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# Antihydrogen formation in antiproton–positronium collisions

Nobuhiro Yamanaka [a,*], Yasushi Kino [b]

[a] *RIKEN (Institute of Physical and Chemical Research), Wako, Saitama 351-0198, Japan*
[b] *Department of Chemistry, Tohoku University, Sendai 980-8578, Japan*

**Abstract**

Rearrangement in antiproton ($\bar{p}$) and positronium (Ps) collisions, $\bar{p} + Ps \rightarrow \overline{H} + e^-$, is a promising process to produce large amount of antihydrogen atom ($\overline{H}$). The formation cross section is calculated by using a time-dependent coupled channel (TDCC) method. Numerical accuracy of the TDCC method is demonstrated in a calculation of Ps-formation cross sections in positron and hydrogen collisions. The present result shows a dominant peak of the cross section around a center of mass collision energy of 10 eV.


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## 1. Introduction

Cold antihydrogen production at the CERN antiproton decelerator (AD) has been recently reported [1,2]. The production of the atomic antimatter stimulates spectroscopic studies for the test of fundamental principles of physics, e.g. the CPT invariance and the weak equivalence principle, and collision studies to reveal interaction between antimatter and matter (see, for review, [3]). Production of large amount of antihydrogen atom is essential to these experiments.

A promising process of large amount antihydrogen ($\overline{H}$) production is rearrangement in antiproton ($\bar{p}$) and positronium (Ps) collisions, i.e. $\bar{p} + Ps \rightarrow \overline{H} + e^-$ [3]. Antihydrogen-formation cross sections were reported in many theoretical works with, e.g., the close-coupling (CC) method [4,5], and the hyperspherical close-coupling (HSCC) method [6], and the classical trajectory Monte Carlo (CTMC) simulation [7]. However, those predictions are inconsistent with each other.

In the present work, we calculate antihydrogen-formation cross sections using a time-dependent coupled channel (TDCC) method [8,9] in which coupled-channel equations for scattering wave functions are time-dependently solved with a wave packet. Numerical accuracy of the TDCC method is demonstrated in a calculation of Ps-formation cross sections in positron–hydrogen collisions, $e^+ + H \rightarrow Ps + p$. The present result is compared with previous theoretical results and the experiment [10] for the charge-conjugate reaction $p + Ps \rightarrow H + e^+$.

* Corresponding author.
*E-mail addresses:* yam@postman.riken.go.jp (N. Yamanaka), kino@mail.cc.tohoku.ac.jp (Y. Kino).

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## 2. TDCC equation and numerical method

The total Hamiltonian for a three-body collision system which consists of an electron, a positron and an antiproton is given by

$$H = \frac{1}{2M}\widehat{\boldsymbol{P}}^2 + \frac{1}{2m}\hat{\boldsymbol{p}}^2 + V(\boldsymbol{R},\boldsymbol{r}) \tag{1}$$

with the interaction

$$V(\boldsymbol{R},\boldsymbol{r}) = \frac{Z_{e^-}Z_{e^+}}{|\boldsymbol{r}|} + \frac{Z_{\bar{p}}Z_{e^-}}{|\boldsymbol{R}+(m/m_{e^-})\boldsymbol{r}|} + \frac{Z_{\bar{p}}Z_{e^+}}{|\boldsymbol{R}-(m/m_{e^+})\boldsymbol{r}|}, \tag{2}$$

where $\boldsymbol{R}$ denotes the position vector of the antiproton from the center of mass (COM) of the electron–positron pair, $\boldsymbol{r}$ the relative position vector between the electron and the positron, $\widehat{\boldsymbol{P}}$ and $\hat{\boldsymbol{p}}$ the conjugate momentum operators of $\boldsymbol{R}$ and $\boldsymbol{r}$, $Z_i$ and $m_i$ the atomic number and mass of $i$-particle ($i = e^-$, $e^+$, or $\bar{p}$), and $M$ and $m$ the reduced masses associated with $\boldsymbol{R}$ and $\boldsymbol{r}$.

The total wave function is expanded into a series of the angular momentum eigenfunction $Y_{Ll}^{JM_J}$ in a form of

$$\Psi^{JM_J}(\boldsymbol{R},\boldsymbol{r},t) = \frac{1}{Rr}\sum_{Ll}\psi_{Ll}^{JM_J}(R,r,t)\,Y_{Ll}^{JM_J}(\widehat{\boldsymbol{R}},\hat{\boldsymbol{r}}), \tag{3}$$

where $J$ is the total angular momentum and $L$ and $l$ the orbital angular momenta associated with $\widehat{\boldsymbol{R}}$ and $\hat{\boldsymbol{r}}$. From these relations and the time-dependent Schrödinger equation: $\mathrm{i}\partial_t\Psi^{JM_J} = H\Psi^{JM_J}$, we have TDCC equations

$$\mathrm{i}\frac{\partial}{\partial t}\psi_{Ll}^{JM_J}(R,r,t) = \sum_{L'l'}\left[T_{Ll}^{JM_J}\delta_{LL'}\delta_{ll'} + V_{LlL'l'}^{JM_J}\right]\psi_{L'l'}^{JM_J}(R,r,t), \tag{4}$$

where

$$T_{Ll}^{JM_J} = -\frac{1}{2M}\frac{\partial^2}{\partial R^2} + \frac{L(L+1)}{2MR^2} - \frac{1}{2m}\frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{2mr^2}, \tag{5}$$

$$V_{LlL'l'}^{JM_J} = \langle Y_{Ll}^{JM_J}|V|Y_{L'l'}^{JM_J}\rangle. \tag{6}$$

The initial radial function is constructed by the product of the ground-state Ps wave function $\phi_{1s}^{Ps}$ and an incoming wave packet of the antiproton $g_{kL}$, $\psi_{Ll}^{JM_J}(R,r,t_0) = g_{kL}(R)\phi_{1s}^{Ps}(r)\delta_{LJ}\delta_{l0}$, where

$$g_{kL}(R) = \frac{1}{(w^2\pi)^{1/4}}\exp\left[-\frac{(R-R_0)^2}{2w^2}\right]h_L^-(kR), \tag{7}$$

where $k = \sqrt{2ME}$ is the wave number with COM collision energy $E$, $R_0$ and $w$ the localization radius and width of the wave packet at $t_0$, and $h_L^-(kR)$ the asymptotic Hankel function. The TDCC equation (4) is fast and stably solved with numerical technique developed in [8].

Antihydrogen-formation cross sections are obtained by projecting bound-state wave functions $\phi_{nlm}^{\overline{H}}$ of antihydrogen atoms on to time-evolved wave functions $\Psi^{JM_J}(\boldsymbol{R},\boldsymbol{r},\infty)$,

$$\sigma_{\overline{H}} = \frac{\pi}{k^2}\sum_J(2J+1)\rho^J \tag{8}$$

with

$$\rho^J = \sum_{nlm}\int \mathrm{d}\boldsymbol{R}'\left|\left\langle\Psi^{JM_J}(\boldsymbol{R},\boldsymbol{r},\infty)|\phi_{nlm}^{\overline{H}}(\boldsymbol{r}')\right\rangle_{\boldsymbol{r}'}\right|^2, \tag{9}$$

where $\boldsymbol{R}'$ and $\boldsymbol{r}'$ are the position vectors of the electron and the positron relative to the antiproton. Ps-formation cross section in positron–hydrogen collisions is also calculated in the similar treatment.

## 3. Result and discussion

Fig. 1 shows Ps-formation cross sections. The present result is in very good agreement with the experiment [11] from the Ps-formation threshold (6.8 eV) to 50 eV, and also with recent CC [12,13] and HSCC [14] calculations. From this agreement, the usefulness of the TDCC method has been demonstrated.

Fig. 2 shows antihydrogen-formation cross sections. The cross section exhibits a dominant peak around a COM collision energy of 10 eV. This peak structure comes from threshold behavior in partial-formation cross sections of excited antihydrogen atoms. The formation channel into the ground state opens for any energy; the threshold energy $E_{th} = E_{nl}^{\overline{H}} - E_{1s}^{Ps}$ is negative for $nl = 1s$. The formation channels into the 2s and 2p states open

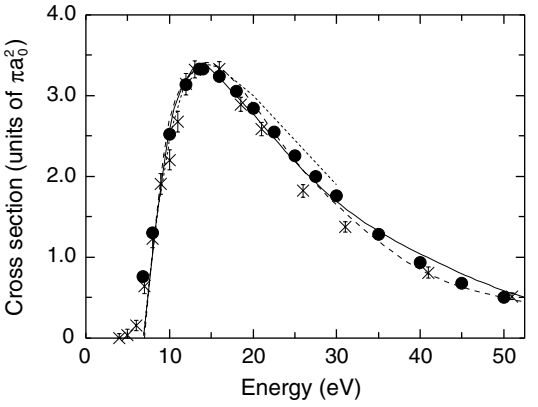


Fig. 1. Ps-formation cross section in positron–hydrogen collisions in units of $\pi a_0^2 = 0.880 \times 10^{-16}$ cm$^2$. Closed circles represent the present result of a TDCC calculation; dash-dotted line, CC(28, 3) [12]; broken line, CC(30, 3) [13]; dotted line, HSCC [14]; crosses, the experiment of Zhou et al. [11].

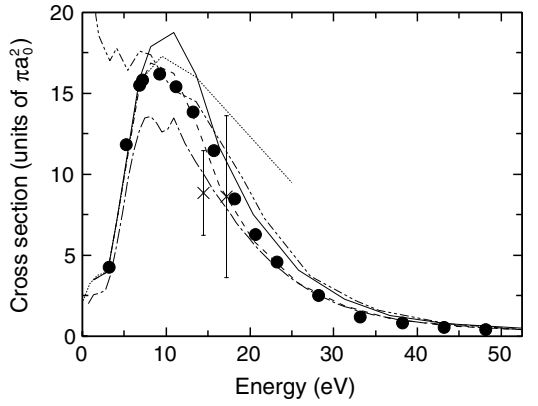


Fig. 2. Antihydrogen-formation cross section in antiproton–Ps collisions in units of $\pi a_0^2 = 0.880 \times 10^{-16}$ cm$^2$. Closed circles represent the present calculation; solid and broken lines, CC(28, 3)–CC(13, 8) with and without the $n^{-3}$ correction [4]; dash-dotted line, CC(3, 3) [5]; dash-double dotted line, CTMC [7]; dotted line, HSCC [6]; crosses, the experiment of Merrison et al. [10]. The collision energy is given in the center-of-mass frame.

at $E_{\text{th}} = 3.4$ eV. The threshold energy $E_{\text{th}}$ shifts toward higher energy for higher excited states. The emergence of the formation channels causes the increase of the cross section from 3.4 eV and the peak around 10 eV. The CTMC simulation [7] fails in reproducing the peak structure. The cross section obtained rises as the energy decreases. This result indicates that the classical mechanical calculation is not appropriate in low energies.

We should mention that the antihydrogen-formation cross section should rise in the low energy limit through an S-wave contribution according to Wigner's threshold law [15], $\sigma^J_{\overline{\text{H}},1s}(E) \propto E^{J-1/2}$, where $\sigma^J_{\overline{\text{H}},1s}$ is the partial-wave $J$ contribution to the ground-state (1s) formation cross section. However, this contribution is negligibly small for $E \gg 0.01$ eV as confirmed from the partial cross sections $\sigma^J_{\text{Ps},1s}$ calculated in [12–14] for Ps formation, through the detailed balance, $\sigma^J_{\overline{\text{H}},1s}(E) = (E + 1/4)/(2E)\sigma^J_{\text{Ps},1s}(E + 6.8\text{ eV})$. Therefore, the previous results [4–6] indicate no appreciable rise of the cross section.

In comparison with the present result, the CC(3, 3) [5] underestimates the cross section. This is caused by non-incorporation of break-up channels into wave functions. This defect has been improved in the CC(28, 3) and CC(13, 8) calculations [4], but somewhat overestimates the cross section. In the CC(28, 3), only six bound states up to $n = 3$ of antihydrogen atom were incorporated into wave functions, where $n$ is the principle quantum number. Contributions of formation channels into higher excited states have been added with an empirical correction in which partial cross sections are proportional to $n^{-3}$. However, this treatment would be overestimation, because it does not satisfy the flux conservation. In fact, the result without the correction is in excellent agreement with the present result (see Fig. 2). The other result of the HSCC calculation [6] is in good agreement for low energies, but significantly deviates from the present result as the energy increases. Finally, the agreement with the experiment [10] is good. However, the precision of the experiment is insufficient to discuss validity of the TDCC method. Hence, we desire more precise experiment.

## 4. Conclusion

We have calculated antihydrogen-formation cross sections using the TDCC method. In a calculation of Ps-formation, its usefulness has been demonstrated. For antihydrogen formation, discrepancy

with previous results was found. The cross section has a dominant peak around 10 eV. The present result is in excellent agreement with the CC calculations [4] without the $n^{-3}$ correction.

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