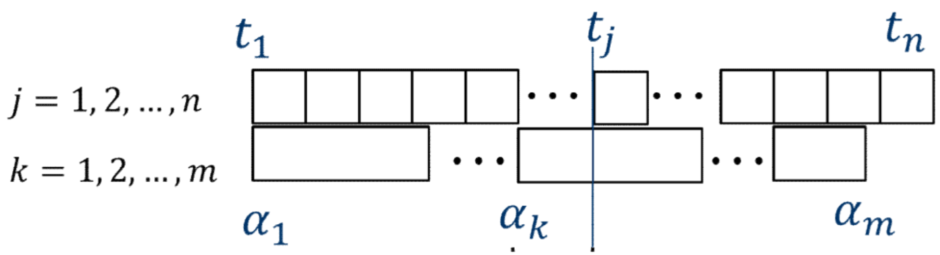
Pixel Coordinates

$$G_\sigma(t_j - t_i) = \begin{cases} \dfrac{G_\sigma(t_i - t_1)}{G_\sigma(t_j - t_1)} & \text{if } t_j \in D_1 \\ \dfrac{G_\sigma(t_j - t_1)}{G_\sigma(t_i - t_1)} & \text{if } t_j \in D_2 \end{cases}$$

$$G_\sigma(t_j - t_i) = \begin{cases} \dfrac{G_\sigma(t_i - \alpha_k)}{G_\sigma(t_j - \alpha_k)} & \text{if } t_j \in \Omega_k^1 \\ \dfrac{G_\sigma(t_j - \alpha_k)}{G_\sigma(t_i - \alpha_k)} & \text{if } t_j \in \Omega_k^2 \\ G_\sigma(t_j - \alpha_k)G_\sigma(t_i - \alpha_k) & \text{if } t_j \in \Omega_k^3 \end{cases}$$







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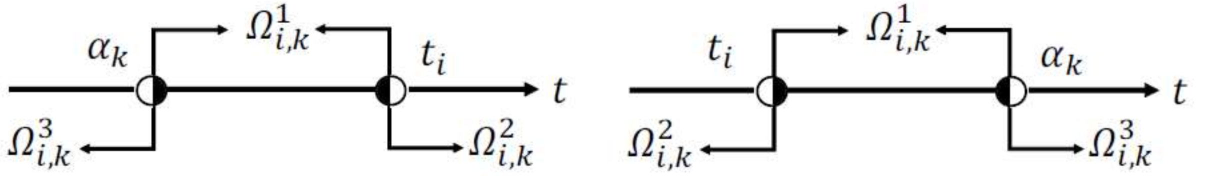
Domain Splitting



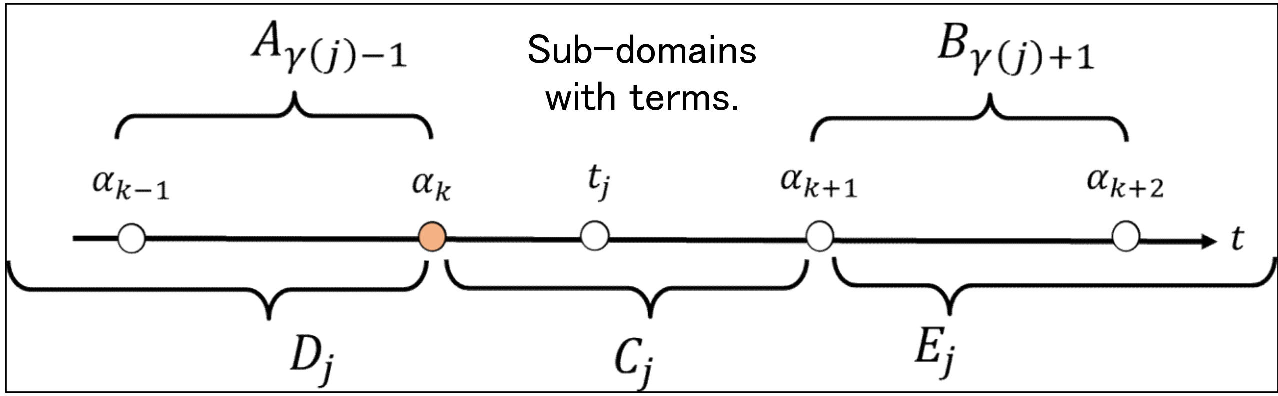
$$G_\sigma(t_j - t_i) = \begin{cases} \dfrac{G_\sigma(t_i-\alpha_k)}{G_\sigma(t_j-\alpha_k)} & \text{if } t_j \in \Omega^1_k \\ \dfrac{G_\sigma(t_j-\alpha_k)}{G_\sigma(t_i-\alpha_k)} & \text{if } t_j \in \Omega^2_k \\ G_\sigma(t_j-\alpha_k)G_\sigma(t_i-\alpha_k) & \text{if } t_j \in \Omega^3_k \end{cases}$$

Sub-domains

$$f(t_j) = \sum_{i=1}^{n} G_\sigma(t_j - t_i)h(t_i)$$

Substitution

Factorization for each sub-domains



$$f(t_j) = h(t_j) + C_j + D_j + E_j,$$

$$C_j = \{G_{(\sigma,j,\gamma(j))} \sum_{i=\gamma_2(\gamma(j))}^{j-1} \frac{h(t_i)}{G_{(\sigma,i,\gamma(j))}}\} +$$

$$+\{\frac{1}{G_{(\sigma,j,\gamma(j))}} \sum_{i=j+1}^{\gamma_2(\gamma(j)+1)-1} G_{(\sigma,i,\gamma(j))}h(t_i)\},$$

$$D_j = \sum_{k=1}^{\gamma(j)-1} G_{(\sigma,j,k)}A_k, \quad E_j = \sum_{k=\gamma(j)+1}^{m} G_{(\sigma,j,k)}B_k,$$

$$A_k = \sum_{i=\gamma_2(k)}^{\gamma_2(k+1)-1} \frac{h(t_i)}{G_{(\sigma,i,k)}}, \quad B_k = \sum_{i=\gamma_2(k)}^{\gamma_2(k+1)-1} G_{(\sigma,i,k)}h(t_i)$$

where $G_{(\sigma,l,k)} \equiv G_\sigma(t_l - \alpha_k)$, $\gamma(j) = k$, and $\gamma_2(k) = \min(j)$ such that $\alpha_k \leq t_j < \alpha_{k+1}$,

$$D_j \approx G_{(\sigma,j,\gamma(j)-1)}A_{\gamma(j)-1} \qquad E_j \approx G_{(\sigma,j,\gamma(j)+1)}B_{\gamma(j)+1}$$

✓ See the paper for detailed equations.

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Domain Splitting

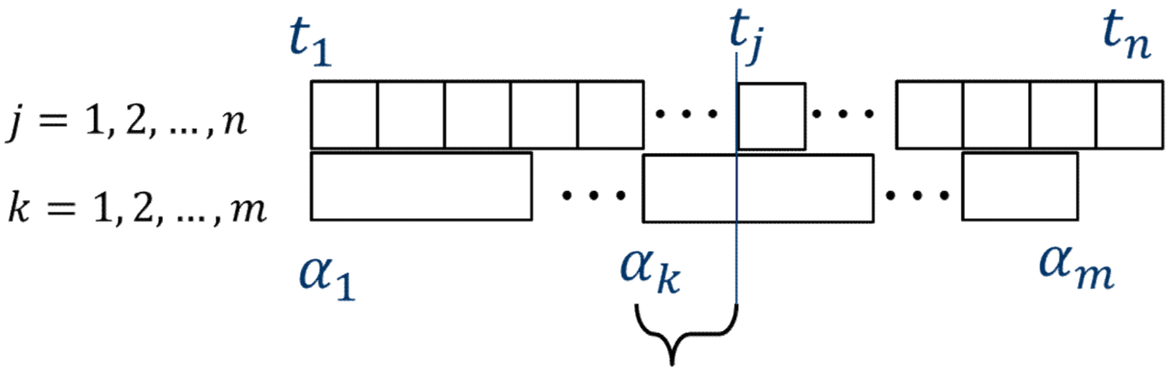
– Stable condition

$$\exp\left(\frac{|\alpha_{k+1} - \alpha_k|}{\sigma}\right) < \delta$$

δ : maximum of double (64 bit C++: DBL_MAX)

$$w = t_n - t_1$$

leads to the representative points $\{\alpha_k\}$: $\{\alpha_k\} = t_1 + \{0, 1, ..., m-1\}\frac{w}{m}, \quad m = [\frac{w}{\varphi\sigma\log(\delta)}]$



– Applicable to non-uniform pixels → transformed domain.

– Theoretically guarantee its stability and precision: approx. 525 σ radius truncation.

$$\delta > \exp\left(\frac{|t_j - \alpha_k|}{\sigma}\right) \Rightarrow \frac{1}{G_\sigma(t_j - \alpha_k)} \rightarrow\neq \infty$$

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Speed & Accuracy: 1D Convolutions

Method	PSNR	$E_{\max}$	Time: $n \in 10^{\{4,5,6\}}$		
EBox	55.79	1.03e-02	0.014	1.77	18
SII	38.88	5.509e-02	0.092	0.86	9.4
Box	31.34	1.535e-01	0.039	0.32	3.4
Deriche	96.37	1.184e-04	0.165	1.62	18
VYV	74.8	1.294e-03	0.167	1.46	14.9
AM	53.6	1.438e-02	0.495	3.8	38.4
Our	291.6	2.842e-14	0.325	3.58	43.2
Our NU	298.7	3.12e-14	0.43	3.57	43

E-Box: *P. Gwosdek et al., SSVM'11.*
SII: *A. Bhatia et al. IEEE ICRA'10.P. E. Elboher & M. Werman, ISDA'12.*
Box: *N. Sochen et al. IEEE TIP, 1998.*
Deriche: *R. Deriche, INRIA-TR, 1993.*
VYV: *L. Vliet et al. ICPR, 1998.*
AM: *L. Alvarez & L. Mazorra, SIAM JNA, 1994.*
Our & Our NU (Non-Uniform): *S. Yoshizawa & H. Yokota. IEEE ICIP'14.*

Time: Millisecond, PSNR (Peak Signal to Noise Ratio), E_{max}: maximum error, n is the number of 1D pixels.

$\sigma \in \{5, 10, ..., 100\}$

- ✓ This table shows the results with small σ such as 0.05–1% of image size (n).
- ✓ Our method achieved very accurate approximation (about 10^{10} times better in Emax) for both uniform and non-uniform cases.
- ✓ Little bit slower than the accurate conventional methods (Deriche and VYV).

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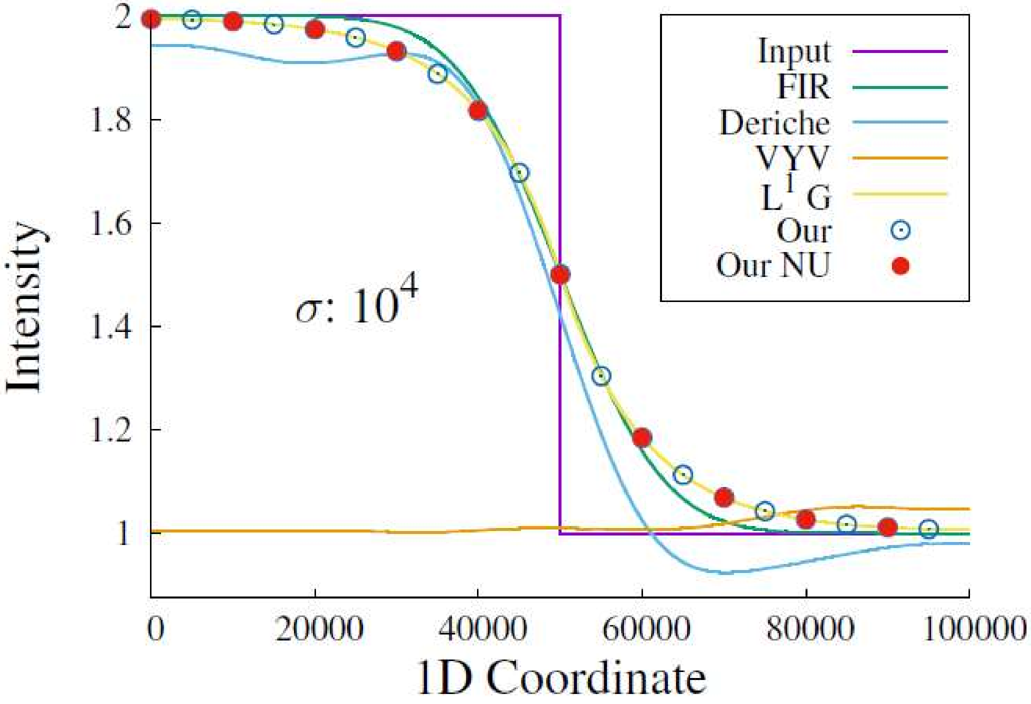
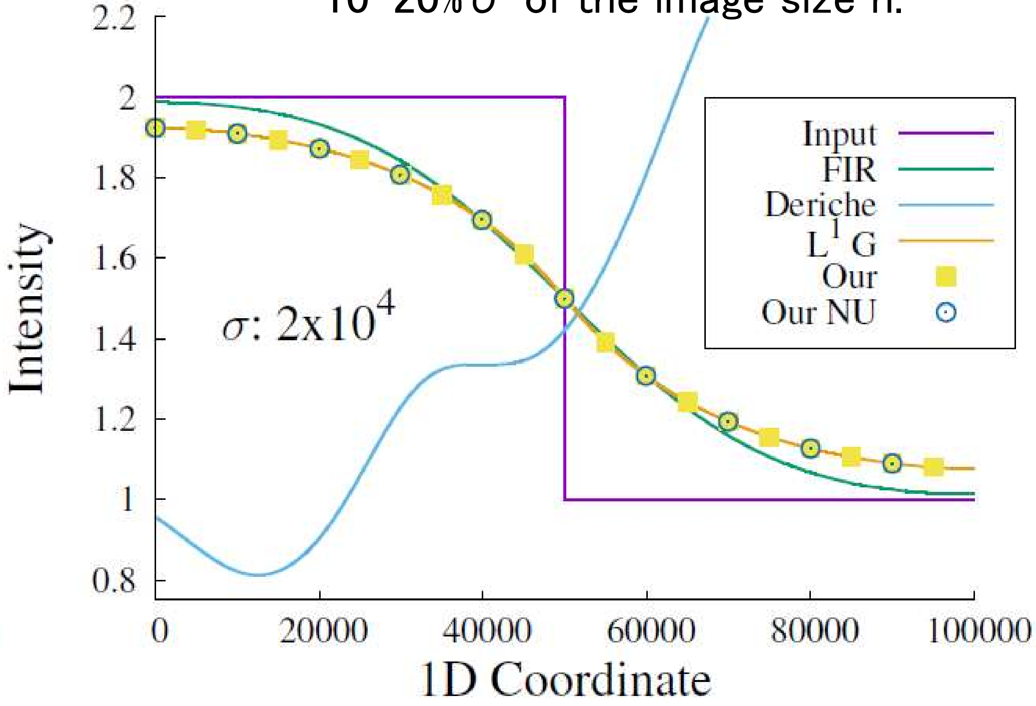
Not Trivial: 1D Convolutions

✓ Deriche and VYV (also most of the box/recursive methods) behave nonlinearly and may fail w.r.t. varying parameters because they optimize their coefficients for fixed parameters (linearly scaled σ and n examples):

10–20% σ of the image size n.





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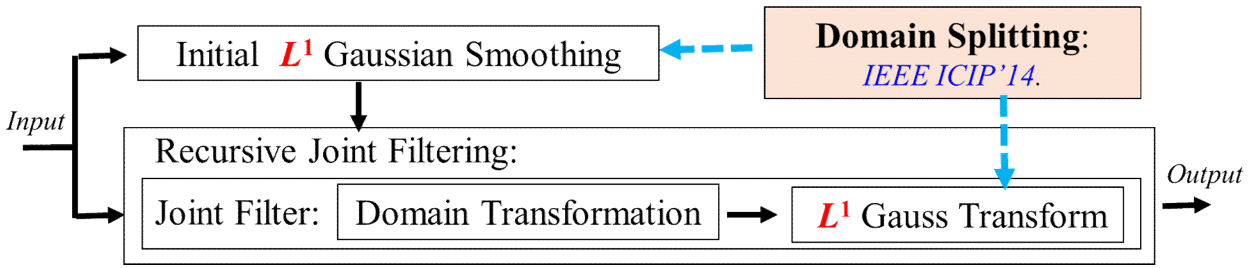
Implemented Scale-Aware Filters



$\boldsymbol{I}$: Input Image
$\boldsymbol{J}^0$: Initial Gaussian Smoothed Image
$\boldsymbol{J}^s$: Filtered Image
s: Number of Iterations

$f(\cdot,\cdot) \equiv f(\text{Guide}, \text{Integrand})$: Joint Filter

✓ RG: Rolling Guidance: *Q. Zhang et al. ECCV'14.*

- Dynamic Guide and Static Integrand: $\boldsymbol{J}^{s+1} = f(\boldsymbol{J}^s, \boldsymbol{I}).$

✓ SiR: Smooth & iteratively Restore: *P. Kniefacz & W. Kropatsch, CoRR 2015.*

- Static Guide and Dynamic Integrand: $\boldsymbol{J}^{s+1} = f(\boldsymbol{I}, \boldsymbol{J}^s).$

✓ AG: Alternating Guided: *A. Toet, PeerJ Comput. Sci. 2016.*

$$\boldsymbol{J}^{s+1} = \text{Median}_{3\text{x}3}\left[\text{SiR}\left(\text{RG}(\boldsymbol{J}^s)\right)\right].$$

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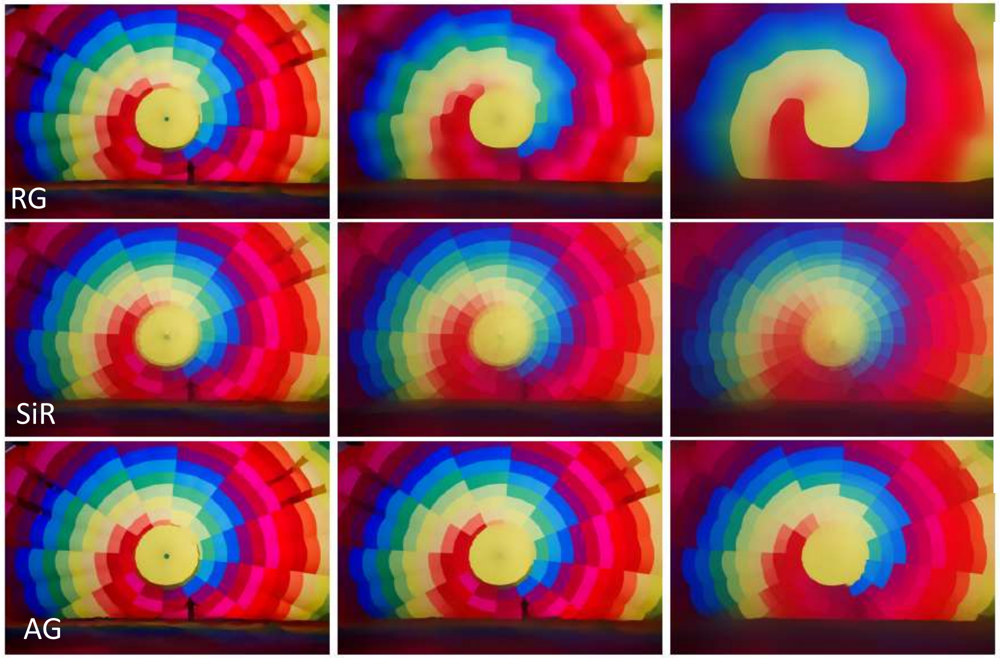
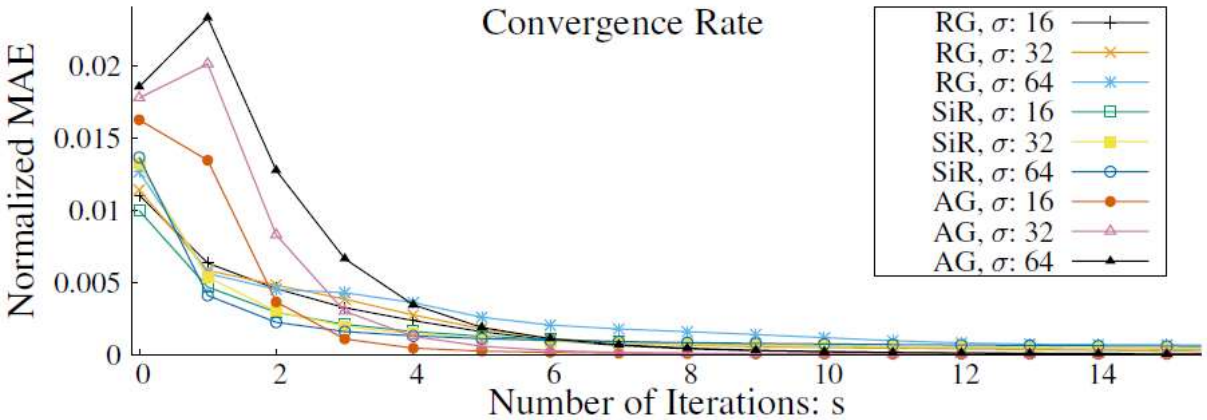
Filtering Results





- ✓ Obtained nice convergence rates.
- ✓ Inherit original filter's characteristics.

$\phi = 1.5$ and $\sigma \in \{16, 32, 64\}$ (left to right).

σ : Scale.

ϕ : Edge parameter.

Input RG $\sigma = 64.$ SiR AG

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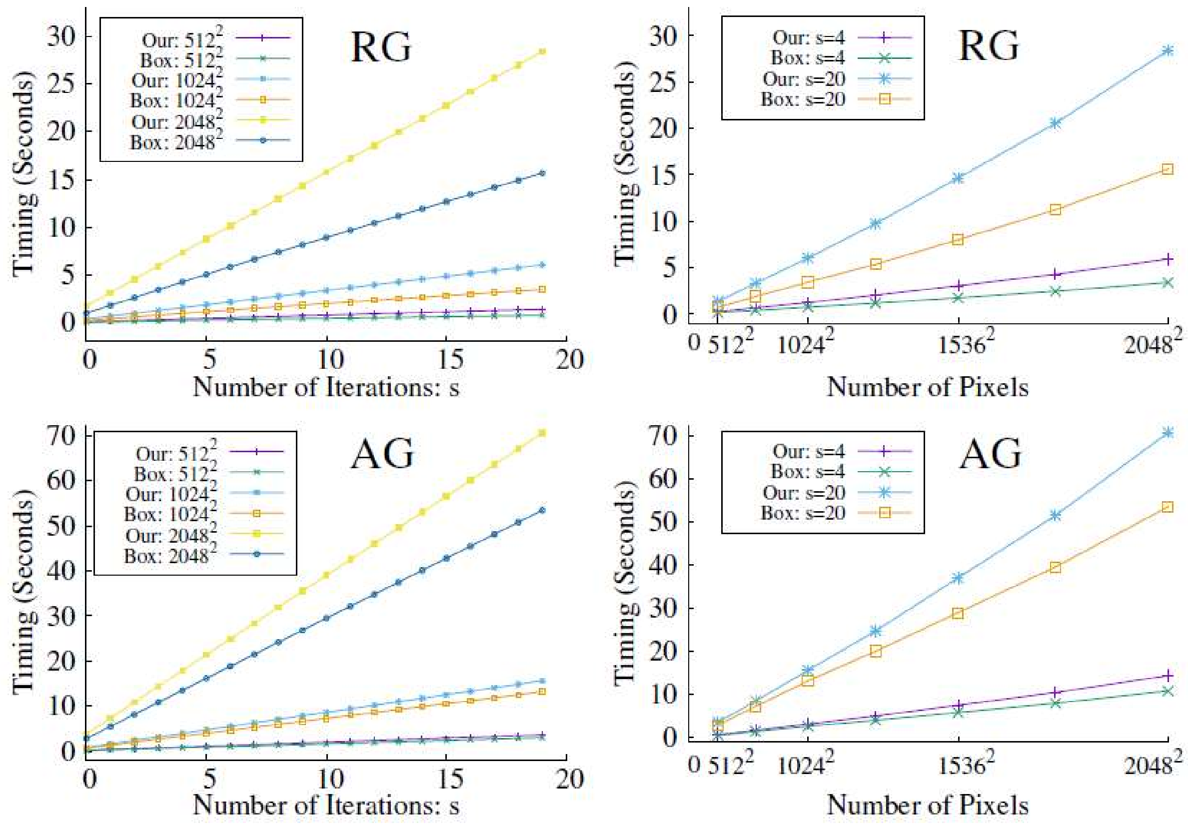
Speed: 2D Scale-Aware

- ✓ Achieved fast & linear speed w.r.t. the pixel & iteration numbers.
- ✓ Slower than box-based (moving average: *N. Sochen et al. IEEE TIP, 1998*) method: less than 2 times.
- ✓ Average performance:

σ={5,10,⋯,50} for 10 times with s={1,2,⋯,20}.



RG-box	RG-our	SiR-box	SiR-our	AG-box	AG-our
5.41/s	3.072/s	3.902/s	3.014/s	1.597/s	1.244/s

Mega pixels per second: s is the iteration number.

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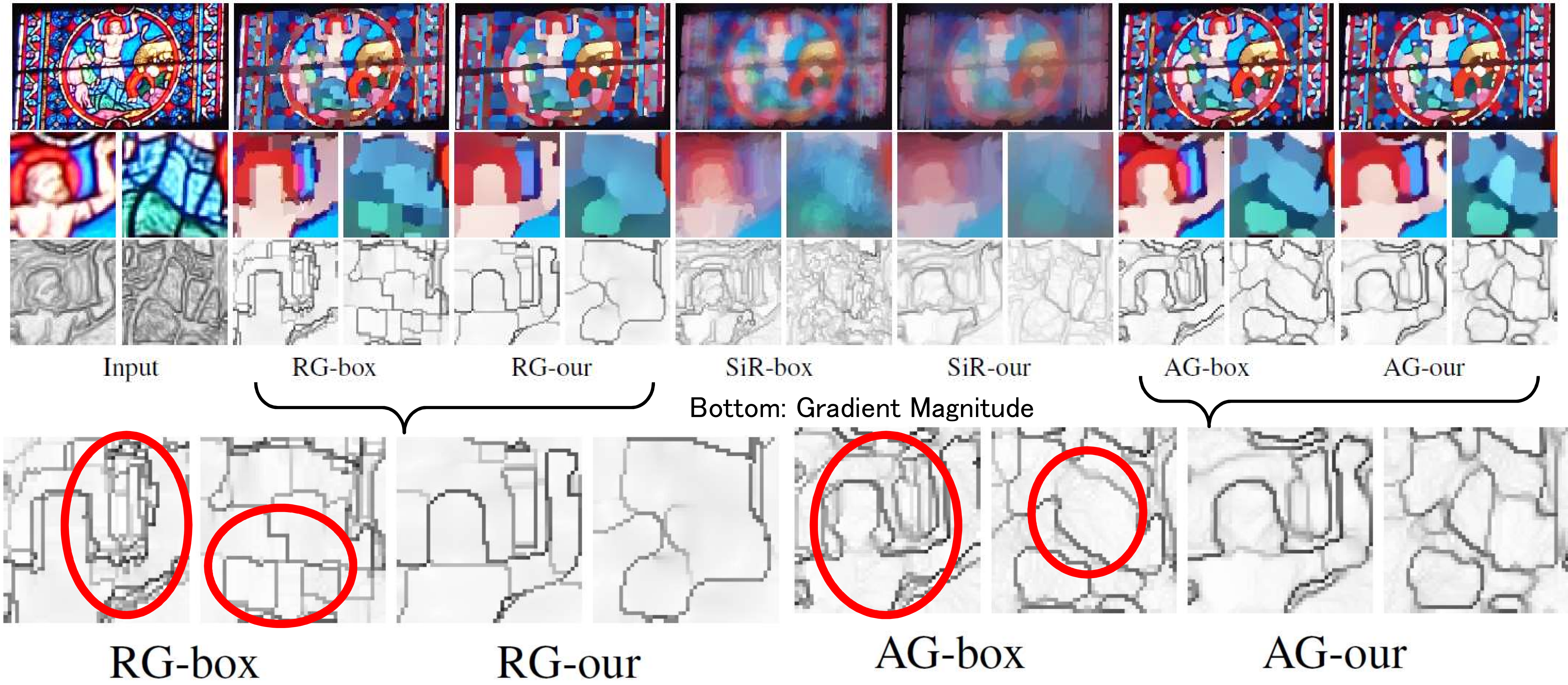
Visual Quality Comparison



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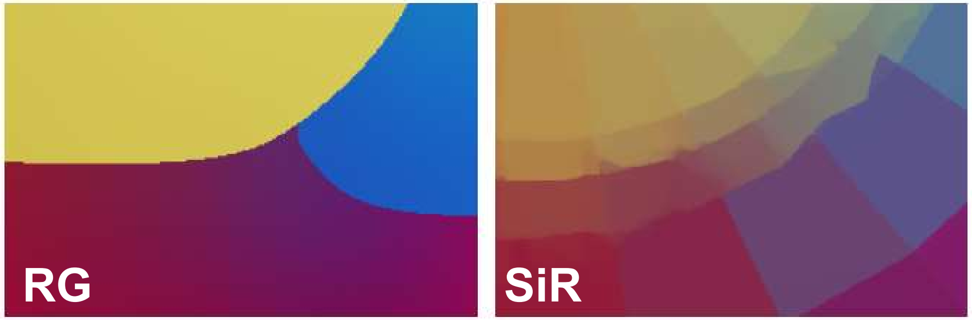
Limitations & Future Work



AG: J^4 AG: J^{20}

✓ Inherits original filter's limitations:

- e.g., smoothing curvature of edges (RG) & reduced intensity (SiR).

✓ Slow convergence of elongated regions:

- because of separable (x-y) domain transformations.

→ Guided filter: *K. He et al. TPAMI 2013.*

✓ Comparison with non-uniform Deriche: *E. Gastal et al. EG'15.*

✓ Applications to computational photography, engineering, and science.

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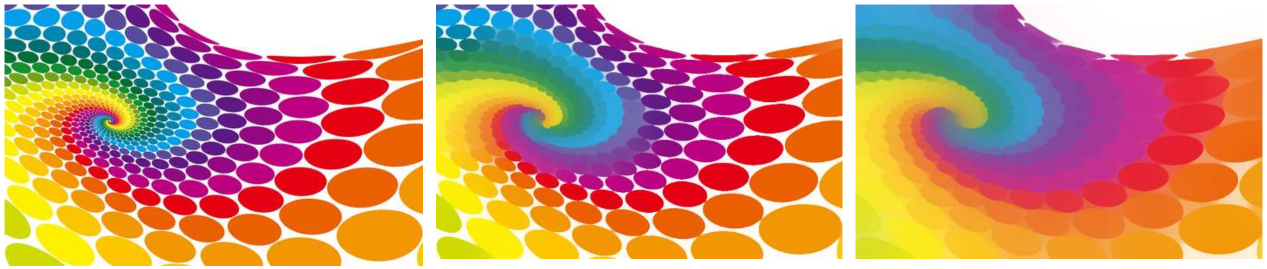
Conclusion

✓ Fast & accurate framework for scale-aware image filters.

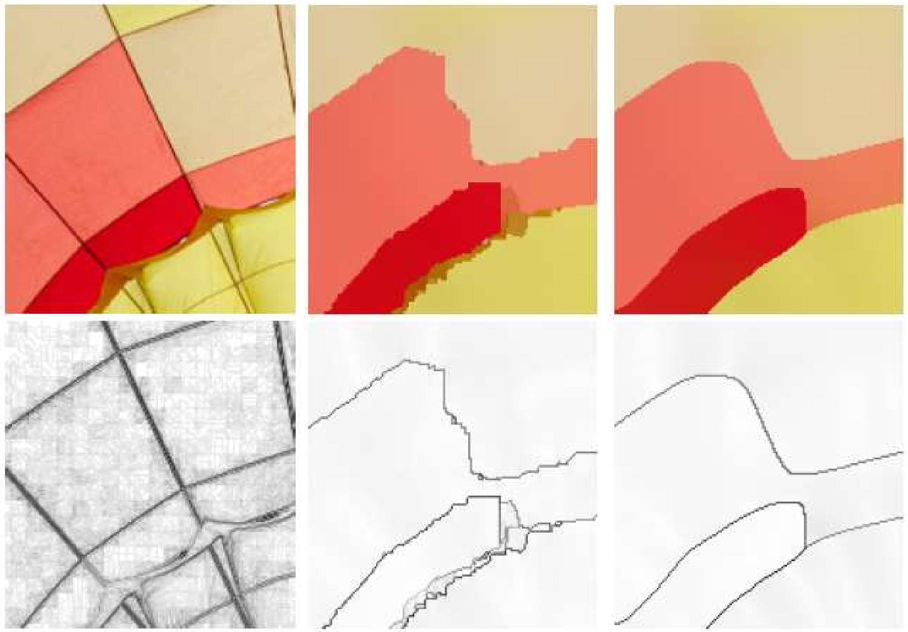
- Idea: Domain Transform & Splitting → joint (guided) Gaussian averaging.



Contributions & Benefits:

- Fast: linear complexity O(n) & O(1/ σ).
- Faithful: Gaussian scale-space.
- Accurate: stable & 525 σ truncation.

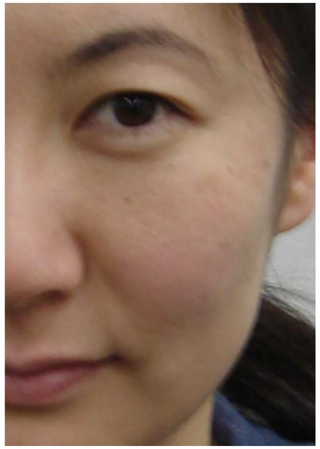
→ Prevent ringing artifact/phantom edges.

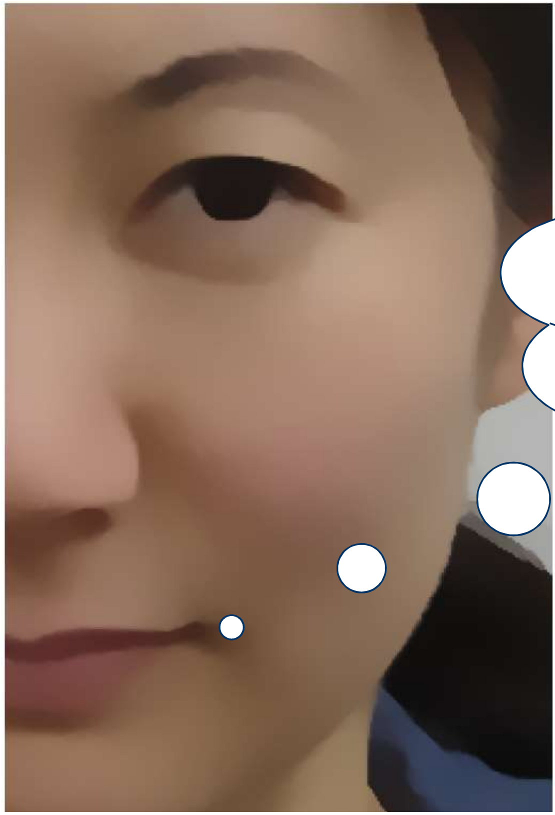


Input RG-Box RG-Our

The End

Thank you very much for your attention !





Prof. T. Kunii, IEEE 2021.

Fast and Faithful
Scale-Aware Image Filters

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Workshop in Homage to Prof. T. L. Kunii, September 2021