Non-Neutral Plasma Confinement In A Cusp-Trap And Possible Application To Anti-Hydrogen Beam Generation

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Abstract. A new scheme for synthesizing antihydrogen by trapping positrons and antiprotons in a field consisting of a magnetic quadrupole and an electric octupole (cusp –trap) is now under investigation. The total electric field of the octupole with the space charge of a nonneutral plasma composed of particles of the same sign of charge, i.e., positrons or mixture of electrons and antiprotons, is expected to form a potential well for particles of the opposite sign of charge. Particles trapped in the well are mixed with the present dense particles, where positrons and antiprotons will combine to produce antihydrogen atoms. A considerable fraction of antihydrogen atoms in low-field seeking states will be transported outside as a beam.

Experiments on electron confinement in the cusp-trap were carried out in a strong magnetic quadrupole (3.8T at the maximum on the axis). The confinement time reached 400s for the trapped electron number $N_0=3.6 \times 10^7$. The time decreased with N_0 but it was still about 100s for $N_0=1.6 \times 10^8$.

An electron plasma initially formed around the zero-field point rapidly expanded and settled down onto a quasi-stable state. Cross-sectional density profiles had shapes like a high volcano with a big crater. Analysis of the density profile shows that a potential well for oppositely charged particles (positive ions in this case) is probably formed inside the trapped electrons.

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INTRODUCTION

Antihydrogen (H) has been synthesized by two working groups at CERN [1,2]. Research related to antihydrogen is now entering a new advanced stage. Many potential candidates for stringent CPT tests will be examined in their practical application, which are high-precision laser spectroscopy, high-precision measurement of hyperfine splitting with rf-cavity and so on. Such a high-precision measurement will be pursued either for \overline{H} atoms trapped in a closed space or for \overline{H} atoms constituting a beam. Antihydrogen atoms in low-field seeking states can be confined in a minimum-B magnetic field. Generation of \overline{H} beams needs a proper spatial distribution of magnetic field gradient for guiding the atoms. In the both cases,

confinement or guide of \overline{H} atoms should be compatible with synthesis of them in the system.

A project named "ALPHA" [3] has recently started aiming at confinement of H atoms in a minimum-B field, which is produced by a magnetic multi-pole and a mirror field. \overline{H} atoms are synthesized in a nested trap locating near the axis where the magnetic field is nearly uniform. This plan is one of solutions to make the synthesis and the confinement compatible together.

Several plans to produce beams of H atoms of low energies have been considered. A simple and certainly realizable plan [4] is to use \overline{H} atoms leaking from slits of a nested trap in which \overline{H} atoms are produced in the similar way taken by ATRAP and ATHENA groups. Charged particles can be confined in an effective potential well formed by ponderomotive forces of radio-frequency fields. A new concept to synthesize \overline{H} atoms are also taken out of the rf-trap through opening gaps of installed electrodes. Emerging \overline{H} atom beams in above two cases are so divergent that collection of them may need large open areas on instruments.

The other scheme to produce \overline{H} beams incorporates a magnetic quadrupole (cusp) and an electric octupole [6] in a trap. This arrangement, referred to as cusp-trap or MCEO, possibly allows extraction of almost fully polarized \overline{H} beams with a considerable flux density. This intense \overline{H} beam will make a high-precision determination of the antiproton magnetic moment feasible. Positrons and antiprotons recombine in the central region of the cusp-trap and synthesized \overline{H} atoms in low-field seeking states are guided by the surrounding magnetic field with strong gradient. Realization of this scheme strongly depends on the dynamics of nonneutral plasma confined in the cusp-trap.

This report mainly describes recent experiments on confinement of nonneutral electron plasma in the cusp-trap.

CUSP-TRAP

Combination of a magnetic quadrupole (cusp) and an electric octupole forms a trap of non-neural plasma. An axisymmetric magnetic quadrupole is simply expressed in terms of the vector potential in cylindrical coordinates (r, θ, z) as

$$A_{\theta}(r,z) = \frac{B_0}{L}rz,$$
(1)

and the magnetic field components are

$$\boldsymbol{B} = \left(-\frac{B_0}{2L}r, \ 0, \ \frac{B_0}{L}z\right) \tag{2}$$

Also, the electric potential of an axisymmetric octupole is given by

$$\phi(r,z) = \phi_0 \left(\left(\frac{r}{L}\right)^2 + \left(\frac{z}{L}\right)^2 \right)^2 P_4 \left[\frac{\left(\frac{z}{L}\right)}{\sqrt{\left(\frac{r}{L}\right)^2 + \left(\frac{z}{L}\right)^2}} \right].$$
(3)

Here, *L* is the scale length, B_0 and ϕ_0 are the magnetic field strength and the potential at *z*=*L* and *r*= 0, respectively, and P_4 is the Legendre function of the first kind and the fourth order. Figure 1 shows magnetic field lines of a magnetic cusp and equipotential surfaces of an electric octupole.

The magnetic cusp is representative of the minimum-B configuration in which a confined neutral plasma is magnetohydrodynamically stable. However, particles of confined neutral plasma are rapidly lost, since their orbital magnetic moments cannot be invariant around the null-field, i.e., the null-field acts as a scattering center for particles.

On the other hand, the cusp-trap, which incorpotates both a magnetic quadrupole and an electric octupole, perfectly confines a single charged particle [7]. That is, the St örmer region that constrains a charged particle is closed. Similarly, nonneutral plasmas are expected to be confined in this field configuration

Equilibrium state of nonneutral plasma in the cusp-trap has theoretically been found at the Brillouin limit for cold plasma [8]. Also, preliminary experiments using electrons in low magnetic fields have proved that cusp-traps are capable of nonneutral confinement [8,9]. However, plasma diffusion processes as well as formation of a well for particles of the opposite sign of charge have not been clear yet and remain to be studied.



FIGURE 1. Field lines of a magnetic quadrupole(cusp) and equi-potential surfaces of an electric octupole.

Application Possibility Of Cusp-Trap To Generation Of Spin-Polarized Antihydrogen Beam

We shall consider a magnetic field depicted in Fig.2. The field has a pure cusp in the central region and extends outwards having an appropriate field distribution. When a mixture of cold positrons and antiprotons condenses in the central region, recombination of antiprotons with positrons is expected to take place. Cold \overline{H} atoms in the ground state enter the nonzero magnetic field region and their quantum states are split to four states as $(F, M_F)=(1,1)$, (1,0), (1,-1), (0,0) where F and M_F are the total spin and its magnetic quantum number, respectively. The magnetic moment in the state (1,1) and (1,0) is antiparallel to the magnetic field, so that \overline{H} atoms in these two states prefer weaker fields (low-field seeker). In other words, low-field seekers feel backward force in magnetic field gradient $\nabla |B|$ as given in Fig.3, a considerable part of produced low-field seekers are transported outside as a spin-polarized beam. Trajectories of \overline{H} atoms in (1,1) state with the initial energy of 0.233meV are shown in Fig.2.

The magnetic fields shown in Fig.2 and Fig.3 are both the same. This field is produced by super-conducting coils and used for experiments described in this report. This magnet will be used for \overline{H} beam generation when the concept is considered to be feasible.

To realize the concept, it becomes an essential theme how to make cold positrons and antiprotons coexist in the central region.

In a magnetic mirror, of which the minimum field is B_L and the maximum one is B_H , the electric potential difference between these two extreme field points appears when a nonneutral plasma is confined therein [10]. This difference $\Delta \phi$ roughly amounts to

$$\Delta \phi \sim \left(\frac{kT}{e}\right) \cdot \beta \quad ; \quad \beta = \frac{B_H}{B_L} - 1, \tag{4}$$

where k is the Boltzmann constant and e is the unit of charge. Real nonneutral plasma is not perfectly cold in any case. Also, a magnetic cusp can be regarded as a bundle of magnetic flux tubes with high mirror ratios and the field strength increases in all directions. It is expected that, when a nonneutral plasma is confined in the cusp-trap, a potential well for particles of the opposite sign of charge is formed in the central region. That is, confined positrons (or mixture of electrons and antiprotons) digs a well for antiprotons (or positrons). Therefore, the coexistence of positrons and antiprotons may become possible.

Formation of the well and its stable sustainment have not yet been studied. Those problems are key subjects of this experiment.



Figure 2. Spin-polarized antihydrogen atoms H flow out of the recombination region located around the center.

EXPERIMENT

Experimental Setup

The experiment is performed by using a trap consisting of a super-conducting quadrupole magnet and a vacuum tube which houses a set of electrodes for generating an octupole, an electron gun and a Faraday cup.

Super-Conducting Quadrupole Magnet

Two pairs of super-conducting coils generate a magnetic cusp with a proper spatial distribution of field gradient necessary for intensity enhancement of outgoing spinpolarized antihydrogen atoms when they are synthesized. These coils are cooled grad|B|r (T/m)



FIGURE 3. Radial and axial components of the magnetic field gradient in the quadrupole magnet.

down to 5K with a cryogenic refrigerator without the use of liquid helium. The magnetic field lines are shown in Fig.2 and the radial and axial components of field gradient at the radius of 1 and 4 cm are shown in Fig.3. The maximum magnetic field strength on the axis is 3.5T at 15cm from the plane of symmetry. Inside the warm bore of 16cm inner diameter is installed a vacuum tube of aluminium alloy.

Generation Of Octupole And Equipments Inside Vacuum Tube

Figure 4 depicts the arrangement of electrodes producing an octupole, an electron gun and a segmented Faraday cup. The space enclosed with the electrodes is in the diameter of 9.2cm and in the axial width of 9.2cm. Shapes of the electrodes and the voltage allocation applied on them are optimized so as to make a wide region of octupole inside the trap region. Two electrodes on the both sides are copper-meshes with 80% transparency in order to enable the passage of electrons at the injection and also at the dump. The system is evacuated down to the vacuum pressure of 1.5x10⁻⁷Pa.

A tiny electron gun with a cathode of Ba-sintered porous tungsten is set on the axis outside the trap as is shown in Fig.4. A burst of pulsed electron beams, each of which has the pulse width of 40μ s typically, are injected into the trap.

The Faraday cup in the diameter of 6cm is segmented to eight parts in the radial direction with the pitch of 0.4mm. Line-density, that is integrated amount of density along magnetic field lines, can be inferred from charges collected on the segments. Also, the total number of the confined electrons, N, is determined by summing up all signals of the segment.

Injection, Confinement And Dump Of Electrons

Synchronously to every pulsed electron beam, the potential on the electron injection side is made shallower to introduce the beam into the trap and, at the pulse end, the potential is again returned back to the former one to keep the injected electrons. Figure 5(a) and (b) show the potential distributions at the injection and the



FIGURE 4. Experimental components inside the vacuum tube: the electrodes, electron gun and the segmented Faraday cup.



FIGURE 5. Equi-potential surfaces of the trap(upper figures) and axial change in the potential (lower figures) for the case (a) At electron beam injection. The potential barrier on the injection side (left side) is lowered to allow the electrons come in, (b) At electron confinement. The octupole field is held during the confinement. (c) At dump of electrons. Electrons are forced to get out of the trap.

confinement, respectively. By repeating this cycle, electrons are stacked and confined. Stacked electron number can be adjusted by choosing the repetition number.

At an elapsed time after the end of electron stacking, the confined electrons are dumped out of the trap by changing the potential distribution as shown in Fig.5(c). In this moment, the electric field pushes and dumps the electrons outside the trap along magnetic field lines. A half of the dumped electrons are collected with the Faraday cup.

Time Evolution Of Confined Electron Number

Trapped electrons are localized around the center of the trap in a quite earlier phase after the electron stacking. The field strength near the center is not so strong to withstand the expansion of the electrons due to their space charge, so that the electrons change their spatial distribution until they settle in a state where the space charge field is balanced with forces exerted by externally applied electric and magnetic fields. However, the plasma gradually expands across the magnetic field, being caused either by collisions with the residual molecules or by fluctuations in the plasma. Finally, the outward plasma edge touches the electrons. Time dependence of N strongly reflects



FIGURE 6. Time variation in the electron number N for different initially stacked number N_0 .

such a macroscopic behaviour of the plasma. Observed spatial change in the density distribution will be discussed later.

Figure 6 shows the change in the total trapped electron number N with time for different initially stacked number N_0 . Here, the magnetic field at the radial inner surface of the electrode, i.e., at r=4.6cm and z=0, is 0.9T and the well depth of the octupole is 33.7V. In the case of $N_0 = 3.6 \times 10^7$, N is nearly constant until the time $t_c \sim 400$ s and then decreases faster. The turning time t_c becomes shorter as N_0 increases, e.g., $t_c \sim 110$ s for $N_0 = 1.1 \times 10^8$. This trend suggests that the plasma periphery moves faster toward the electrode wall for larger N_0 .

Density Profile And Electric Potential Of Plasma

In order to determine the density distribution of the plasma n(r,z) from experimentally obtained line-densities, it is necessary to assume a shape of constant density surface. In the equilibrium state of cold plasma at the Brillouin limit in this field configuration [8], the density is constant at a surface ρ =const as

$$n = n(\rho), \quad ; \quad \rho^2 = r^2 + 2z^2.$$
 (5)

Whence, each equi-density surface is the surface of an oblate spheroid with the aspect ratio of $\sqrt{2}$. This relation is used to determine density profiles although electron densities in this experiment are much less than that in Brillouin limit. Line densities estimated from a temporary function $n(\rho)$ are compared with experimentally obtained ones on the same field lines, and thereby a proper $n(\rho)$ that corresponds to the observation can be found.

Density Profile At Early Phase After Electron Stacking

Electrons are injected into the trap and, at a time t_D after the injection, trapped electrons are dumped. Time variation in the density distribution is inferred from the obtained line-densities, using the model eq.(5). Electrons during the injection and immediately after its end would be not so cold to allow the use of eq.(5) given for cold plasma and the practical profile itself would differ from it. However, by using the modeled profile, we may get a rough picture of cross-field motions of the trapped electrons.

Figure 7 shows examples of obtained density profiles. Here, the magnetic field at r=4.6cm and z=0 was 0.7T and the well depth of the octupole was 33.7V. The duration of the electron injection was 800µs. The total number of stacked electrons was N_0 =6x10⁷ at the end of the pulse. As noticed in Fig.7(a) (t_D =0), expansion of electrons across the magnetic field had already occurred during the electron injection and the profile became like a crater where the density near the center was much less. This expansion rapidly proceeded thereafter as seen in Fig.7(b) (t_D =1ms). Though the expansion was fast at the early phase, it gradually slowed down as the plasma approached an equilibrium state.



FIGURE 7. Density profiles inferred from the relation eq.(5) at (a) the end of the electron injection : t_D and (b) t_D =1ms. Upper figures are axial changes in the density and lower ones are density profiles on a *r*-*z* plane. Shown densities are in an arbitrary scale.

Calculation of Potential Distribution

When the plasma density distribution is expressed by eq.(5), we can regard the plasma as a superposition of many spheroidal plasmas with the same aspect ratio, each of which has its own uniform density and size. To make such a spheroidal plasma, we divide the plasma density profile on a r-z plane into thin slices parallel to the plane. Each slice, which has a density and a size in the r- and z-directions, corresponds with a spheroidal plasma with a uniform density. Potentials produced by individual spheroidal plasmas $\phi_j^s(r,z)$ can be estimated analytically. If a slice is across the concave part of the profile as shown in Fig.7, the slice possesses an elliptic hole. This hole is considered as an additive slice having the same density but with the opposite charge. The potential for the hole is noted as $\phi_i^{sn}(r,z)$. Then, the electric potential of the plasma $\phi^s(r,z)$ is found by summing up all potentials of the sliced plasma components as

$$\phi^{s}(r,z) = \sum_{j} \phi^{s}_{j}(r,z) - \sum_{i} \phi^{sn}_{i}(r,z).$$
(6)

The externally applied electric potential $\phi^{ex}(r,z)$ is numerically found from the voltages applied to the electrodes. Also, the potential caused by image charges on the electrode surfaces $\phi^i(r,z)$ is numerically obtainable using $\phi^s(r,z)$. Summing these potentials, we have the potential distribution inside the trap region $\phi^t(r,z)$ as

$$\phi^{i}(r,z) = \phi^{s}(r,z) + \phi^{i}(r,z) + \phi^{ex}(r,z).$$
(7)

Density Profile And Internal Potential Distribution Of Confined Plasma

The number of trapped electrons N was nearly constant until a time t_c as shown in Fig.6, while the plasma changed its shape in this period. Within a few seconds after the completion of electron stacking, the plasma rapidly expanded outwards. Exampled density profiles during this expansion have been shown in Fig.7. After then, the change in the density distribution became slow and the associated potential structure also gradually varied with time.

Figure 8 shows radial profiles of the density and the potentials in a typical case of a large N_0 . The experimental conditions were as follows; the magnetic field at r=4.6cm and z=0 was 0.9T, $N_0 = 2.8 \times 10^8$, the potential well depth of the octupole was 33.7V. The data was taken at 10s after the electron stacking. Here, the referring potential is on the inner wall of electrode on the midplane, i.e., at r=4.6cm and z=0. It should be noted that all profiles shown in Fig.8 were obtained by using eq.(5). The plasma density *n* radially increased till r=2.4cm and steeply fell. At this time the plasma periphery did not contact with the electrode surface. The plasma potential ϕ^s decreased from the electrode surface and also the potential due to the image charges ϕ^i did similarly. These two potentials ϕ^s and ϕ^i were negative to the external octupole ϕ^{ex} , so that the resultant net potential ϕ^i became lower than ϕ^{ex} . The figure on the right and the lower side in Fig.8 shows an enlarged one of ϕ^i , where the

potential difference from the center $\Delta \phi^i$ is shown. There was a potential well, noted by W^s, for particles of a positive charge. Its well depth was about 40mV. This well formation is what we have expected. In this experimental condition the well continued for about 20s but the well depth became shallower with time.

Of course, the well: W^s might be easily formed and its well depth would become deeper if N_0 is increased to make ϕ^s large enough. However, at larger N_0 , the plasma diffuses faster across the magnetic field and the well: W^s itself disappears in a shorter time. In the best case so far, the well: W^s is maintained for 40s for $N_0 = 1.5 \times 10^8$.



FIGURE 8. Radial density distribution and variation of electric potentials on the z=0 plane at 10s after the end of electron stacking, where *n* is the density, ϕ^{ex} the external octupole, ϕ^s the self-field of the plasma, ϕ^i the potential caused by the image charge, ϕ^t the potential inside the trap region, and $\Delta \phi$ is the potential difference measured from the potential at the center. The experimental conditions are noted in the text.

SUMMARY

Experiments on electron confinement in the cusp-trap with super-conducting coils were performed in order to see the possibility to generate a spin-polarized antihydrogen atomic beam.

Confinement of electron plasma continued for a long time, keeping the plasma isolated from the surrounding wall. There was dependence of the confinement time τ on the trapped electron number N_0 . The time τ became shorter for larger N_0 , e.g., τ -400s for N_0 = 3.6x10⁷ and τ -100s for N_0 = 1.6x10⁸. This dependence shows there are some mechanisms to enhance cross-field diffusion of plasma at larger N_0 . Explication of the dependence remains for further study.

Formation of the well for positively charged particles in trapped electrons has been one of the key issues of antihydrogen synthesis in the cusp-trap. It is said from the data analysis that this potential well is probably formed inside electron plasma, although the applied model on the density distribution is an assumption. The well lasted for 40s in a proper condition. It is necessary to find out the condition for elongating the lasting time.

There are many subjects to be investigated further more. Electron plasmas in this experiment were confined in a weak magnetic field region. To realize efficient synchrotron cooling, the plasma should be extended to a higher magnetic field. This extension is accompanied with the deformation of plasma shape. Application of additive electric fields with quadrupole component may elongate the plasma shape in the axial direction. As pointed out by ATHENA group [11], sufficiently cold antiprotons should combine with positrons to make antihydrogen atoms cold enough to ease manipulation of them by realizable magnetic field gradient. Then, cooling of mixture of electrons and antiprotons may become necessary. How to load such different kinds of particles into the cusp-trap is also an important subject to be studied.

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